POSITIONING THE TERRESTRIAL EPHEMERIS ORIGIN IN THE INTERNATIONAL TERRESTRIAL REFERENCE FRAME

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ABSTRACT. Resolution B1.8 adopted by the XXIV General Assembly of the International Astronomical Union (Manchester, august 2000) recommends the use of the Non-Rotating Origin (Guinot 1979) on the moving equator both in the International Celestial Reference System (ICRS) and in the International Terrestrial Reference System (ITRS). The Non-Rotating Origin (NRO) in the ITRS is designated the Terrestrial Ephemeris Origin (TEO). Resolution B1.8 is to be implemented on 1 January 2003 by the International Earth Rotation Service (IERS) which is required to provide the position of the TEO in the ITRS. This paper is devoted to the calculation of this position. Because the TEO depends on the polar motion, which is poorly modeled, its position has been derived from observational data. This has been compared to an analytical model based on a simplified representation of polar motion. We propose then a numerical expression for the displacement of the TEO to be used in the new coordinate transformation from the Celestial Reference System to the Terrestrial Reference System according to the resolution.

1. INTRODUCTION

The study has been realized in the framework of the resolutions adopted by the XXIV General Assembly of the International Astronomical Union (Manchester, August 2000) concerning the transformation between the Celestial Reference System (CRS) and the Terrestrial Reference System (TRS), which are to be implemented in the International Earth Rotation Service (IERS) procedures on 1 January 2003.

Resolution B1.8 recommends the use of the Non-Rotating Origin (NRO) on the moving equator to reckon the angle of rotation of the Earth. The NRO (also called departure point) in the Celestial Reference System (CRS) is defined by the kinematical condition of non-rotation of this point around the rotation axis when the rotation pole moves in the CRS (Guinot 1979). Similarly, the NRO in the Terrestrial Reference System (TRS) is defined by the kinematical condition of non-rotation around the rotation axis when the rotation pole moves in the TRS. The properties of these two points have already been studied in Capitaine *et al.* (1986 and 2000).

In practice, the determination of the Earth orientation refers to the Celestial Intermediate Pole (CIP), defined by IAU 2000 resolution B1.7, which is the pole correponding to an axis closed to the rotation axis (difference smaller than 20 mas). Therefore :

• The NRO as realized by taking into account the motion of the CIP in the Celestial Reference

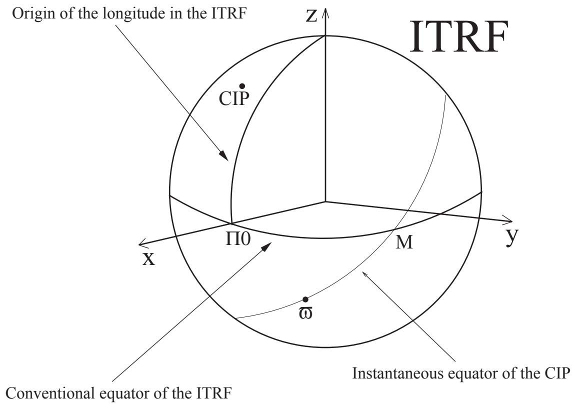


Figure 1: The Non-Rotating Origin with respect to the ITRF.

System (CRS) gives a point on the moving equator of date noted σ and designated by the Celestial Ephemeris Origin (CEO).

• The NRO as realized by taking into account the motion of the CIP in the Terrestrial Reference System (TRS) gives a point on the moving equator of date noted ϖ and designated as the Terrestrial Ephemeris Origin (TEO).

Astrometric and geodetic data analysis software involve the computation of the transformation between the CRS and the TRS. In the framework of the IAU 2000 recommendations, they need an expression for the displacement of the Terrestrial Ephemeris Origin both for the classical transformation and for transformation based on the Non-Rotating Origin representation.

The study is focused on the computation of the position of the TEO in the ITRS.

2. FORMULATION

Consider a displacement of the CIP between dates t_0 and t, the point ϖ along the moving equator defines the broken arc s' (see Figure 1) :

$$s' = \varpi M - \Pi_0 M - (\varpi_0 M_0 - \Pi_0 M_0). \tag{1}$$

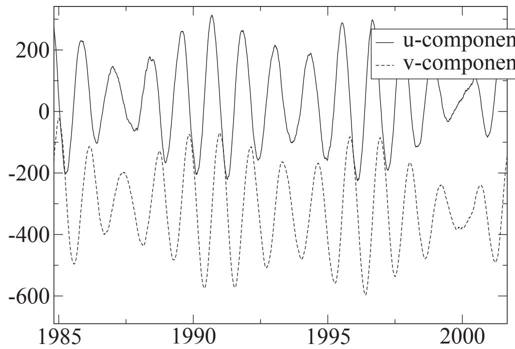
In this definition, ϖ and ϖ_0 represent the position of the departure point at dates t and t_0, Π_0 is the origin of the longitude in the ITRF, and M and M_0 are the nodes between the conventional equator of the ITRF and the equators of dates t and t_0. Because of the fact that the kinematical condition does not constrain the position of ϖ_0 along the moving equator, it is convenient to take, by convention (Capitaine *et al.* 1986) :

$$\varpi_0 M_0 - \Pi_0 M_0 = 0. \tag{2}$$

The displacement of the TEO in the ITRS, s', is related, for a date t, to the CIP coordinates u and v in the TRS by the formula :

$$\begin{aligned} s' &= \varpi M - \Pi_0 M \\ &= -\int_{t_0}^{t} \frac{u\dot{v} - \dot{u}v}{2} \mathrm{d}t, \end{aligned} \tag{3}$$

where u corresponds to the coordinate x_p of the CIP in the ITRS, v refers to $-y_p$, and the dot is for the time derivative. The integration is made between dates t_0 and t.



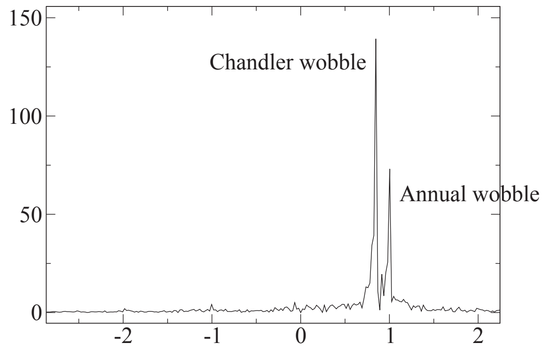


Figure 2: Left side : bserved polar motion during the last two decades from the IERS C04 series, amplitudes are in mas, u-component (x_p) is in full line, v-component ($-y_p$) is in dotted line, the time is given in years. Right side : complex spectrum of polar motion (Fast Fourier Transform) over the total duration of the IERS C04 series (1962-2002), amplitudes are in mas, frequencies are in cycles per year (cpy).

3. POSITIONING FROM GEODETIC DATA

The computation of s' is based upon combined time series C04 for polar motion which is provided by the Earth Orientation Parameters Product Center of the International Earth Rotation Service (IERS EOP-PC) based at the Paris Observatory, France. Such a series is obtained by the combination of individual polar motion series from various spatial and geodetic techniques such as Very Long Baseline radio Interferometry (VLBI), Global Positioning System (GPS, GLONASS), Satellite Laser Ranging (SLR) or Lunar Laser Ranging (LLR). This series give daily averaged values for the CIP coordinates x_p, y_p, the values for $UT1-UTC$ and length-of-day LOD, and the Celestial pole offsets $d\psi$ and $d\epsilon$ which are the corrections to the nutation model IAU 1980. For a sampling of one day (at 0h UTC) the data covers the period from 1962 january 1st until 2002 (see Figure 2).

Let $u = x_p$ and $v = -y_p$ be the CIP coordinates in the ITRS. From time series of u and v, we can compute numerically the quantity s' given by equation (3). The result of the computation is plotted in Figure 3.

We notice that the curve of s' shows a periodic variation with period 2334.4 days with an amplitude that reaches 1 μas and which is also present in the polar motion (see Figure 2). Smaller oscillations with Chandlerian and annual periods are also visible. The trend of the curve is varies during the 40 years depending on the variations of the amplitudes of the chandlerian and annual wobbles. It is a well known fact that the Chandler wobble has undergone some drastic variations in amplitude and phase during the twentieth century. For details see (Guinot 1972) and (Vondrák 1985).

4. POSITIONING BASED ON MODEL FOR POLAR MOTION

It is well known that the polar motion, at low frequencies (periods longer than a few days), is

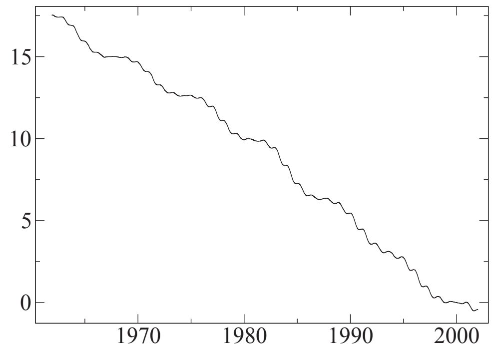


Figure 3: Displacement s' of the Terrestrial Ephemeris Origin over the total duration of the IERS C04 series (1962-2002), the time is given in years, amplitudes are in μas.

mostly a combination of the Chandler wobble, with a period of about 433 days (Lambeck 1988), and the annual oscillation. The combination of these two oscillations causes a modulation of 2334.4 days (about 6.4 years) which is clearly visible in Figure 2. In addition, we observe a linear trend in both coordinates.

In this section, we assume that the observed polar motion is well represented by two prograde circular waves at the above periods and by a linear trend :

$$\begin{aligned} u &= A_c \cos(\sigma_c t + \phi_c) + A_a \cos(\sigma_a t + \phi_a) + u_0 + u_1 t, \\ v &= A_c \sin(\sigma_c t + \phi_c) + A_a \sin(\sigma_a t + \phi_a) + v_0 + v_1 t, \end{aligned} \tag{4}$$

where A_c and A_a are the real amplitudes of the Chandler and annual wobbles respectively and are considered as constant over the span of the series, σ_c and σ_a are the Chandler and annual frequencies, with the respective phases ϕ_c and ϕ_a refered to epoch J2000.0. The non-periodic terms are represented by u_0, v_0, u_1 and v_1. After an analytical integration, we obtain a model for s' in the form :

$$s'(t) = L(t) + B(t) + P(t), \tag{5}$$

where the first term is the linear trend

$$L(t) = -\frac{1}{2}\left[\sigma_c A_c^2 + \sigma_a A_a^2 + v_1 u_0 - u_1 v_0\right] t. \tag{6}$$

The second term contains periodic terms of period 2334.4 days :

$$B(t) = -\frac{1}{2}\frac{\sigma_c + \sigma_a}{\sigma_c - \sigma_a} A_c A_a \sin((\sigma_c - \sigma_a)t + (\phi_c - \phi_a)). \tag{7}$$

The last term is less significant and more complex. It contains Chandler and annual periods and combined terms $t \times \sin$ or $t \times \cos$ (see Lambert and Bizouard 2002).

As we mentionned above, Chandlerian and annual wobbles amplitudes have known variations. To illustrate these variations, least-squares fits of Chandler, annual and linear terms over three windows spanning twenty years were performed. Then we have computed the amplitudes of the main terms expressed in above equations, L and B, for three periods of 20 years. Results are displayed in Table 1. The periodic term $B(t)$ is, with these values, below the desirable accuracy,

Table 1: Values of the main terms of s' as presented in equations (1), (6) and (20) for three different periods and different amplitudes of the Chandler wobble A_c and the annual wobble A_a.

	1962-1982	1972-1992	1982-2002
A_c mas	140	160	178
A_a mas	92	87	80
L μas pjc	-38	-44	-51
B μas	0.4	0.4	0.4

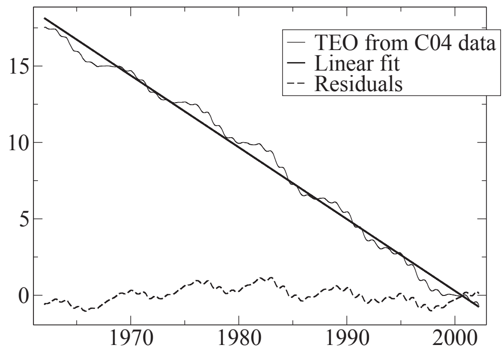


Figure 4: Displacement of the Terrestrial Ephemeris Origin s' over the total duration of the IERS C04 series (1962-2000) and the model given by expression 9. The time is given in years, displacements are in μas.

and should be neglected. At 1 μas accuracy over the last 40 years, the expression for s' is reduced to a linear form :

$$s'(t) - s'(t_0) = -\frac{1}{2}\left[\sigma_c A_c^2 + \sigma_a A_a^2\right] t. \tag{8}$$

The diurnal and subdiurnal tidal variations for polar motion provided by Ray (McCarthy 1996 and Ray 1994) should be added for a more rigourous study due to possible effects from their very short periods in the time integration. These high-frequency contributions are very small compared to the amplitude of the long periodic polar motion : they can reach a maximum of 0.5 mas. The effect is only about 0.06 μas per century and is therefore negligible (Lambert and Bizouard 2002).

5. EXPRESSION FOR POSITIONING THE TEO IN THE ITRF

As a conclusion of this study, we propose a numerical expression for the quantity s'. Although such a model is limited in time because of unpredictable changes in the wobbles amplitudes, it is possible to provide a numerical expression for s' which represents the motion of the TEO in the ITRS with an accuracy of 1 μas over the last 40 years. On this span, variations in the amplitudes of the wobbles will not produce an error larger than 1 μas. The following expression is fitted on the curve obtained from C04 data :

$$s' = -47.0 \times t, \tag{9}$$

where s' is in μas. The parameter t is the Terrestrial Time (TT) expressed in julian centuries from epoch J2000.0 :

$$t = (\mathrm{TT} - 2000 \text{ january 1d 12h TT})/36525, \tag{10}$$

with TT in days.

If the Chandler amplitude in the next years does not present variations larger than during the span 1962-2002, then according to Table 1, the uncertainty on the linear trend is 13 μas per julian century. We can ensure that the 1 μas accuracy linear model can be extended for the 10 next years.

Figure 4 shows the linear model of s' corresponding to expression (9) together with the numerical computation from observations. If unexpected variations in the amplitudes of the wobbles occur in the coming years, such a model will have to be updated.

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