MARTIAN ROTATION INFLUENCE ON ECCENTRIC TRAJECTORIES OF ORBITERS

V. MIOC and M. STAVINSCHI
Astronomical Institute of the Romanian Academy
Str. Cuţitul de Argint 5, RO-752121 Bucharest, Romania
e-mail: vmioc@aira.astro.ro, magda@aira.astro.ro

ABSTRACT. The modelling of perturbing effects is of particular importance in the theoretical study of a spacecraft motion, in view of a better knowledge of the future trajectory.

This paper deals with the influence of Mars' rotation on the dynamics of an orbiter via the rotation and oblateness of its atmosphere (considered separately from the general atmospheric drag). For the density distribution, we have adopted the nominal density profile proposed by Sehnal, valid within the height range $100-1000$ km.

Starting from the Newton-Euler equations, we have estimated analytically the variations of the orbital elements over one nodal period. These changes were determined to first order in the magnitude of the perturbing factors and to second order in eccentricity.

1. INTRODUCTION

While studying theoretically the motion of a spacecraft, of first importance is the modelling of as much perturbing effects as possible. Better known the influence of the various perturbing factors is, and better known the future trajectory of the cosmic vehicle will be.

Among the effects that act on the motion of a planetary orbiter, the planet's rotation was less studied. Its influence can be tackled from many standpoints, considering for instance: the even zonal harmonics of the planetary gravitational potential, the relativistic effect of the quadrupole momentum, the Lense-Thirring effect, the atmospheric rotation and/or oblateness (due, obviously, to the planet's rotation).

We approach here the last situation for the concrete case of Mars. The motion of an orbiter in the Martian atmosphere was first studied analytically by Sehnal and Pospišilová (1988). Using the data provided by Moroz et al. (1988), they modelled the density distribution by the law

$$\rho = \exp(a_{j1} + a_{j2}/h), \qquad j = \overline{1,3}, \tag{1}$$

where the numerical value of the density ρ results in kg/m^3 for the altitude h, expressed in km, above Mars' surface. The constants a_{j1}, a_{j2} are separately determined for the minimal ($j = 1$), nominal ($j = 2$) and maximal ($j = 3$) density profiles. In the nominal model, used by us for numerical estimates, Sehnal (1990) gave $a_{21} = -37.936$, $a_{22} = 2376.1$. Expression (1) is valid for the altitude range 100 km $\leq h \leq$ 1000 km.

The quoted papers did not consider the atmospheric rotation and oblateness. Further analytic approaches took into account these effects (e.g., Mioc et al. 1991, 1992), but imbedded in the general atmospheric drag effect, or went deeper in the Martian atmospheric drag problem, but without considering rotation and oblateness (e.g., Mioc and Radu 1991b). In this paper we are interested in the way in which the *separate* influence of the rotation and oblateness of Mars' atmosphere affects the motion of an orbiter. Such a study was performed by Mioc and Stavinschi (2001), but only for initially circular orbits. Here we resume this problem for eccentric orbits, using expansions to second order in eccentricity. We determine the changes of five independent orbital parameters over one nodal period under the following hypotheses:

(i) the atmosphere rotates with the same angular velocity ω_M as Mars, and is oblate (the surfaces of equal density having the same oblateness ε as the planet);

(ii) the initial orbits lies entirely in the height range 100−1000 km above Mars' surface;

(iii) the perturbations are estimated to first order in the magnitude of the perturbing factors and to second order in eccentricity.

2. BASIC EQUATIONS

We start from the Newton-Euler equations written with respect to the argument of latitude (u):

$$\begin{aligned}
p' &= 2(\gamma/\mu)r^3T, \\
\Omega' &= (\gamma/\mu)r^3BN/(pD), \\
i' &= (\gamma/\mu)r^3AN/p, \\
q' &= (\gamma/\mu)\{r^3kBCN/(pD) + r^2T[r(q+A)/p + A] + r^2BR\}, \\
k' &= (\gamma/\mu)\{-r^3qBCN/(pD) + r^2T[r(k+B)/p + B] - r^2AR\}, \\
t' &= \gamma r^2/\sqrt{\mu p},
\end{aligned} \tag{2}$$

where $' = d/du$, p = semilatus rectum, Ω = longitude of the ascending node, i = inclination, r = planetocentric radius vector, μ = Mars' gravitational parameter, $q = e\cos\omega$, $k = e\sin\omega$ (e = eccentricity, ω = argument of periastron), $(A,B) = (\cos,\sin)u$, $(C,D) = (\cos,\sin)i$, $\gamma = (1 - r^2C\dot{\Omega}/\sqrt{\mu p})^{-1}$, (R,T,N) = radial, transverse, and binormal components of the perturbing acceleration, respectively.

Considering only the influence of Mars' atmospheric rotation (oblateness entailed) on the orbiter motion, and expanding the orbit equation

$$r = p/(1 + Aq + Bk) \tag{3}$$

to second order in q and k (hence in eccentricity), the perturbing acceleration components read (cf. Mioc and Radu 1991a):

$$\begin{aligned}
R &= 0, \\
T &= \rho\delta\sqrt{\mu p}C[1 - ABqk + (1-A^2)q^2/2 + A^2k^2/2]\omega_M, \\
N &= -\rho\delta\sqrt{\mu p}D[1 - ABqk + (1-A^2)q^2/2 + A^2k^2/2]A\omega_M,
\end{aligned} \tag{4}$$

where δ is the drag parameter of the orbiter.

Let us retain the first five equations (2) and consider, as usual, that the perturbations of the orbital elements over one revolution are small, such that the parameters $\{y_i\}_{i=\overline{1,5}} = \{p, \Omega, i, q, k\}$ may be considered constant (and equal to their initial values) in the right-hand side of the motion equations. These ones may then be integrated separately, and the variations of the orbital elements over one nodal period can be deduced from

$$\Delta y_i = \int_0^{2\pi} y' du, \qquad i = \overline{1,5}. \tag{5}$$

The integrals will be estimated by successive approximations, with $\gamma \approx 1$, observing hypothesis (iii).

Replacing (4) in (2), and using the same expansions of r, the integrands in (5) are

$$\begin{aligned}
p' &= \rho p L C[2 - 6Aq - 6Bk + (1 + 11A^2)q^2 + (12 - 11A^2)k^2 + 22ABqk], \\
\Omega' &= -\rho LAB[1 - 3Aq - 3Bk + (1 + 11A^2)q^2/2 + (12 - 11A^2)k^2/2 \\
&\quad +11ABqk], \\
i' &= -\rho LDA^2[1 - 3Aq - 3Bk + (1 + 11A^2)q^2/2 + (12 - 11A^2)k^2/2 \\
&\quad +11ABqk], \\
q' &= \rho LC[2A + (1 - 5A^2)q - 6ABk + 2A(4A^2 - 1)q^2 + A(12 - 11A^2)k^2 \\
&\quad +B(19A^2 - 3)qk], \\
k' &= \rho LC[2B - 4ABq + (5A^2 - 4)k + B(1 + 5A^2)q^2 + 2B(3 - 4A^2)k^2 \\
&\quad +A(10 - 13A^2)qk],
\end{aligned} \tag{6}$$

where we abridged $L = \gamma\delta\omega_M p^{5/2}/\sqrt{\mu}$.

It is clear that equations (6) can be brought to the very concentrated form

$$y_i' = \rho \sum_{t=0}^{4} (G_{it}A^t + H_{it}A^t B), \qquad i = \overline{1,5}, \tag{7}$$

in which the coefficients G_{it}, H_{it} depend on μ, ω_M, δ, and the initial values of p, i, q, k.

3. VARIATIONS OF THE ORBITAL ELEMENTS

Let us now express the density (1) in terms of quantities considered constant over one nodal period, and of u (via A and B). To this end, we use

$$h = r - R_M(1 - \varepsilon \sin^2 \varphi), \tag{8}$$

where R_M = mean equatorial Martian radius, φ = latitude. With $\sin\varphi = DB$, and expanding r and the exponential in (1) to second order in q and k, the expression of the density can be brought to the concentrated form

$$\rho = \sum_{j=0}^{6} (R_j A^j + S_j A^j B), \tag{9}$$

in which the coefficients R_j, S_j depend on a_{21}, a_{22}, ε, R_M, and the initial values of p, i, q, k.

By (7) and (9), equations (5) become

$$\Delta y_i = \int_0^{2\pi} \left[\left(\sum_{j=0}^{6} R_j A^j \right) \left(\sum_{t=0}^{4} G_{it} A^t \right) + \left(\sum_{j=0}^{6} S_j A^j B \right) \left(\sum_{t=0}^{4} H_{it} A^t B \right) \right] du, \qquad i = \overline{1,5}, \tag{10}$$

where we used $\int_0^{2\pi} KA^n B du = 0$ (K = constant, $n \in \mathbf{N}$).

Denoting $U_{ik} = \sum_{j+t=k} R_j G_{it}$, $V_{ik} = \sum_{j+t=k} S_j H_{it}$, $j = \overline{0,6}$, $k = \overline{0,10}$, $X_{ik} = U_{ik} + V_{ik} - V_{i,k-2}$, $k = \overline{0,12}$, $i = \overline{1,5}$ (where we have artificially introduced $U_{i,11} = U_{i,12} = V_{i,11} = V_{i,12} = V_{i,-2} = V_{i,-1} = 0$), performing the calculations in the integrand of (10), then performing the integrations, we obtain the expressions of the variations of the orbital elements over one nodal period in the general form

$$\Delta y_i = \pi \left(2X_{i0} + X_{i2} + \frac{3}{4}X_{i4} + \frac{5}{8}X_{i6} + \frac{35}{64}X_{i8} + \frac{63}{128}X_{i,10} + \frac{429}{1024}X_{i,12} \right). \tag{11}$$

Of course, this expression can be particularized to each considered orbital element, by assigning the corresponding numerical value to the index i.

4. CONCLUDING REMARKS

4.1. Integrating the motion equations (6), we have obtained analytical expressions (with a second-order accuracy in eccentricity) for the variations of five orbital parameters caused by Mars' rotation over one nodal period of the orbiter. Expressions for such changes can be obtained for any other orbital element (semimajor axis, eccentricity, argument of periastron, etc.).

4.2. The choice of u as independent variable allows the study of the perturbations for very low eccentric orbits (even circular). This becomes impossible if one of the anomalies is taken as independent variable (the anomalistic period being considered as the basic time interval).

4.3. The final formulae (11) can serve as a departure point for the study of the evolution of such elements over large time intervals, via either averaging-type methods or numerical integration.

4.4. Integrating (5) between u_0 (initial) and u (current), instead of 0 and 2π, the results can subsequently be used to determine the perturbations of the nodal period itself.

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