RELATIVITY FOR ASTROMETRY AT THE MICROARCSECOND LEVEL

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ABSTRACT. The current state of relativistic modeling of positional observations with microarcsecond accuracy is reviewed. The requirements which should satisfy a reasonable relativistic model as well as the structure of a standard relativistic model are both discussed. Some subtle relativistic effects playing a role in the model are elucidated. Additional relativistic effects due to gravitational fields produced outside of the solar system which potentially could be larger than 1 μas are also briefly mentioned.

1. INTRODUCTION

Fifteen years after the idea of positional measurements with an accuracy of a microarcsecond was seriously discussed for the first time microarcsecond astrometry is gradually becoming reality. Two space missions GAIA and SIM approved by ESA and NASA should be launched and produce first results within the next decade. Being rather different in goals and technical means these two missions have one thing in common – the accuracy of ~ 1 μas. One microarcsecond is the apparent thickness of a sheet of paper as seen from the other side of the Earth. This amazing technical accuracy requires very careful theoretical modeling. It is clear that very complicated instrumental calibrations and models are necessary here. It is also clear that in many cases the sources themselves (quasars, stars, solar system bodies, etc.) have angular extensions exceeding 1 μas and, thus, adequate astrophysical models of the sources are needed to interpret the photocenters of the corresponding images with required accuracies. This paper deals with what lies between these two parts of the overall modeling: here we briefly describe a standard relativistic model of positional observations of point-like sources with microarcsecond accuracy.

2. SOME REQUIREMENTS TO A RELATIVISTIC MODEL OF POSITIONAL OBSERVATIONS

Let us first formulate some simple requirements which any reasonable relativistic model of positional observations should satisfy.

A. The model should be compatible with state-of-the-art relativistic models of other kind of observations (i.e. geodetic VLBI observations which are used to construct the ICRS). This indispensably means that the model must be compatible with the IAU 2000 Resolutions on relativity (Soffel *et al.* 2003).

B. The model should be complete. The suggested relativistic framework should not only give the possibility to model the positional observations, but also should provide the community with relativistic models for translational and rotational motion of the satellite, etc.

C. The model should be valid for any trajectory of the satellite (although the types of the orbits for GAIA and SIM are fixed, actual orbits will be affected by various non-gravitational effects and other factors and cannot be calculated in advance).

D. The model should take into account a realistic model of the solar system (i.e. all gravitating bodies producing a light deflection of 1 μas and more should be taken into account).

E. The model should be formulated in such a way that any model of the motion of solar system bodies can be used (i.e. any ephemeris can be plugged into the model).

F. The model should be valid for any light source: from a near-earth asteroid to a quasar.

G. The model should contain all effects in light propagation which may amount to 1 μas in real observations.

H. The model should be as simple as possible (i.e. any formulas and calculations affecting the results at levels below 1 μas should be avoided).

I. The model should be configurable. Since the accuracy of the observations will crucially depend on the brightness of the source it is important to efficiently calculate the observed quantities with a given accuracy (1 μas or worse) and not just with a fixed accuracy of 1 μas. Furthermore, for detailed investigations of the observational data it is necessary to switch off and on particular physical effects.

J. The model should be formulated in such a way that we could test general relativity against at least some class of alternative theories of gravity.

3. THE STRUCTURE OF THE MODEL

A model satisfying all the requirements formulated above was elaborated and refined by several authors during more than a decade. After the pioneering works of Brumberg, Klioner and Kopeikin (1990) and Klioner and Kopeikin (1992) the model was substantially refined and simplified by Klioner (2000, 2002, 2003a). Full details of the model are given in Klioner (2003a). Here we just give an overview of its structure. The model uses the Barycentric Celestial Reference System (BCRS) of the IAU to describe the translational motion of the satellite and the solar system bodies as well as the light propagation from the source to the satellite. The Geocentric Celestial Reference System (GCRS) of the IAU is used only for the purpose of satellite orbit determination (this implies the use of Earth-based observations of the satellite itself). The model also uses a local reference system of the satellite (or a tetrad attached to the satellite) kinematically nonrotating relative to the BCRS to describe the observable direction of the light propagation and any local physical processes in the satellite (e.g. its rotational motion). Let us introduce five vectors:

1. $\mathbf{s}$ is the unit observed direction (the word "unit" means here and below that the formally Euclidean scalar product $\mathbf{s} \cdot \mathbf{s} = s^i\, s^i$ is equal to unity). This vector is defined with respect to the tetrad attached to the satellite's center of mass and kinematically non-rotating with respect to the spatial axes of the BCRS.

2. $\mathbf{n}$ is the unit tangent vector to the light ray at the moment of observation.

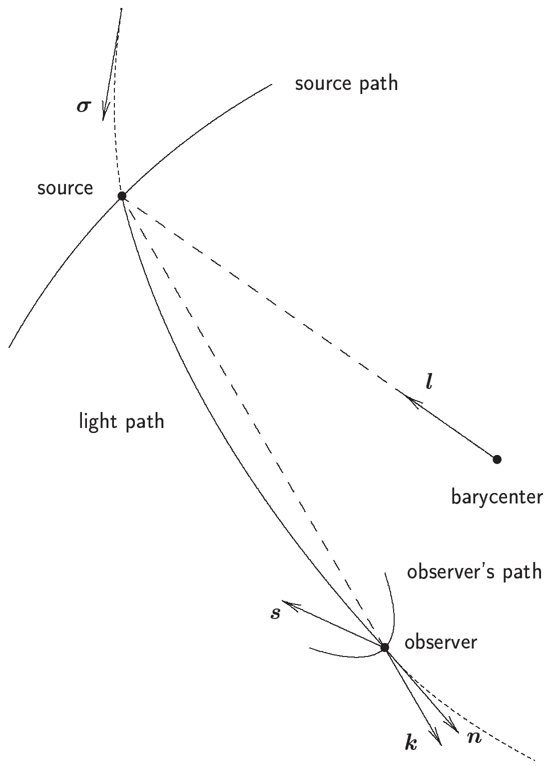

Figure 1: Five principal vectors used in the model: $\mathbf{s}$, $\mathbf{n}$, $\boldsymbol{\sigma}$, $\mathbf{k}$, $\mathbf{l}$.

3. $\boldsymbol{\sigma}$ is the unit tangent vector to the light ray at $t = -\infty$.

4. $\mathbf{k}$ is the unit coordinate vector from the source to the observer.

5. $\mathbf{l}$ is the unit vector from the barycenter of the Solar system to the source.

Note that the last four vectors are defined formally in the coordinate space of the BCRS. The five vectors used in the model are illustrated on Fig. 1. The calculations constituting the model lead to subsequent transformations of the five vectors mentioned above into each other. The physical content of these transformations can be formulated as follows:

$\mathbf{s} \leftrightarrow \mathbf{n}$ Aberration and gravitational effects affecting the observable light direction as compared to the coordinate direction of light propagation at the point of observation.

$\mathbf{n} \leftrightarrow \boldsymbol{\sigma}$ Gravitational light deflection for infinitely distant sources.

$\mathbf{n} \leftrightarrow \mathbf{k}$ Gravitational light deflection for sources situated at finite distances.

$\mathbf{k} \leftrightarrow \mathbf{l}$ Parallax.

The transformation sequence for a solar system source is $\mathbf{s} \to \mathbf{n} \to \mathbf{k}$. A number of vectors $\mathbf{k}$ derived for several epochs of observations allows one to determine (or refine) the BCRS orbit of the observed body. The transformation sequence for a source outside of the solar system is $\mathbf{s} \to \mathbf{n} \to \boldsymbol{\sigma} = \mathbf{k} \to \mathbf{l}$. In general the vector $\mathbf{l}$ depends on time. A reasonable parametrization of that time dependence allows one to obtain the proper motion of the source (and, additionally, orbital elements for binary stars, etc.).

The most complicated part of the transformations is the calculation of the gravitational light deflection. At the level of 1 μas this must include

1. post-Newtonian effects of motionless mass monopoles (in principle, not only the major planets should be taken into account here, but also a number of satellites and even Ceres),
2. post-Newtonian effects of motionless mass quadrupoles (important only for the four giant planets of the solar system), and
3. effects of translational motion of the gravitating bodies.

The effects of rotational motion of Jupiter are also marginally important. Moreover, although the post-post-Newtonian effects in the framework of general relativity are expected to play no role at the level of 1 μas (accounting for the minimal Sun avoidance angle of at least 35 degrees), one has to include a reasonable parametrization of post-post-Newtonian effects in the light propagation in the framework of a class of alternative theories of gravity. The reason is that the expected accuracy of the determination of the PPN parameter γ (of order $\sim 10^{-7}$) is close to the validity limit of the post-Newtonian approximation.

4. A SUBTLE POINT OF THE MODEL: LIGHT PROPAGATION IN THE FIELD OF MOVING BODIES

One of the most complicated points in the whole relativistic model of positional observations is the effect of translation motion of gravitating bodies on the light propagation. Hellings (1986) was probably the first who treated the problem from a rather intuitive point of view: he recommended to use the standard post-Newtonian formulas for the light propagation in the gravitational field of a motionless body and to substitute in those formulas the position of each gravitating body at the moment of closest approach of that body and the photon. The next step has been done by Klioner (1989) where the problem has been solved rigorously for the bodies moving with a constant velocity in the first post-Newtonian approximation. The effects of accelerations of the bodies have been further treated by Klioner and Kopeikin (1992) where it was shown that if the coordinates and velocities of the bodies are computed at the moments of closest approach of the corresponding body and the photon, the residual terms of the solution are in some sense minimized. The complete solution of the problem for arbitrarily moving bodies in the first post-Minkowskian approximation was found by Kopeikin and Schäfer (1999) who succeeded to integrate analytically the post-Minkowskian equations of light propagation in the field of arbitrarily moving mass monopoles. The Kopeikin-Schäfer solution has been rewritten explicitly by Klioner (2003b) who also showed how to get the Kopeikin-Schäfer solution in a very clear and straightforward way for a uniformly moving body as well as how to generalize the Kopeikin-Schäfer solution for bodies with full multipole structure. Recently, Klioner and Peip (2003) performed a series of numerical simulations to investigate the practical accuracy of various analytical formulas for the case of a GAIA-like observing satellite. The authors concluded that if an accuracy of 0.2 μas is sufficient, the initial suggestion of Hellings (1986) can be used. If higher accuracy is needed both the Klioner-Kopeikin and Kopeikin-Schäfer solutions can be used (the difference between them being less than 0.002 μas).

5. BEYOND THE STANDARD MODEL

The model sketched above can be called a "standard model". That model guarantees the accuracy of 1 μas provided that all gravitational effects in the light propagation come from the Solar system itself. In reality this cannot be always assumed and the standard model should be extended to include some additional gravitational effects in the light propagation in some specific cases. Just to give some examples, let us mention that a more complicated model is necessary to describe the motion of light sources which are members of nearly edge-on binary systems. Gravitational light deflection caused by stars near the line of sight (microlensing) or by whole galaxies (macrolensing) is an important issue (Belokurov, Evans, 2002) as is the gravitational light deflection caused by gravity waves (Kopeikin *et al.* 1999). Finally, the expansion of the universe has to be considered for light-sources at cosmic distances (larger than say a few 100 Mpc). To this end the BCRS metric has to be 'matched' to the cosmic metric that contains the scale factor $a(t)$ of the universe (Soffel, Klioner, 2003).

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