THEORY OF NUTATION OF THE NON-RIGID EARTH WITH THE ATMOSPHERE

V.E. ZHAROV, S.L. PASYNOK
Sternberg State Astronomical Institute
119992, Universitetskij pr.,13,Moscow,Russia
e-mail: zharov@sai.msu.ru

ABSTRACT. The nutation series ZP2002 based on geophysical theory of the non-rigid Earth with the atmosphere was obtained. The modeling of precession and nutations is based on a fit to the VLBI observational data. A brief description of the series is presented.
The series ZP2002 was obtained by convolution of the rigid Earth nutation series with the transfer function. The resonance parameters and frequencies of the transfer function are determined by the Earth structure. They were calculated by taking into account physical limitations related to the Earth's inner structure parameters. Another important step is an incorporation of additional effects in order to create a more accurate theory. In particular, we investigate the role of the atmosphere as well as viscosity of the liquid core on nutation. The wrms errors of residuals in $\Delta\varepsilon$ is 0.29 mas, and in $\sin\varepsilon_0\Delta\psi$ is 0.20 mas (the FCN model is not included in ZP2002 series).

1. INTRODUCTION. The IAU Resolution B1.6 (2000) states that beginning on 1 January 2003, the IAU 1976 Precession Model and IAU 1980 Theory of Nutation will be replaced by the precession-nutation model IAU 2000A (MHB2000, based on the transfer functions of Mathews, Herring and Buffett(2000)). Besides this, the IAU recommends to continue of theoretical developments of non-rigid Earth nutation series for more accurate account of some processes which are difficult to model.
In this paper account of the atmosphere, viscosity of the fluid outer core (FOC) with radial distribution and non-isotropic heating of the FOC are discussed. The solid inner core (SIC) and electro-magnetic coupling were taken into account according Mathews et.al.(1998). The mantle non-elasticity (frequency-dependent number k and others) was taken into account according Wahr and Bergen (1986). For ocean corrections the model of Huang et. al. (2001) was used.

2. THE ATMOSPHERE EFFECTS ON NUTATION. There are two methods of account of the atmosphere: angular momentum and torque approaches. The torque approach was developed in many papers (see e.g. de Viron and Dehant (1998), Bizouard, de Viron and Dehant (1999)).

We used the angular momentum approach that has been applied for modeling of the atmospheric effects on nutation in papers of Sasao and Wahr (1981), Zharov and Gambis (1996), Bizouard et al. (1998). It allows to avoid direct calculation of the friction torques. Besides, angular momentum approach allows to describe the rotation of the atmosphere by the dynamical equation and add it to system of other equations.

The atmosphere moments of inertia are very small ($\cong 1.4{\cdot}10^{32}$kg·m^2) compared with the moments of inertia of the whole Earth. Thus the ratio of moments of inertia of the atmosphere C_a and whole Earth C is equal $C_a/C \approx 1.8 \cdot 10^{-6}$. However, statement, that rotation of the atmosphere does not influence on nutation, is not true at that level of accuracy, which is required from the nutation theory now.

Rotation of the atmosphere as whole layer relative to the mantle is necessary to take into account due to possibility of the tides resonant strengthening. Such amplification of amplitudes of some nutation terms can be caused by new Earth's normal modes connected with the free rotation of the atmosphere relative to the mantle, if the frequencies of these modes lay in diurnal frequency band.

The dynamical equations for the Earth with the atmosphere can be written as

$$\frac{\partial \overline{H}}{\partial t} + \overline{\Omega} \otimes \overline{H} = \overline{L}, \quad \frac{\partial \overline{H}_f}{\partial t} - \overline{\omega}_f \otimes \overline{H}_f = 0$$

$$\frac{\partial \overline{H}_s}{\partial t} + \overline{\Omega} \otimes \overline{H}_s = \overline{L}_s, \quad \frac{\partial \overline{H}_a}{\partial t} + \overline{\Omega} \otimes \overline{H}_a = \overline{L}_a$$

where $\overline{H}, \overline{H}_f, \overline{H}_s, \overline{H}_a$ are the angular moments of whole Earth with the atmosphere, the FOC, the SIC and the atmosphere, $\overline{L}, \overline{L}_s, \overline{L}_a$ are the torque acting on the Earth with atmosphere, on the SIC and on the atmosphere.

For description of the Earth's rotation and motion of the layers six coordinate systems are entered. Every coordinate system has beginning in the Earth's center of mass. First Cartesian coordinate system is the inertial system, second coordinate system rotates in the inertial space with the constant angular velocity Ω_0. The axes of the terrestial coordinate system $\bar{i}_1, \bar{i}_2, \bar{i}_3$ are the Tisserand mantle axes. The axes of other coordinate systems are the Tisserand FOC, SIC and the atmosphere axes.

It is supposed that the instantaneous angular velocity vector of the Earth Ω is connected with angular velocity vectors of the FOC $\overline{\Omega}_f$, SIC $\overline{\Omega}_s$, and atmosphere $\overline{\Omega}_a$ by equations

$$\overline{\Omega} = \overline{\Omega}_0 + \overline{\omega} \equiv \Omega_0 \left(\bar{i}_3 + \overline{m}\right),$$

$$\overline{\Omega}_f = \overline{\Omega} + \overline{\omega}_f \equiv \Omega_0 \left(\bar{i}_3 + \overline{m} + \overline{m}_f\right),$$

$$\overline{\Omega}_s = \overline{\Omega} + \overline{\omega}_s \equiv \Omega_0 \left(\bar{i}_3 + \overline{m} + \overline{m}_s\right),$$

$$\overline{\Omega}_a = \overline{\Omega} + \overline{\omega}_a \equiv \Omega_0 \left(\bar{i}_3 + \overline{m} + \overline{m}_a\right).$$

The dimensionless vectors $\overline{m}, \overline{m}_f, \overline{m}_s, \overline{m}_a$ characterize the layers' velocities variations.

If we consider that the atmosphere is in state of hydrostatic equilibrium and connect coordinate system with the Tisserand axis then torque can be expressed through the atmospheric excitation functions χ:

$$\tilde{L}_a = L_{a1} + iL_{a2} = iUeA\left(\chi_1^p + i\chi_2^p\right)$$

where A is the Earth's principle equatorial moment of inertia, e is the dynamical ellipticity of the Earth, U is parameter depending on topography, $\chi_{1,2}^P$ are the pressure term components. We did not use the hypothesis of inverted barometer here. The contribution of atmospheric winds in torque in the Tisserand axes system is equal zero.

Table 1: The largest terms in parameter U.

j(degree)	m(order)	$U_{j,m}$
2	-1	($0.508 \cdot 10^{-9}$, $-0.116 \cdot 10^{-7}$)
2	0	($0.101 \cdot 10^{-7}$, $-0.173 \cdot 10^{-11}$)
4	-2	($-0.532 \cdot 10^{-9}$, $-0.581 \cdot 10^{-10}$)
4	-1	($-0.314 \cdot 10^{-9}$, $0.718 \cdot 10^{-8}$)

Parameter U is the proportional coefficient between atmospheric torque and the atmospheric excitation function. It is appeared because of we used angular momentum approach. Parameter U can be calculated analytically only for ellipsoidal surface of the Earth. In order to take into account the topography we used the spherical harmonic coefficients of the pressure field and topography of the Earth, and U is sum of terms depending from degree and order of them. The most important terms of U are shown in the Table 1.

For calculation of U we used complex spherical decomposition of the pressure field and topography, so order of harmonic can be negative.

Note that value of coefficient U_{20} exactly corresponds the ellipsoidal Earth's surface and imaginary part of coefficient $U_{2,-1}$ approximately equal to real part of $U_{2,0}$ but with opposite sign.

Effect of the atmosphere on nutation consists not only in corrections for some nutation terms but in appearance of new normal modes, frequencies of which are equal to

$$\sigma_{PFAN} \approx -1 + \left(U/\Omega_0^2\right) e_a \quad , \quad \sigma_{AW} \approx \left(1 - U/\Omega_0^2\right) e_a$$

where σ_{AW} is the Atmosphere Wobble frequency and σ_{PFAN} is the Prograde Free Atmosphere Nutation frequency. Value of U is changed during year and is complex quantity. It means that the frequencies of the normal modes depend on time, and topography not only determines the frequencies but determine dissipation of energy too. The frequencies σ_{PFAN}, σ_{AW} are proportional the dynamical ellipticity of the atmosphere e_a. As our calculation shown, ellipticity e_a was variable with period of one year.

The Bizouard et al. (1998) amplitudes of the atmospheric excitation function were used for numerical calculation effect of the atmosphere.

3. VISCOSITY AND ELECTROMAGNETIC COUPLING. Role of viscosity of the fluid core was discussed in Sasao et al. (1980), Getino and Ferrandiz (1997). It was proposed that viscosity did not depend on radial distance and had very small value in these works. But on base of experiments (Brazhkin and Lyapin, 2000) more complicated model of viscosity was proposed. We used this model and incorporated the electro-magnetic torque in the nutation theory too.

In order to take into account viscosity of the liquid core, we used radial distribution of viscosity which was proposed on base of experiments and measurements of the viscosity of melted iron under a high pressure by Brazhkin and Lyapin (2000).

Viscosity is increased exponentially in the thin layer near the SOC according this results. This conclusion allows to simplify solution of hydrodynamical equations in order to determine the velocity as function of radius and calculate the viscous torque. The torque depends on parameter W and proportional to difference of angular velocities of the FOC and SIC that are characterized by non-dimensional vectors $\overline{m}_f, \overline{m}_s$:

$$\overline{L}_f^{(\eta)} = -\Omega_0^2 W \left(\overline{m}_f - \overline{m}_s\right).$$

Here $\overline{L}_f$ is the torque acting on the FOC from the SIC. One can show that for distribution of viscosity shown on Fig.1 the torque acting on the FOC from the mantle is significantly less then Γ_f and

$$W \equiv \eta_s \frac{8\pi r_s^4}{3\delta r \Omega_0}$$

where η_s is viscosity on the FOC – SIC boundary; $\delta r = 100$ km is thickness of the layer of high viscosity; r_s is radius of the Earth's solid inner core.

The electromagnetic coupling was incorporated into our model in accordance with Mathews et al. (1998). Then

$$S_{22} = e_f - i\frac{W}{A_f} + S_{22}^{(e)}, \quad S_{23} = i\frac{W}{A_f} + S_{23}^{(e)},$$

$$S_{32} = i\frac{W}{A_s} + S_{32}^{(e)}, \quad S_{33} = -i\frac{W}{A_s} + S_{33}^{(e)},$$

where $S_{ij}^{(e)}$ is term depending on electromagnetic coupling.

4. THE SYSTEM OF EQUATIONS. Forced nutations can be found from equation:

$$Mx = y$$

where:

$$M =$$

$$\begin{pmatrix}
\sigma(1+\kappa)+\kappa-e & (1+\sigma)(\frac{A_f}{A}+\xi) & (1+\sigma)(\frac{A_s}{A}+\zeta) & (1+\sigma)\alpha_3 e_s\frac{A_s}{A} & (1+\sigma)\frac{A_a}{A} & (1+\sigma)\frac{A_a}{A}e_a(1+\kappa') \\
\sigma\left(1+\gamma\left(1-\frac{U_f}{\Omega_0^2}\right)\right) & 1+\sigma(1+\beta)+S_{22}-\frac{U_f}{\Omega_0^2} & \sigma\delta\left(1-\frac{U_f}{\Omega_0^2}\right)+S_{23} & -\sigma\alpha_1 e_s\frac{A_s}{A} & 0 & \sigma\left(\frac{\xi}{\tau}+h_f-\frac{U_f}{\Omega_0^2}\right)\frac{A_a}{A_f}e_a \\
\sigma(1+\theta)-e_s\alpha_3-\theta\frac{U_s}{\Omega_0^2} & \sigma\chi+\alpha_1 e_s+S_{32}-\chi\frac{U_s}{\Omega_0^2} & \sigma\nu+1+\sigma+S_{32}-\nu\frac{U_s}{\Omega_0^2} & (1+\sigma-\alpha_2)e_s & 0 & \left(\sigma-\frac{U_s}{\Omega_0^2}\right)(\frac{\zeta}{\tau}+h_s)\frac{A_a}{A_s}e_a \\
0 & 0 & 1 & \sigma & 0 & 0 \\
\sigma-e_a & 0 & 0 & 0 & 1+\sigma & (1+\sigma)\sigma+\left(1+\sigma-\frac{U}{\Omega_0^2}\right)e_a \\
0 & 0 & 0 & 0 & 1 & \sigma
\end{pmatrix}$$

$$x = \begin{pmatrix} \widetilde{m} \\ \widetilde{m_f} \\ \widetilde{m_s} \\ \widetilde{n_s} \\ \widetilde{m_a} \\ \widetilde{n_a} \end{pmatrix}, \qquad y = \begin{pmatrix} (\kappa-e+\sigma\kappa)\widetilde{\phi}-(1+\sigma)(1+\kappa')\frac{\widetilde{c_3^a}}{A} \\ \sigma\gamma\widetilde{\phi}\left(1-\frac{U_f}{\Omega_0^2}\right)-\sigma\left(\frac{\xi}{\tau}+h_f-\frac{U_f}{\Omega_0^2}\right)\frac{\widetilde{c_3^a}}{A_f} \\ \left(\sigma\theta-\alpha_3 e_s-\theta\frac{U_s}{\Omega_0^2}\right)\widetilde{\phi}-\left(\sigma-\frac{U_s}{\Omega_0^2}\right)(\zeta/\tau+h_s)\frac{\widetilde{c_3^a}}{A_s} \\ 0 \\ -\left(1+\sigma-\frac{U}{\Omega_0^2}\right)\frac{\widetilde{c_3^a}}{A_a} \\ 0 \end{pmatrix}.$$

The parameters A, A_f, A_s, A_a are the equatorial moments of inertia, e, e_f, e_s, e_a are the dynamical ellipticities of the whole Earth, the FOC, the SIC and the atmosphere, σ is the frequency of harmonics of the tidal potential $\widetilde{\phi}$. Other greek symbols are the compliancies representing the deformations of the Earth and the core. From this equation the transfer function was obtained. Forced nutations of the Earth without effects of the mantle anelasticity and oceans were obtained by multiplication of the transfer function on the rigid Earth nutation series RDAN97.

The mantle anelasticity corrections were estimated according formulae from Wahr and Bergen (1986) for QMU model with the $\alpha = 0.15$. For ocean corrections the model Huang et. al. (2001) was used. The final nutation amplitudes were estimated as the nutation of the Earth without effects of the mantle anelasticity and oceans plus mantle anelasticity corrections and ocean corrections.

The frequencies of the resonance modes were obtained from equation:

$$Mx = 0. \tag{1}$$

5. THE NEGATIVE IMAGINARY PART OF THE RESONANCE FREQUENCY PROBLEM AND NON-ISOTROPIC HEATING EFFECT.

The CW resonance frequency is:

$$\sigma_{CW} = \frac{A}{A_m}(e-k) + \Delta \tag{2}$$

where Δ is small factor. Frequency σ_{CW} is obtained as solution of equation (1). Imaginary part of the Δ arises from the electro-magnetic coupling in the MHB2000 theory. The eigenfrequency of CW of the Earth differs from those in (2) because frequency dependence of the k leads to the correction to k. This correction can be calculated according formulae from Wahr and Bergen (1986); it is complex value and arises from the anelastic dissipation in mantle.

Sign of the imaginary part of the Δ is negative in the MHB2000 theory. It means that Δ can not arise from the electro-magnetic forces because electro-magnetic forces can not leads to the generation of the energy. The correction to the imaginary part of the k is positive and large. Therefore resulting imaginary part of the CW frequency is correct and positive. But it can not explain the negative sign of the imaginary part of Δ. For explanation of negative sign of the imaginary part of Δ it is necessary to find process that can provide energy for it.

At first we built model with the super-rotation of the SIC. It was proposed that the SIC has non-zero mean angular velocity relative to the mantle. This hypothesis is based on results of work Song and Richards (1996) (see also Whaler and Holme 1996; Vidale et al., 2000). As our calculations shown it was possible to fit theoretical and observed nutations with rms error of 0.24 mas. It is possible to get positive parts of the normal mode frequencies, but differential rotation of the SIC has to be very large $\sim 15^0$/day (the velocity of the differential rotation of the SIC was estimated as $0.15^0/year$ in the latter paper). So this model was rejected.

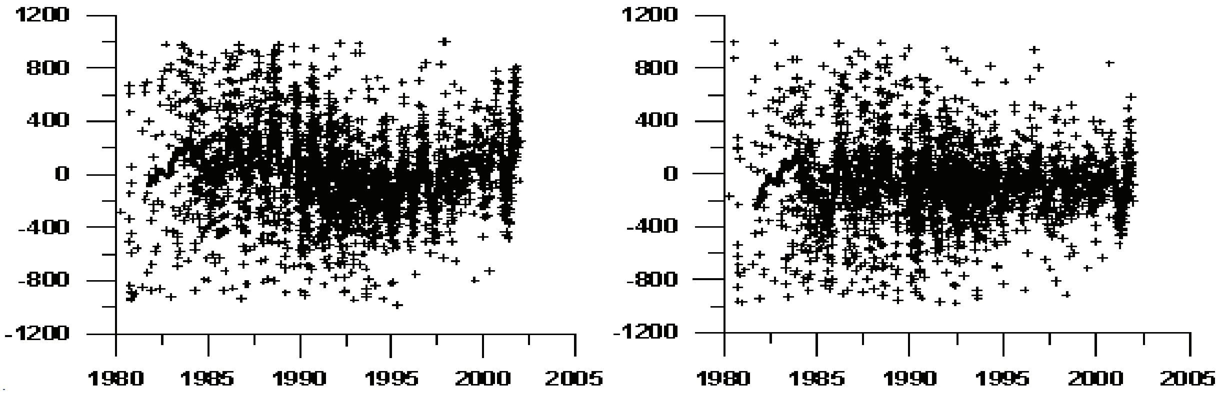

Figure 1: Comparison with the VLBI observations. The corrections $d\varepsilon$ to the theoretical nutation angle ε (in μas) for ZP2002 without FCN (left) and MHB2000 without FCN (right). The solid line is the running average.

Model that is suggested can explain the negative sign of the imaginary part of Δ. It was based on assumption that there are sources of heat in the Earth's deep interior. This situation is similar to the atmospheric excitation owing to absorption of solar energy. If we use the excitation functions formalism then we can write:

$$\tilde{L}_f^h = iU_f e_f A_f \left(\chi_{1f}^p + i\chi_{2f}^p\right) \quad \tilde{L}_s^h = iU_s e_s A_s \left(\chi_{1s}^p + i\chi_{2s}^p\right)$$

where the excitation functions $\widetilde{\chi}_f^p$ and $\widetilde{\chi}_f^s$ can be expressed through the components of the inertia tensors fluctuations $\widetilde{c}_3^f + A_f e_f \widetilde{n}_f$ and $\widetilde{c}_3^s + A_s e_s \widetilde{n}_s$. The U_f and U_s are considered as complex constants which has to be determined from the fitting procedure.

6. THE NUMERICAL RESULTS.

The package OCCAM5.0 was used for comparison our series ZP2002 with VLBI observations and with the MHB2000 theory. The VLBI observations from 1980 to 2002 were processed and corrections to nutation angles for the theories MHB2000 and ZP2002 were obtained. On Fig.1 we show corrections $d\varepsilon$ for the nutation angle ε.

The wrms errors are 0.29 mas for $\Delta\varepsilon$ and 0.20 mas for $sin\varepsilon_0\Delta\psi$ for the ZP2002 theory without FCN. The same quantities for the MHB2000 theory without FCN are 0.22 and 0.17 mas. It is interesting that if we assume that ZP2002 corrections arise from FCN than wrms errors will be 0.17 mas in $\Delta\varepsilon$ and 0.18 mas in $sin\varepsilon_0\Delta\psi$. The same quantities for the MHB2000 theory with FCN are 0.19 and 0.12 mas.

It is preliminary result because the non-linear terms in gravitational torques are not included in our theory. The final conclusions and comparison of theories will be made when these terms will be took into account.

This work was supported by grant 01-02-16529 of the RFBR.

7. REFERENCES

Brazhkin, V. V., and Lyapin, A.G.,2000, *Journal "Uspehi phizicheskih nauk"* 170, 5, 535-551(in russian).

Getino, J., and Ferrandiz, J.M., 1997,*Geophys. J. Int.*, 130, 326-334.

IAU Resolutions 2000,24th General Assembly,Manchester, August, 2000.

Mathews, P.M., Herring, T.A., Buffet, B.A.,2002,*J. Geophys. Res.*,(in press).

Mathews, P.M., Buffet,B.A., Herring,T.A., Fessel M.,1998, In: *Proc. of Journées 1998, Systèmes de référence spatio-temporels*, Paris, 86-91.

Bizouard, Ch., de Viron, O., and Dehant, V., 1999, In: *Proc. of Journées 1999, Systèmes de référence spatio-temporels*, Dresden, Allemagne,157-166.

Bizouard Ch., Brzezinski, A., PETROV, S.,1998, *Journal of Geodesy*, 72, 561-577.

Huang,C.L.,Jin,W.J.,Liao,X.H.,2001, *Geophys.J.Int.* ,146, 126-133.

de Viron, O., and Dehant, V., 1998, In: *Proc. of Journées, Systèmes de référence spatio-temporels*, Paris,146-147.

Sasao, T.S., Okubo and Saito, M., 1980, In: *Proc. of IAU Symposium on Nutation and Earth's Rotation*, 78, 165-183.

Sasao, T.S., and Wahr, J.M., 1981,*J. R. Astr. Soc.*, 64,729-746.

Song, X., and Richards, P.G., 1996, *Nature*, 382, 221.

Vidale, J.E., Dodge, D.A., and Earle, P.S.,2000, *Nature*, 405, 445 - 448.

Wahr, J., and Bergen, Z.,1986,*Geophys. J. R. Astron. Soc.*,87,633-688.

Whaler, K. A., and Holme, R., 1996, *Nature*, 382, 205.

Zharov, V.E., and Gambis, 1996, *Journal of Geodesy*, 70, 321-326.