

# COMPARISON OF VLBI NUTATION SERIES WITH THE IAU2000A MODEL

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ABSTRACT. Three most long and dense VLBI nutation series obtained at the Goddard Space Flight Center, Institute of Applied Astronomy, and U. S. Naval Observatory, containing about 3000 estimates of the nutation angles were used for investigation of systematic differences between observations and IAU2000A model. Bias and secular trends (precession and obliquity rate) were estimated together with main periodical terms for three periods of observations. It is shown that result substantially depends on period of observations used in analysis. Corrections to some IAU2000A nutation terms were also estimated and found to be at the level up to several tens microarcseconds. A new Free Core Nutation model with variable amplitude and period (phase) is developed. Comparison of this model with observations shows better agreement than existing one.

## 1. INTRODUCTION

New precession-nutation model IAU2000A (MHB2000, Mathews et al., 2002) is officially implemented in the astronomical practice starting from Jan 1, 2003. This model is intended to provide the accuracy at the level of 0.2 mas. Several modern VLBI EOP series provided by the International VLBI Service for Geodesy and Astrometry (IVS) allow us to estimate a disagreement of the IAU2000A model with observations. Those VLBI series are listed in Table 1.

Table 1: Available VLBI nutation series (01 Aug 2003).

Series	Software	Period	Number of points	Number of accepted points
GSF2003C	Calc/Solve	1979–2003	3424	3295
IAAN0307	OCCAM	1979–2003	3233	3091
USN2003A	Calc/Solve	1979–2003	3013	2921
CGS2002A	Calc/Solve	1979–2001	2708	2639
BKG00005	Calc/Solve	1984–2003	2645	2636
AUS00002	OCCAM	1983–2003	1229	1224
SPU0002M	OCCAM	1994–2003	542	532

Three most long, dense and independent nutation series GSF, IAA and USN were selected for detailed analysis. Since only the IAA nutation series provides estimation of celestial pole offset w.r.t. the IAU2000A model, GSF and USN series, containing estimation of celestial pole offset w.r.t. the IAU1976/1980 model, were transformed to the IAU2000A system.

Main results were obtained with averaged series GSF+IAA+USN hereafter referred to as mean series. These four series were compared with the IAU2000A model. The differences between observed nutation series and the model are shown in Figure 1, and spectrum of the differences is presented in Figure 2.

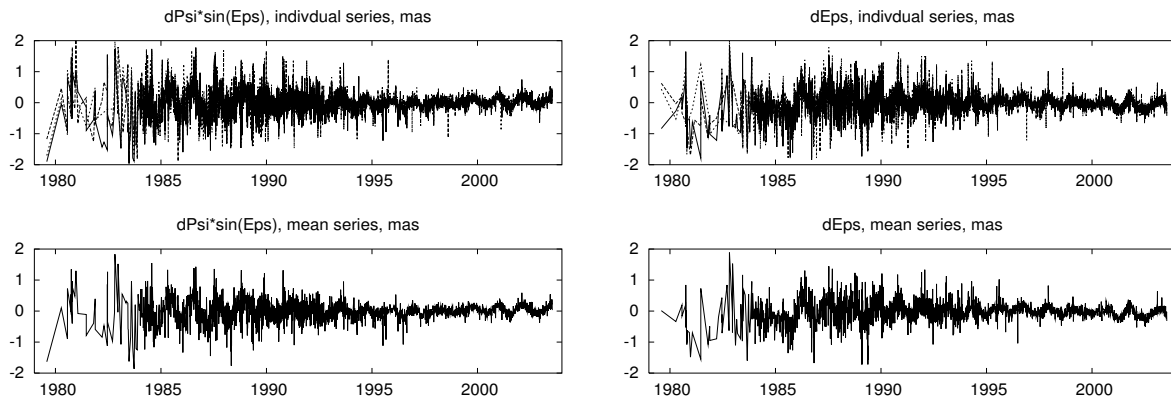


Figure 1: Differences between observed nutation series and the IAU2000A model.

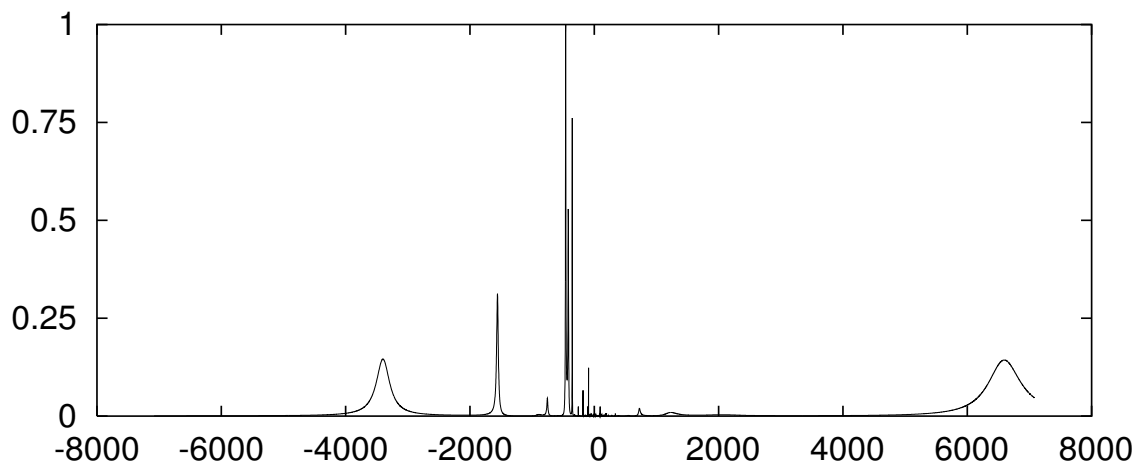


Figure 2: Spectrum of the differences between observed nutation series and the IAU2000A model.

The present investigation of the discrepancies between observations and the model was focused on the following topics.

1. Bias and trend.
2. Corrections to IAU2000A nutation terms.
3. Free Core Nutation (FCN) contribution.

This paper presents some results of this study.

## 2. BIAS AND RATES

Bias in celestial pole offset, precession constant and obliquity rate were estimated as linear trend along with largest long-period terms  $6798.38^d$ ,  $3399.19^d$ ,  $365.26^d$ ,  $182.62^d$ ,  $121.75^d$ . It is well known that the accuracy of the VLBI results had two significant improvement at the epochs *approx*1984.0 and *approx*1990.0 (see e.g. Malkin, 2002). So, bias and rate was estimated for three intervals 1979–2003, 1984–2003 and 1990.0–2003 (in the latter case the term with period  $6798.38^d$  was not included in the adjustment procedure).

The results of computation are presented in Table 2 For more detailed comparison we compute those both for individual and mean series.

Table 2: Bias in  $\Delta\psi$  and  $\Delta\epsilon$ ,  $\mu\text{as}$ .

Series	1979–2003		1984–2003		1990–2003	
	$\Delta\psi$	$\Delta\epsilon$	$\Delta\psi$	$\Delta\epsilon$	$\Delta\psi$	$\Delta\epsilon$
GSF	−76	+10	−84	+5	−30	−56
	±23	±9	±23	±9	±9	±4
IAA	+46	+30	+38	+26	+71	−31
	±23	±9	±24	±9	±10	±4
USN	−65	+44	−73	+39	+1	−30
	±24	±9	±25	±10	±9	±4
Mean	−29	+30	−37	+25	+13	−38
	±22	±8	±22	±8	±9	±4

Table 3: Rate in  $\Delta\psi$  and  $\Delta\epsilon$ ,  $\mu\text{as}/\text{yr}$ .

Series	1979–2003		1984–2003		1990–2003	
	$\Delta\psi$	$\Delta\epsilon$	$\Delta\psi$	$\Delta\epsilon$	$\Delta\psi$	$\Delta\epsilon$
GSF	−7	+8	−9	+7	+15	−8
	±5	±2	±5	±2	±2	±1
IAA	−2	+5	−4	+4	+17	−5
	±5	±2	±5	±2	±2	±1
USN	−14	+10	−16	+9	+14	−9
	±5	±2	±5	±2	±2	±1
Mean	−7	+8	−9	+7	+15	−8
	±4	±2	±5	±2	±2	±1

One can see that there is no evident systematic differences between OCCAM (IAA) and CALC/SOLVE (GSF, USN) results for precession constant and obliquity rate, however such a difference obviously exists for the bias, especially in  $\Delta\psi$ .

It's remarkable that the results presented here differ substantially, at the level of several tens microarcseconds, from those presented in the previous study (Malkin, 2002). It should be mentioned that both results were obtained using the same software and strategy, and only difference is in VLBI data used in the analysis. Direct comparison of the VLBI series used in previous and present analysis show that the difference (bias) between them also may be as large as several tens microarcseconds. Taking into account well known dependence of VLBI EOP results on station network, CRF realization, software, and other factors (MacMillan and Ma., 2000; Sokolskaya and Skurikhina, 2000), we can conclude that it is hardly possible to obtain the biases and rates with an accuracy better than 10–20  $\mu\text{as}$  and 5–10  $\mu\text{as}/\text{yr}$  from the current set of observations.

## 3. PERIODICAL TERMS

An estimation of the amplitude of the IAU2000A nutation terms have been obtained by Least Squares. The results are shown in Table 4, which includes the same set of harmonics as investigated in (Herring et al., 2002). The estimation was made both for original data and after removing the FCN contribution computed as described in the next section.

Table 4: Corrections to amplitude of nutation terms,  $\mu\text{as}$ . Formal errors are about 4  $\mu\text{as}$ .

Period	Original data				After removing FCN			
	$\Delta\psi \sin(\epsilon)$		$\Delta\epsilon$		$\Delta\psi \sin(\epsilon)$		$\Delta\epsilon$	
	sin	cos	sin	cos	sin	cos	sin	cos
6798.38	33	22	-31	-40	30	21	-41	-34
3399.19	21	22	-54	-22	30	24	-49	-13
1615.75	6	-7	-3	45	3	-12	6	46
1305.48	-7	1	4	28	-9	-6	14	15
1095.18	-8	10	7	-5	-11	-2	13	-13
386.00	17	55	-33	6	6	8	4	2
365.26	15	15	-30	13	-4	10	-2	0
346.64	13	3	-14	8	-9	11	-8	1
182.62	-13	-3	-19	-13	-11	1	-11	-12
121.75	-11	-13	-6	-5	-2	-9	-8	-1
31.81	7	1	-2	14	2	-4	-7	8
27.55	10	2	-11	4	17	7	-14	4
23.94	7	-8	2	2	5	-3	5	-4
14.77	-3	0	-7	-1	0	-2	-3	-3
13.78	-14	6	-3	-3	-10	4	-5	-6
13.66	-4	-29	6	0	8	-26	0	9
9.56	2	-8	-2	-2	-3	-9	-3	-4
9.13	-11	17	-2	-10	-14	12	-2	-5
9.12	-11	12	-13	-5	-10	9	-11	1
7.10	-14	20	12	27	9	1	-4	-6
6.86	-3	10	7	-5	4	-1	3	-3

One can see that most of the harmonics with periods close to FCN are affected, as expected. However, evidently due to wide spectrum of the new FCN model, other amplitudes are also influenced. In any case, this effect should be investigated more carefully.

#### 4. FREE CORE NUTATION

To investigate the FCN contribution we used smoothed data to eliminate a noise in the investigated data. Figure 3 shows smoothed differences between observed nutation series and the IAU2000A model. From Figures 1–3 one can see that a signal at the FCN frequency band prevails in the spectrum of the differences, which is also known from previous studies (e.g. Shirai and Fukushima, 2001a).

Several models are proposed for the FCN contribution (e.g. Herring et al., 2002; Shirai and Fukushima, 2001b). All existing models suppose that FCN is an oscillation with constant period of about 430 days and variable amplitude. However, newest investigations (Malkin and Terentev, 2003a, 2003b) show that the FCN period is also variable, which may be explained by variable FCN phase though.

Let us consider how a model with variable amplitude and period (phase) can be used in practice. We can describe the FCN term as

$$\begin{aligned}\Delta\psi \cdot \sin \epsilon_0 &= A(t) \sin(\Phi(t)), \\ \Delta\epsilon &= A(t) \cos(\Phi(t)).\end{aligned}$$

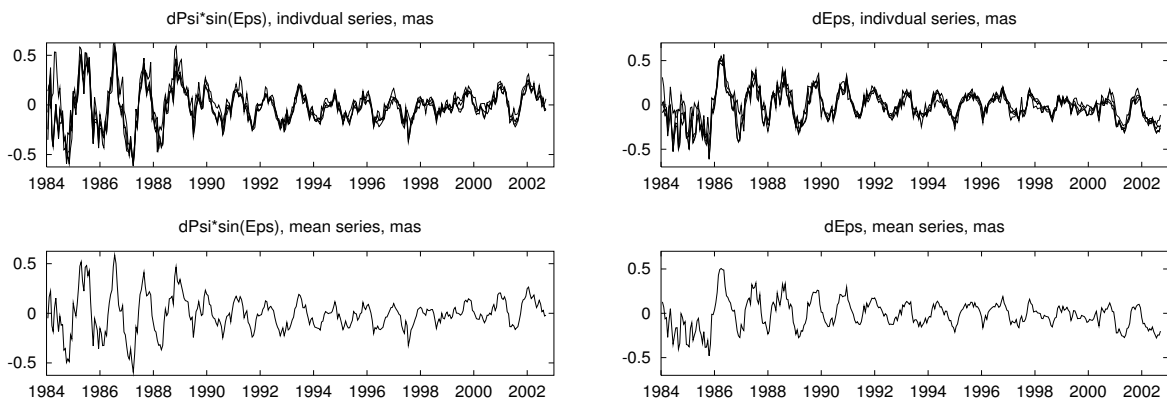


Figure 3: Smoothed differences between observed nutation series and the IAU2000A model.

Mathematically (not geophysically, indeed!), we can suppose three equivalent models for the FCN phase  $\Phi(t)$

$$\Phi(t) = \begin{cases} \frac{2\pi}{P(t)} t + \Phi_0, \\ \frac{2\pi}{P_0} t + \Phi(t), \\ \frac{2\pi}{P(t)} t + \Phi(t), \end{cases}$$

where  $P$  is the FCN period. In other word we can suppose variable period with constant phase, variable phase with constant period, or variable both period and phase. Of course, this is a subject of geophysical consideration, but this doesn't matter for an empirical FCN model using time variations of the FCN parameters found from analysis of the observed data. In practice we can compute  $\Phi(t)$  as

$$\Phi(t) = \int_{t_0}^t \frac{2\pi}{P(t)} dt + \varphi_0,$$

where  $\varphi_0$  is a parameter to be adjusted. Amplitude variations  $A(t)$  can be easily computed from the differences between observed series and model as

$$A(t) = \sqrt{(d\psi * \sin \epsilon)^2 + d\epsilon^2},$$

where  $d\psi$  and  $d\epsilon$  are the differences in longitude and obliquity at epoch  $t$ . Indeed, using such an approach we suppose that all differences in the FCN frequency band can be attributed to the FCN, but this seems to be a good approximation to reality.

Variations of the FCN amplitude  $P(t)$  and phase  $\Phi(t)$  are shown in Figure 4 along with the corresponding FCN parameters included in the MHB2000 model which is, in fact, also a model with variable phase and amplitude, though this is not stated explicitly (we used the text of the FCN\_NUT routine included in the MHB\_2000 code to extract the FCN(MHB) amplitude and phase variations). One can see that both models show similar behavior of the FCN parameters, however new approach allow us to get more smooth and predictable functions  $A(t)$  and  $\Phi(t)$ . Comparing these two models one should keep in mind that MHB2000 model is developed only till epoch 2001.4, and after this epoch the difference between models grows rapidly.

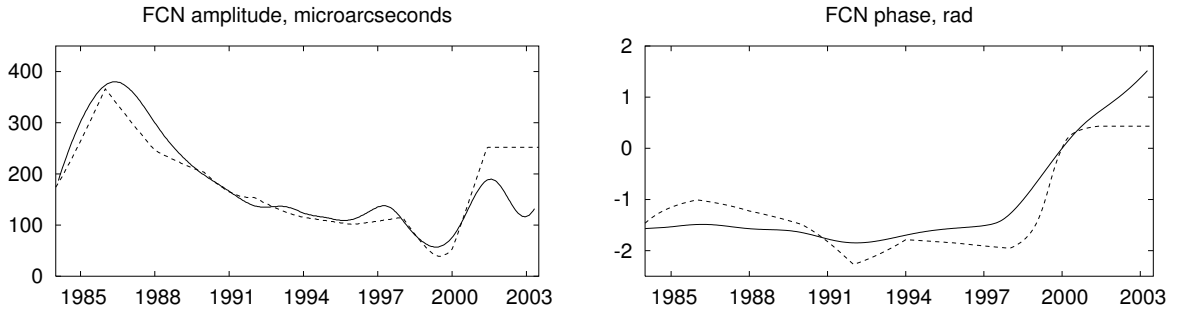


Figure 4: The FCN amplitude and phase variations found in this study (solid line), and a comparison with the MHB2000 model (dashed line).

Figure 5 shows spectra of the differences between observed nutation series and the IAU2000A model computed for raw differences and after removing FCN contribution. One can see that the FCN signal is completely eliminated.

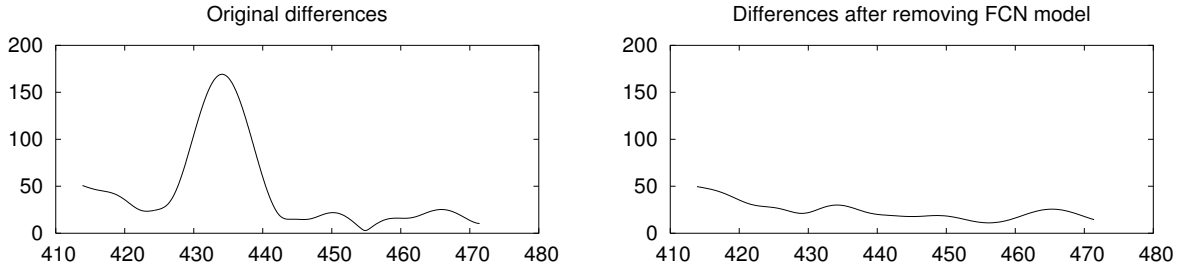


Figure 5: Spectrum of the differences between observed nutation series and the IAU2000A model, period in days, amplitude in  $\mu\text{as}$ .

However, the differences between observed nutation series and model have a noise of various origins with the rms compatible with the FCN contribution. To estimate actual contribution of the FCN model to this noise we computed rms of differences between observations and model with three different accounting for the FCN term: no FCN (raw differences), extracting FCN term according to the MHB2000 model, and extracting the FCN term according to new model described here. The results are shown in Table 5. One can see that accounting for the FCN contribution leads to decreasing of differences. Especially interesting is the last part of the table corresponding to period of observations 2002–2003. Using MHB2000 FCN model for this period leads to degradation of differences between observations and the IAU2000A model.

Table 5: WRMS of differences with two FCN models,  $\mu\text{as}$ .

Series	All sessions			NEOS			R1R4		
	FCN model			FCN model			FCN model		
	No	MHB	New	No	MHB	New	No	MHB	New
GSF	166	146	138	138	122	120	134	150	102
IAA	170	152	144	140	123	123	138	154	111
USN	161	144	136	138	122	122	136	156	107
Mean	156	136	126	131	113	112	129	146	97

A FCN model with variable period and phase allow us to try a new approach to FCN prediction. We can consider two possibilities. The first one is a prediction of actual FCN contribution, which is developed e.g. in (Brzezinski and Kosek, 2003). Another possibility is to predict functions  $A(t)$  and  $\Phi(t)$  separately, and then use predictions to construct the FCN contribution using the formulas given above. Figure 6 presents a variant of such a prediction obtained using ARIMA method. It is interesting to compare both approaches of FCN prediction in details.

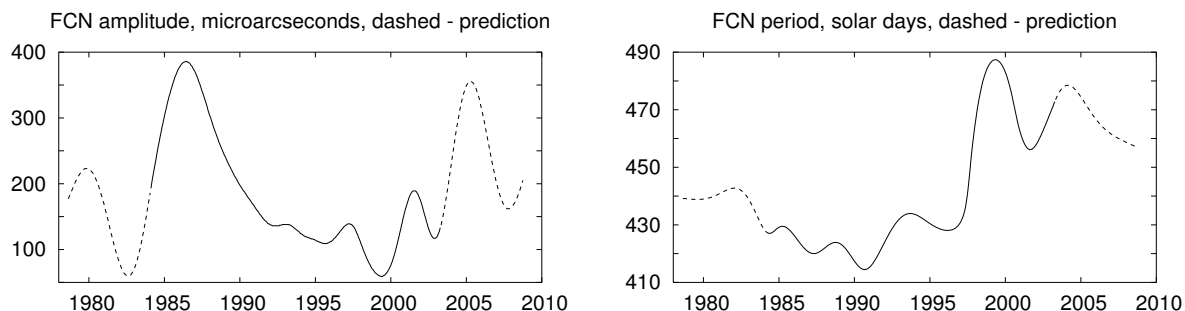


Figure 6: Examples of predictions of the FCN amplitude and phase.

## 5. CONCLUSIONS

The results of this study allow us to make some conclusions.

1. The IAU2000A model represents the modern VLBI observations with an accuracy at the level of 100–120  $\mu\text{as}$ , i.e. about twice better than intended.
2. Bias between observed and modelled celestial pole offset is found to be at the level 20–30  $\mu\text{as}$ , and, evidently cannot be obtained with an accuracy better than 10–20  $\mu\text{as}$  from the current set of observations. The same can be said about precession constant and obliquity rate which seems to be accurate at the level of several microarcseconds per year, and hardly can be significantly improved using the current set of observations.
3. Free Core Nutation heavily contributes to the differences between the observed nutation series and the IAU2000A model. Latest investigations show that the FCN oscillation has not only variable amplitude, but also variable period or phase. A new FCN model with variable amplitude and phase was found to be in better agreement with observations than existing one.

## 6. REFERENCES

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