

# DEFLECTION OF THE VERTICAL IN BUCHAREST DERIVED FROM GEODETIC ASTRONOMICAL OBSERVATIONS

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## ABSTRACT.

In order to develop a zeroth order network of deflection of the vertical, the Astronomical Institute joined with the Faculty of Geodesy has started in Bucharest a scientific project. There were used two very precise instruments: the CCD astrolabe and Leica TC 2002 theodolite. The used software package based on FK5 star catalogue has been designed to work both with classical mechanical theodolite and CCD astrolabe.

This paper emphasizes the methods of observation, reduction techniques, and results. The value of the determined vertical deflection in Bucharest, reported at WGS84 ellipsoid, is about  $11''.5$ , while the variance for each component lies under  $0''.4$ .

## 1. DEDUCTION OF VERTICAL DEFLECTION

### 1.1 Introduction

The connection between space-related observations using GPS technology, and ground based measurements such as VLBI or SLR, demands a better knowledge of the parameters related to the equipotential surface at sea level. These parameters lead to the problem of local vertical deflection reported to the reference ellipsoid. In astronomical geodesy, the local plumb line is related to the astronomical coordinates  $(\Phi, \Lambda)$ , derived from star observations. In exchange, geodetic coordinates  $(B, L)$ , obtained by means of GPS technology, give the direction of the normal to the ellipsoid. In the same Earth surface point, the disagreement between the two series of data, i.e. the differences:  $\Phi - B$  and  $\Lambda - L$ , particularly reflect the unparallelism between the local vertical, and the normal reported to the surface. Transitions regarding these differences in several different local points on Earth's surface could be explained as anomalies in tectonic mass distribution.

### 1.2 Astrogeodetic method

The main drawback of the gravimetric method is not being straight. The vertical deflection is not immediate, but results after a great amount of calculus performed upon the detected values

of gravitational acceleration. Besides, the method implies expensive hardware (high precision gravimeters).

In comparison with the gravimetric method, the astrogeodetic method is direct, allowing the immediate deduction of the orthogonal components of vertical deflection. That can be easily performed by comparing the astronomical values of coordinates with those obtained by GPS technology.

The first component denoted by  $\xi$  is in the plane of local meridian, while the second component  $\eta$  is in the plane of prime vertical

The following relationship gives the angle of vertical deflection ( $u$ )

$$u = \sqrt{\xi^2 + \eta^2} \quad (1)$$

The  $\xi$  component can be written as the difference between astronomical  $\Phi$  and geodetic  $B$  latitudes, in the same point of observation. Identically,  $\eta$  represents the difference between the two kinds of longitude ( $\Lambda$  and  $L$ ) corrected of meridian convergence:

$$\xi = \Phi - B, \eta = (\Lambda - L) \cos \Phi \quad (2)$$

## 2. INTERNATIONAL PROJECTS AND JOINT VENTURES

The beginning of the 9th decade of the last century brought a revolutionary concept in satellitary technique: the GPS. For almost ten years, the scientific community pointed out toward this new domain, according less interest to the other geodetic technologies

Nevertheless, in the last years one have assisted to sudden technological changes in the domain of terrestrial geodesy (electronic theodolits, gravimeters, EDM instruments), so we can now talk about a link between GPS technology and ground based techniques.

At the beginning of 2000, one have been developed new observational technologies, as a result of a close cooperation between several European research institutes and private enterprises (Swiss Geodetic Commission; Swiss Federal Office of Topography; Swiss Federal Institute of Technology; Leica Heerbrugg; Department of Mathematical Sciences, University of Trieste, Italy; Istituto Geografico Militare Italiano; Institut für Geodäsie, Universität der Bundeswehr, Neubiberg, Deutschland; Department of Mathematics, Science Faculty, University of Lisbon, Portugal; etc.).

New extensive software packages such as *ICARUS* or *DIADEM*, based on the FK5 star catalogue have been released in order to determine the local vertical deflection, as keystone of geoid fine structure determination. (Bürki, 2002).

These new projects have been tested in several European countries, such as: Swiss, Italy, France, Spain, Greece, Germany, Portugal.

## 3. ROMANIAN CAMPAIGN OF OBSERVATIONS

### 3.1 *The start of a joint project in Bucharest*

The Astronomical Institute joined the Faculty of Geodesy, in the attempt to develop a network of deflections of the vertical. Hence, a new scientific project has been started at Bucharest.

### 3.2 *Observations: sites, instruments and reduction methods*

For the very beginning of this project, there have been chosen to points of observation: the first point is located at AIRA, on the astrolabe pilaster and have the GPS coordinates ( $B=44^\circ 24'$

43". 059, L=26°05' 38". 024). The second site, whose GPS coordinates are (B = 44° 27' 50". 247, L = 26° 07' 32". 993), is located at TUCE, on the roof of the Faculty of Geodesy.

The distance between the observational sites is 6 km. There were used two very precise instruments: the CCD astrolabe and Leica TC 2002 theodolite. Astrolabe observations were performed only at AIRA, while visual observations with the mechanical theodolite were performed in both sites.

The used software package, based on FK5 star catalogue, has been designed to work both with classical mechanical theodolite and CCD astrolabe. Usually, the reduction of astronomical data achieved at ground asks for resolution, the classic method of *equal altitude*. All the astronomical data obtained with CCD astrolabe have been reduced by this method. In exchange, the set of observational data obtained with the mechanical theodolite has been reduced by means of a new method called: *measurements checked by unknowns*.

Here, the depart relationship remains as in the classical procedure, the well known expression:

$$F = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H - \cos z = 0 \quad (3)$$

Beside  $d\lambda$ ,  $d\varphi$ , and  $dr$ , the unknown vector also contains two additional unknowns ( $\nu_t$ ,  $\nu_z$ ), related to the directly measured parameters, i.e. timing and zenithal distance of stars passage.

The condition equation can be written:

$$\left(\frac{\partial F}{\partial t}\right) \nu_t + \left(\frac{\partial F}{\partial z}\right) \nu_z + \left(\frac{\partial F}{\partial \lambda}\right) d\lambda + \left(\frac{\partial F}{\partial \varphi}\right) d\varphi + \left(\frac{\partial F}{\partial r}\right) dr \quad (4)$$

In order to solve the system, one have to introduce a special matrix, called the pound matrix. The pound for  $\nu_t$  associated to each observed star is given by formula:

$$p_t = (\cos \delta \sin \omega)^2 \quad (5)$$

while for  $\nu_z$ , the pound is set to the unity.

Atudorei (1993), and Badescu (2002) present a rigorous demonstration of this method.

### 3.3 Statistical analysis concerning the results obtained at AIRA (astrolabe pilaster), with both instruments

The observational situation is syntactically described in table 1. Unless both of them are based on astronomical observations, there are a lot of differences between the two methods (different instruments, number of nights and observed stars, different dates, and even different mode of synchronizing the time)

As remark, one have to mention that  $\xi$  and  $\eta$ , obtained by both methods, are obtained as results of astronomical observations, corrected of polar movement.

Preliminary tests with the CCD astrolabe have begun in 1997 (Popescu et al. 1997), but reliable observational data concerning the vertical deflection in Bucharest, have been obtained only since 1998 (Popescu 1999).

The averaged values of obtained data, and their dispersions are shown in table 2. It worth to remark the similarity between both sets of data.

Concerning the observational correctness denominated by  $\sigma_\xi$ , and  $\sigma_\eta$ , we can say it is 3-4 times better in the case of CCD observations. That can be explained first, by the great quantity of observed stars during the night; as soon as the number of observed stars rises toward 100 by night,  $\sigma$  falls down under 0.2". On the other hand, the computer aided observations with accurate timing (time GPS), increases the accuracy.

Figures 1 and 2 show the variation of  $\xi$  and  $\eta$ , obtained with both instruments at the same location, during the period of observation (1998-2002):

Characteristics	Method#1	Method#2	Observations
Location	Astrolabe pilaster A.I.R.A	Astrolabe pilaster A.I.R.A	In the same point have been used both methods of reduction
Period of observation	1998 -2002	2002	Different perriods of observation
Observers	AIRA	TUCE	Different observators.
Instruments	CCD astrolabe (set)	Leica TC 2002 (portable)	Completely different observing instruments with different accuracy
Number of nights of observation	34	9	no observations
Star catalogue	FK5	FK5 GSC	no observations
Observations	Zenithal: almucantharat of $45^\circ$	Both azimuthal and zenital any value	no observations
Time	UTC $\Rightarrow$ GPS + time card	UTC $\Rightarrow$ Internet, portable chronometer	no observations
Average number of observed stars by night	$\approx 30$	$\approx 18$	Method #1:great variation of observed stars at different nights.; Method #2: no variation of observed stars at different nights
Reduction method	Equal altitudes	Measurements cheked by unknowns	no observations
One observed star $\Rightarrow$	one ecuation	two ecuations	#1 : n stars $\Rightarrow$ n ecuations #2 : n stars $\Rightarrow$ 2n ecuations
Number of unknowns	3	3	no observations
Geodetic coordinates	WGS 84	WGS 84	no observations

Table 1:

CCD	$\xi_{avr}$	$d_\xi$	$\eta_{avr}$	$d_\eta$	Leica2002	$\xi_{avr}$	$d_\xi$	$\eta_{avr}$	$d_\eta$
( " )	11.679	0.349	4.776	0.323	( " )	11.415	0.378	4.578	0.457

Table 2:

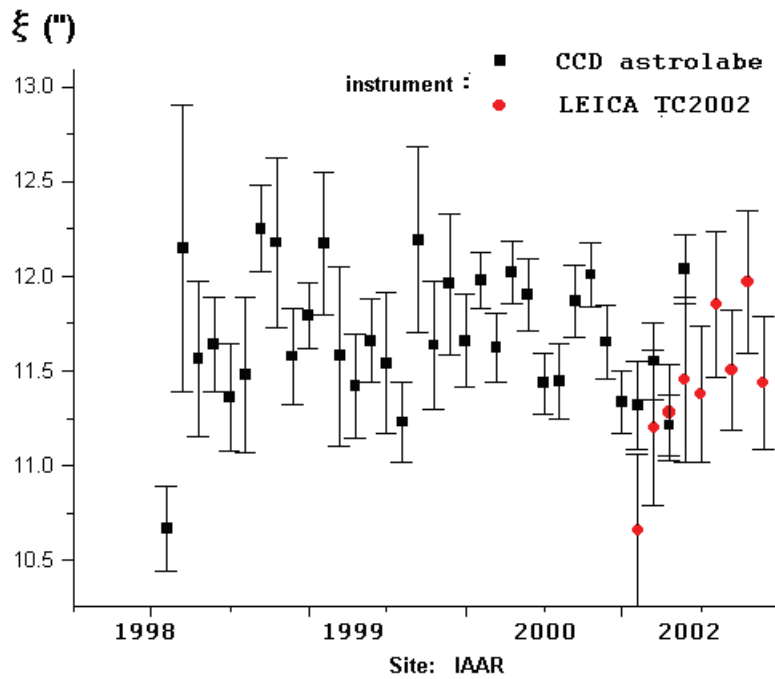


Figure 1: Time variation of  $\xi$  since 1998-2002

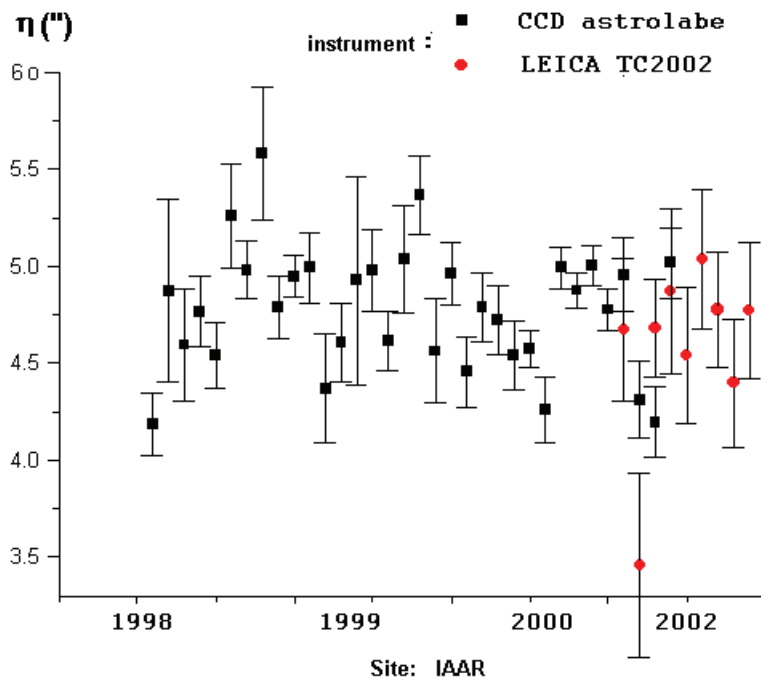


Figure 2: Time variation of  $\eta$  since 1998-2002

#### 4. CONCLUDING REMARKS

- A statistical study concerning the obtained values shows the homogenousness of standard deviations in both cases of  $\xi$  and  $\eta$ , no matter what method was used. That means no significant differences in obtained precisions at different nights of observations. The Barlett test applied on data set shows  $\chi^2 = 0.3576$  ;  $\chi^2$  (P=95%, f=3-1 = 2) = 5.95 for meridian component ( $\xi$ ) and  $c2 = 1.9566$  ;  $c2$  (P=95%, f=3-1 = 2) = 5.95 for prime vertical component ( $\eta$ ).

- The comparison between the two method can be done only at level of data associated dispersions. It consist in verifying the differences (if there exist) between both statistic populations. After applying the F test (small number of determinations) it results a significant difference. First method is more precise, as we already attempted. On the other hand, a small number of determinations can't be characteristic and we hope to continue the series of observations.

- None of applied statistic tests confirmed the presence of factors acting sistematicly upon the results

- The mean external accuracies ( $1\sigma$ ) for the components of the deflection in latitude and longitude are estimated to be ;  $0''.2$ , and about  $0''.4$  in case of theodolite observations. The results seems to be reasonable, comparable with those obtained in Netherland ( ;  $0''.3$ ), Italy, and Swiss.

#### 5. REFERENCES

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