# POLAR MOTION PREDICTION BY DIFFERENT METHODS IN POLAR COORDINATES SYSTEM

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#### 1. INTRODUCTION

Prediction errors for a few days in the future of the pole coordinate data determined from the new space techniques is several times greater than their determination errors, which are of the order of 0.1 mas. The current prediction method of polar motion data carried out in the IERS Rapid Service/Prediction Center is the least-squares extrapolation of a Chandler circle, annual and semiannual ellipses and a bias fit to the last 1 year of the combined pole coordinate data (McCarthy and Luzum 1991, IERS 2000). Previously, the length of polar motion data from which this extrapolation model was computed was equal to three years however this increase caused an increase of the mean polar motion prediction errors especially during the time of El Niño events (Kosek et al. 2001a,b). Any improvements made to the polar motion forecast using the autocovariance prediction procedures (Kosek et al. 1998, 2000) were not effective especially in the time of the El Niño event in 1997/98. In this paper, the autocovariance and least-squares prediction were applied to the pole coordinate data transformed into polar motion radius and angular distance (Kosek 2002).

### 2. DATA

The analysis used the USNO pole coordinate data in the years 1973.0 to 2002.7 with a sampling interval of 1 day (USNO 2002), the IERS EOPC01 pole coordinate data in the years 1846.0 to 2000.0 with the sampling interval of 0.05 years and the IERS EOPC04 pole coordinate data in the years 1962.0 to 2002.7 with the sampling interval of 1 day (IERS 2002). Additionally, the monthly sea surface temperature anomalies Nino 1+2 and Nino 4 in the years 1976.0 to 2002.7 from the Climate Prediction Center (NOAA 2002) were used.

## 3. THE IERS LEAST SQUARES PREDICTION ERRORS

The distances between polar motion data and their least-squares predictions at different starting prediction epochs from 1 to 50 days in the future are shown in Figure 1. The polar motion data from which the extrapolation model of the Chandler circle, annual and semiannual ellipses and a bias was computed was equal to one and three years. The increase of the length of polar motion data going into the least-squares extrapolation model increases the polar prediction errors (Kosek et al. 2001b). The reasons for these increased errors are irregular short period oscillations of pole coordinate data (Kosek and Kolaczek 1995, Kosek 2000) as well as the phase and amplitude variations of the annual oscillation (Kosek et. al. 2000, 2001a,b). The amplitude

and phase variations of the Chandler and annual oscillations computed by the least-squares method in one year and three year time intervals for the USNO x pole coordinate data are shown in Figure 2. The amplitude variations of these oscillations computed from the y pole coordinate are very similar (Kosek et. al. 2001a,b).

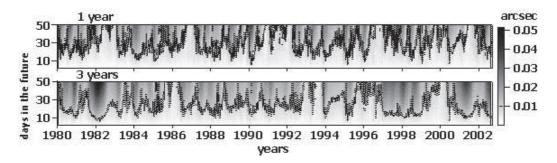


Figure 1: The distances between polar motion data and their least-squares predictions computed at different starting prediction epochs for the time span of polar motion data going into the least-squares extrapolation model equal to one and three years (contour lines at 0.01 arcsec).

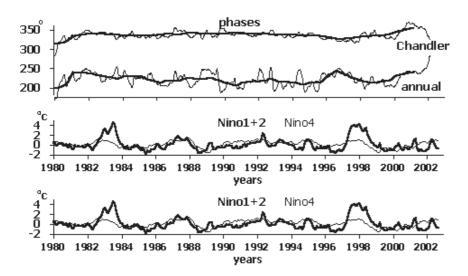


Figure 2: The amplitude and phase variations of the Chandler and annual oscillations computed by the least-squares method in the one (thin line) and three (heavy line) year time intervals and the Nino 1+2 (heavy line) and Nino 4 (thin line) data.

The amplitudes and phases of the Chandler and annual oscillations become smoother when the interval of data going into the least-squares extrapolation model becomes longer. The phases computed by the least-squares using the three year intervals are smoother for the Chandler than the annual oscillation. This means that poor accuracy of the least-squares polar motion predictions are caused by irregular variations of the annual oscillation phases. The two biggest maxima of the annual oscillation phase and amplitude preceded the two biggest El Niño events in 1982/83 and 1997/98 by about 1.5 and 0.5 years, respectively. The increase of the phase and amplitude of the annual oscillation in the year 2000 suggests that another El Niño is expected in the end of this year. The Nino 4 index corresponding to the sea surface temperature difference in the central Pacific has already begun to increase.

# 4. TRANSORMATION OF POLE COORDINATE DATA BETWEEN CARTESIAN AND POLAR COORDINATE SYSTEM

In order to transform pole coordinate data from the Cartesian to the polar coordinate system in which the polar radius is stationary we must refer the pole coordinate data to the mean pole positions. The radius and angular distance are computed by the following formulae:

$$R_t = \sqrt{(x_t - x_t^m)^2 + (y_t - y_t^m)^2}, \quad t = 1, 2, ..., n$$

$$L_t = \sqrt{(x_t - x_{t-1})^2 + (y_t - y_{t-1})^2}, \quad t = 1, 2, ..., n$$
(2)

$$L_t = \sqrt{(x_t - x_{t-1})^2 + (y_t - y_{t-1})^2}, \quad t = 1, 2, ..., n$$
 (2)

where:  $x_t, y_t$  are the pole coordinates data and  $x_t^m, y_t^m$  are the mean pole coordinates data.

The transformation from the polar into the Cartesian coordinate system is carried out when the first predictions of the radius  $R_{n+1}$  and angular distance  $L_{n+1}$  are known. Time-frequency amplitude spectra computed by the Fourier transform band pass filter (FTBPF) of the complexvalued pole coordinate data show that the amplitudes of oscillations with positive periods are bigger than the amplitudes of oscillations with the negative ones which indicates that oscillations in polar motion are mostly counterclockwise (Kosek 1995). Assuming, that polar motion is counterclockwise, the coordinates of the first prediction point are computed by the linear intersection formulae (Fig. 3):

$$\left\{ \begin{array}{c} x_{n+1} \\ y_{n+1} \end{array} \right\} = \frac{\left\{ \begin{array}{c} x_n \\ y_n \end{array} \right\} \left( \begin{array}{c} \frac{R_{n+1}^2 + R_n^2 - L_{n+1}^2}{4P} \end{array} \right) + \left\{ \begin{array}{c} -y_n \\ x_n \end{array} \right\} + \left\{ \begin{array}{c} x_n^m \\ y_n^m \end{array} \right\} \left( \begin{array}{c} \frac{R_{n+1}^2 + R_n^2 - R_{n+1}^2}{4P} \end{array} \right) + \left\{ \begin{array}{c} y_n^m \\ -x_n^m \end{array} \right\}}{2R_{n+1}^2 + 2R_n^2 - L_{n+1}^2 - R_{n+1}^2/4P} \tag{3}$$

where P is the area of the triangle shown in Figure 3.

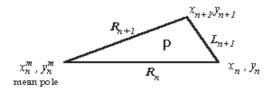


Figure 3: Linear intersection - computation of the first prediction point from the prediction of the radius and angular distance

## 5. PREDICTIONS OF THE MEAN POLE, RADIUS AND ANGULAR DISTANCE

In order to compute predictions of the radius and angular distance the autocovariance (Kosek 1993, 1997) and least-squares prediction methods were applied. It has been shown that the autocovariance prediction of the model pole coordinate data similar to the observed polar motion data does not predict these data as accurately as the forecast computed from the predictions of the radius and angular distance (Kosek 2002).

One of the problems in polar motion prediction through transformation of pole coordinates into a polar coordinate system is the determination of the mean pole and its prediction. The mean pole coordinate data were computed by the Ormsby (Ormsby 1961) and Butterworth (Otnes and Enochson 1972) low pass filters (LPF) with the cutoff period of 18 and 7 years, respectively. The Ormsby LPF cuts off the beginning and the end of time series outputs due to filter length by 3 years, so in order to have the mean pole positions at the end of time series they must be predicted. Three-year least-squares predictions of the mean pole computed at different starting prediction epochs are shown in Figure 4. The agreement between the mean pole coordinates and their predictions is good and of the order of the differences between outputs of the different LPFs. The systematic difference between the mean pole and its prediction will produce an oscillation with a period approximately equal to one year in the computed polar motion radius (eq. 1). The radius and angular distance computed from the EOPC01 extended by the EOPC04 pole coordinate data are shown in Figure 5. The reason for longer period variations in the polar motion radius is the variable amplitude of the Chandler oscillation (Schuh et al. 2001). Time-frequency amplitude spectra computed by the FTBPF of the polar motion radius and angular distance show that the beat period of the Chandler and annual oscillations is not constant. Since 1960s the oscillation with a period equal to about 3 years can be seen as a beat period between the semiannual and semi-Chandler oscillations (Kosek and Kolaczek 1997).

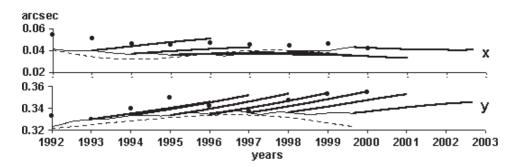


Figure 4: Three-year least-squares predictions computed at different starting prediction epochs (heavy line) of the mean pole computed by the Ormsby LPF (thin line) and the mean pole computed by the Butterworth LPF (dashed line) and by the IERS (dots) (IERS 2002).

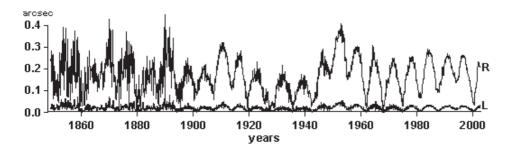


Figure 5: The radius and angular distance computed from the IERS C01 pole coordinate data.

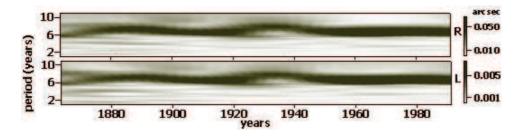


Figure 6: Time-frequency FTBPF amplitude spectra of the polar motion radius and angular distance.

The beat period of the Chandler and annual oscillations computed from the minima and maxima of the polar motion radius and angular distance is shown in Figure 7. The beat period of the Chandler and annual oscillation can be also computed from the least-square phase variations  $\Delta \varphi$  of the Chandler and annual oscillations shown in Figure 2. The change of the period  $\Delta T$  of the Chandler or annual oscillations can be computed from the change of the their least-squares phase variations  $\Delta \varphi$  according to the formula:

$$2t\pi/T + \varphi + \Delta\varphi = 2t\pi/(T + \Delta T) + \varphi = const \tag{4}$$

The beat period of the Chandler and annual oscillation can be computed from variable periods of these oscillations according to the following formula:

$$1/T_{beat} = 1/(T_{An} + \Delta T_{An}) - 1/(T_{Ch} + \Delta T_{Ch})$$
(5)

Since the Chandler phase is not fixed in time a robust method (Priestley 1981) was applied to eliminate its drift before the change of the Chandler period  $\Delta T_{ch}$  was computed.

One-year autocovarince and least-squares predictions of polar motion radius and angular distance computed at different starting prediction epochs agree well with the future data (Fig. 7). In the least-squares prediction the beat period was equal to 6.1 years in the extrapolation model and this model shows a good agreement with the future data only from 1995 to 1998. Outside this time interval, this beat period does not have an appropriate value in the extrapolation model.

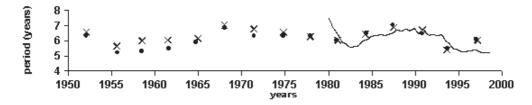


Figure 7: The beat period of the Chandler and annual oscillations computed from the smoothed minima and maxima of the polar motion radius (crosses) and angular distance (dots) and computed from the least-squares phase variations  $\Delta \varphi$  of the Chandler and annual oscillations

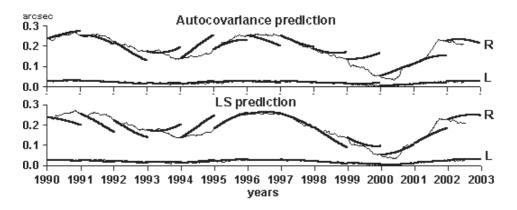


Figure 8: One-year autocovarince and least-squares predictions (heavy lines) of polar motion radius and angular distance (thin lines) computed at different starting prediction epochs.

The distances between the pole coordinate data and their autocovariance predictions computed at different starting prediction epochs are shown in Figure 9. The prediction errors depend

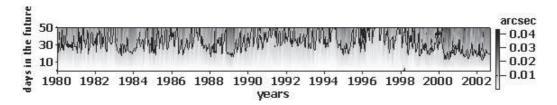


Figure 9: The distances between polar motion data and their autocovariance predictions computed at different starting prediction epochs (contour line at 0.01 arcsec).

on starting prediction epochs but they are not as big as the predictions carried out by the IERS Rapid Service/Prediction Center (Fig. 1) especially for short prediction times.

### 5. CONCLUSIONS

Polar motion least-squares prediction errors depend on irregular phase and amplitude variations of the annual oscillation that had maximum values before the El Niño events in 1982/83 and 1997/98. The increase of the annual oscillation phase and amplitude in 2000 indicates that another El Niño is expected in the end of this year.

Transformation of pole coordinate data from the Cartesian to a polar coordinate system transforms the Chandler and annual as well as the semi-Chandler and the semiannual frequencies into their beat frequencies which helps to solve the frequency resolution problems. Accuracy of polar motion prediction by the method of autocovariance through the transformation to polar coordinate system depends on predicting accurately the mean pole, radius and angular distance. The period of the most energetic oscillation in polar motion radius and angular distance representing the beat period of the Chandler and annual oscillations is variable mainly due to variable phase or period of the annual oscillation.

The error of the autocovariance prediction for a few days in the future is less than the prediction error of the current polar motion forecast carried out by the IERS Rapid Service/Prediction Center.

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