PRECISE ANALYSIS OF EOP SERIES: AN ATTEMPT TO DISTINGUISH CHAOTIC AND NON-STATIONARY PROCESSES.

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ABSTRACT. Recently some authors applying independent mathematical techniques - Lyapunov exponents and Volterra-Wiener-Korenberg (VWK) predictor - have detected chaotic properties in EOP and AAM series. Using extreme VWK robustness and sensitivity we processed short (less than 1 per cent data length fitted for the classic Lyapunov method) and variable EOP and AAM series to perform time decomposition of chaotic signal. Method validation and verification are discussed also.

Last two years were reported about results of new approach to Earth Orientation Parameters (EOP) series analysis [1], [2],[3]. The new interpretation of EOP data with nonlinear dynamics techniques as a chaotic values had a definite success. But usually applied methods such as Lyapunov exponents and correlation integral having strong restriction - data length can not be shorter than 10000 points, and weak sensibility gave only definite confidence results for limited number of series. Two years ago we applied to EOP series recently developed in system analysis Volterra-Wiener-Korenberg (VWK) method further study of which showed its unic properties [4]. VWK provided reliable detection of chaotic signal in contaminated by noise and distorted by other processes data. But the main property that makes it most powerful for the real series analysis is the possibility to process small number of points data. For its mathematical foundations please refer to [5], [6].

In chaotic data analysis VWK works as one-step-ahead predictor that consider a system as a close-loop version of black box when output y_n feeds back as delayed input . Discrete Volterra-Wiener-Korenberg form of degree d and memory k is constructed to calculate the predicted time series y_n^{calc}:

$$\begin{aligned} y_n^{calc} = \quad & a_0 + a_1 y_{n-1} + a_2 y_{n-2} + \ldots + a_k y_{n-k} + \\ & a_{k+1} y_{n-1}^2 + \ldots + a_{k+2} y_{n-1} y_{n-2} + a_{M-1} y_{n-k}^d \end{aligned} \tag{1}$$

where all distinct combinations of $(y_{n-1}, y_{n-2}, \ldots, y_{n-k})$ up to degree d is composed.

Prognosis quality are estimated as normalized error squared

$$\varepsilon^2(k,d) = \sum_{n-1}^{N} (y_n^{calc}(k,d) - y_n)^2 / \sum_{n-1}^{N} (y_n - \bar{y})^2$$

Predictions are performed until optimal degree d and memory k minimizes information criterium C(r)

$$C(r) = \log \varepsilon(r) + r/N \tag{2}$$

where r- number of polynomial terms of truncated Volterra expansion for a certain pair {k,d}.

Srandard Fisher criterium serves to reject the hypothesis that nonlinear model is not better that linear one as one-step-ahead predictor. For the simulated chaotic system it is easy to choose what model is better due to the large difference - some orders of value, between them. Another situation arises when real EOP data are processed , because both curves becomes fluctuative, not so distant one from other and their prediction level is much closer to zero . Fisher criterium vary over the signifince level, but conclusion about models superiority becomes marginal [4] .

To find the origin of distortions we performed a study of method constraints and tested it on large diversity of simulated models as chaotic and stochastic ones. First of all we looked for shortest data length the method can reliably process. As it was occured VWK do not have practically such a constraint - even about 20-30 points can be enough if the system is more or less simple. If so, then what about reliability of Fisher criterium on such short time span? Most crucial moment here is the whitness of prognosis residuals. To verify it one used Pirson criterium compared with carefully simulated white noise signal . Realibility was confirmed with full confidence.

Tne next step - simulation stochastic signals . We applyed theory of probability theorem as generator of arbitrary complex stochastic signal x(r).If random value r has constant distribution on the [0,1] span then the relation between it and random value x(r) with dencity function f(t) is:

$$r = \int_{-\infty}^{x(r)} f(t)dt \tag{3}$$

Solving the function equation one can provide process x(r) stationary or not depending on are the Gaussian function f(t) has stationary or not parameters. An analysis of large diversity of series generated such a way displayed that prediction power of both linear and nonolinear models are low and practically equal one to another but at the same time much more fluctuative.

Last step - series generation which contain strong harmonics contaminated by noise. Here we found main restriction of the method - it fails when beating processes are in data, may be due to their proportionality to linear correlation function involved in computing of Volterra form coefficients.

So after short summary of above mentioned results it was chosen the optimal way for Volterra-Wiener-Korenberg method application. First of all - to return to tradional nonlinear analysis processing -preliminary data filtration and work in spectral bands . And second - to perform VWK on short time span of about hundred points which moves along the series. In fact we provide such a way temporal evolution of chaotic signal in the spectral domain.For the analysis we have taken the same frequency bands for all series: low frequency band with periods T more than 2 years, high frequency band - seasonal oscillations with T less than 100 days, and Chandler band - 2 months wide near 1.2 year value. After that innovation the analysis results changed drastically.

IPMS polar motion leaves no doubts in chaotic behaviour in all frequency domains. The picture is held for EOP C01 series to the exclusion of low frequency band . From our point of view chaotic signal disappearance here could happen due to the plate motion exclusion from the series. The hypothsis must be verified in the future investigation.

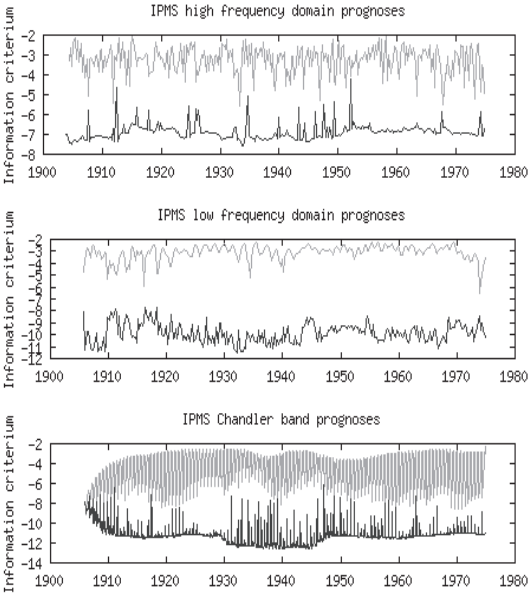

Figure 1: IPMS polar motion prediction by VWK . Frequency bands from up downward:high,low,Chandler. Everywhere polar motion is recognized as confidently chaotic. Vertical axes scale is logarithmical so results presented here about 100 times more confident then in [4]

SPACE 2002 polar motion, LOD and Atmospheric Angular Momentum (AAM) x,y preassure terms showed chaoticity with big confidence also. Fisher criterium was 1000 times stronger than theoretical value for the level 99 % for all series.

CONCLUSIONS

1. Volterra-Wiener-Korenberg is the first technique that enabled to compute temporal evolution of chaotic processes with good resolution.

2. The analysis results in definite spectral bands assures that polar motion, LOD and some components of AAM are chaotic values.

3. Disapearance of low frequency chaotic signal in EOP C01 and SPACE 2002 series were happened may be due to plate motion exclusion during data preprocessing.

4. To be the explicit parameter chaoticity of polar motion, LOD and some AAM components must be approved by careful investigation of possible perturbations which mathematical preprocessing procedures can do in chaotical series. Classical astrometry and new technique observations are not direct mesuarements of EOP. Polar motion, length of day and AAM components are the result of big number mathematical procedures and reduction application. And as preliminary study revealed [4] some of them inevitably distort chaotic values

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