THE LUNAR THEORY ELP2000 REVISITED

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ABSTRACT. The construction of the complete lunar theory ELP goes back to the 1980s. Among all the components which form the solution, planetary perturbations contribute mainly to its deficiency. A new solution has been build that makes use of the planetary perturbations (MPP01) constructed recently by (Bidart 2000). After several transformations and tests, this new solution is called ELP/MPP02.
Fitting the constants and the reference frame, ELP/MPP02 has been extensively compared to various JPL ephemerides and mainly DE405/DE406 (Standish 1998) to test its accuracy on a short time interval of one century and on a long time interval covering several millennia. Compared to ELP, over a few centuries, a significant improvement of the precision - in particular on the radius vector - is put in evidence. On the long range, the planetary contributions with long periods are noticeably improved.
Taking advantage of the partials included in ELP, this new solution is fit directly to LLR observations. Our future ephemerides shall be based on this contribution using our LLR fits.

1. THE SOLUTION ELP

ELP is a semi-analytical solution for the orbital motion of the Moon. Its construction, under a complete form, containing all sensible perturbations, goes back to the 1980s. It is named ELP2000-82 (Chapront-Touzé, Chapront 1983). The main components of the solution includes:
• *The Main Problem.* It represents the motion of Earth, Moon and Sun where the Earth-Moon barycenter EMB is moving along a keplerian orbit; it includes partials with respect to various lunar and planetary parameters which are used when fitting to observations.
• *The Earth's figure perturbations* including the nutational motion of the Earth.
• *The direct and indirect planetary perturbations.* The direct perturbations are due to the action of the planets on the Earth; indirect perturbations are induced by the deviation of EMB from a keplerian orbit. Planetary perturbations contain in particular the secular motions of EMB (eccentricity and perihelion) and of the ecliptic. The motions of the planets come from VSOP82 (Bretagnon 1982).
• *The relativistic effects.*
• *The tidal perturbations.*
• *The Moon's figure perturbations and coupling with libration.*
The first version of ELP has not been fit directly to observations but via the JPL lunar ephemeris DE200-LE200. VSOP82 used the same source of comparisons and fits. A brief description of the various versions of ELP and derived ephemerides is given in Table 1.

Table 1. The different versions of ELP

Version	*Date*	*Fit*	*Characteristics*
ELP2000-82	1983	DE200	Planetary motions: VSOP82 (Bretagnon 1982)
ELP2000-85	1988	DE200	Secular motions of high degree n in time: t^n
ELP2000-82B	1996	DE245	Improved masses, gravitational parameters, tides etc...
ELP2000-96	1997	LLR	Numerical complements ρ_{245}
Lunar librations			Moon's lunar libration theory completed
ELP/MPP02	2002	LLR	Planetary motions VSOP2000 (Moisson 2000)

Before the present solution, our last version was ELP2000-96 obtained by adding numerical complements to ELP on the basis of the JPL ephemeris DE245; a complete analysis of Lunar Laser Ranging observations from 1972 till 1998 has been performed using this solution and the analytically completed Moon's theory of the lunar libration (Chapront et al. 1999).
ELP provides the polar coordinates σ (longitude V, latitude U and distance r) under the general formulation:

$$\sigma = \sum_{n\geq 0} t^n \sum_{i_1,i_2,\ldots,i_p} A^{(n)}_{i_1,i_2,\ldots,i_p} \times \sin\left(i_1\lambda_1 + i_2\lambda_2 + \ldots + i_p\lambda_p + \phi^{(n)}_{i_1,i_2,\ldots,i_p}\right) \tag{1}$$

In the case of V one has to add to the previous formula the secular mean longitude $w_1 = w_1^{(0)} + w_1^{(1)}t + w_1^{(2)}t^2 + \ldots$.. $A^{(n)}_{i_1,i_2,\ldots,i_p}$ are numerical coefficients, $\phi^{(n)}_{i_1,i_2,\ldots,i_p}$ are numerical phases and λ_j are literal arguments standing for polynomial functions of the time: $\lambda_j = \sum_{k\geq 0}\lambda_j^{(k)}t^k$. In the case of the Main Problem, we have also at our disposal the derivatives of the coefficients $A^{(n)}_{i_1,i_2,\ldots,i_p}$ and the mean motions $\lambda_j^{(1)}$ with respect to several constants (sidereal mean motions of Moon and Sun, lunar and solar eccentricities, inclination, ratios of masses,...). The Main Problem depends on 4 arguments, D, F, l and l' (Delaunay's arguments). For Earth's figure perturbations, we add the argument $\zeta = w_1 + pt$, where p is the precession constant for J2000. For planetary perturbations the components λ_j are Delaunay's arguments, and planetary secular mean longitudes known from a planetary theory. The reference plane of the theory is the mean dynamical ecliptic at J2000.

2. A NEW SOLUTION ELP/MPP02

We knew that in ELP the main limitation in precision resulted from the computation of the series for direct and indirect planetary perturbations. A new solution for planetary perturbations in the orbital motion of the Moon has been elaborated by P. Bidart. It is named MPP01 and described in (Bidart 2000, 2001). It has been constructed within the framework of ELP solution and the perturbation method is inspired by Brown's lunar theory whose basic concepts are discussed in (Chapront-Touzé and Chapront 1980). The aim of MPP01 was to improve the accuracy taking advantage of two recent progresses: the availability of numerical tools able to handle very large Poisson series (Software GREGOIRE) and the appearance of a new semi-analytical planetary theory, VSOP2000 (Moisson 2000).

VSOP2000 is more precise than VSOP82 which has been used in ELP and introduces a recent set of planetary masses (IERS92). It contains formal developments similar to ELP: the planetary coordinates (osculating elements) are developed under the form of Poisson series as in (1). The numerical values of the coefficients depend on the masses but also on the values of the osculating elements for a given epoch (J2000). These elements have not been obtained directly from a fit to observations but via a comparison of the analytical solution to the JPL ephemeris DE403. The angles are linear combinations of the planetary mean longitudes which have been derived with a better accuracy than in VSOP82. MPP01 takes advantage of this improvement, in particular in the integration process of arguments with long periods.

Fig 1. Longitude.

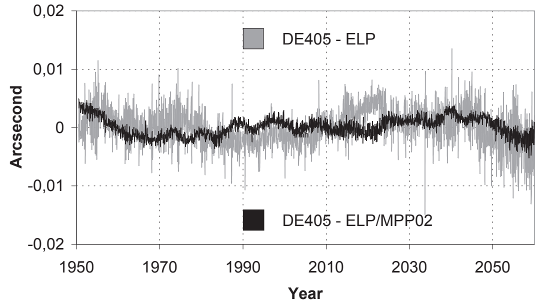


Fig 2. Latitude.

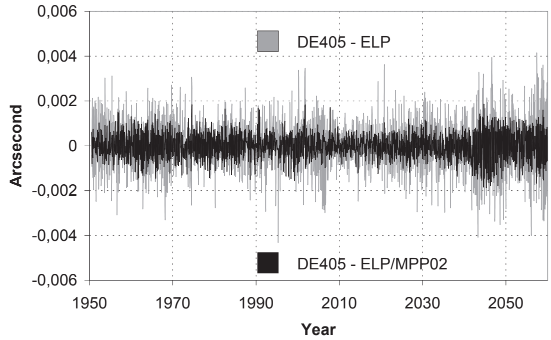
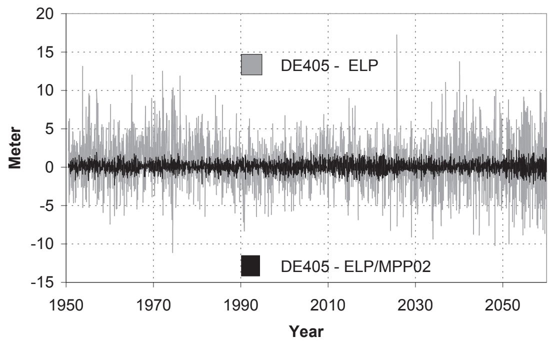


Fig 3. Distance.



On the basis of MPP01 we have derived a new solution ELP/MPP02. Its planetary component MPP02 has the following characteristics:

• *Number of terms*: after various numerical tests, compared to MPP01, the number of terms has been diminished to reduce the round-off errors of large sums (for example, in longitude we kept 16000 arguments instead of 128000)

• *Moving perigee and plane of orbit*: we have used the original series used in ELP2000-82B, more precise than in MPP01.

• *Secular motions in lunar perigee, node and mean longitude*: we have used the original polynomials of ELP2000-82B in t, t^2, t^3, t^4 since Bidart's solution has been achieved only in t^2.

After fitting the constant to DE405 over one century $[+1950; +2060]$, we have made a comparison of the differences between ELP/MPP02 and DE406 on this time interval. Figs 1 to 3 illustrate this comparison. We observe in particular the sensible improvement of ELP/MPP02 (in dark) compared to the original solution ELP (in light).

3. A FIT TO LLR OBSERVATIONS

Fig 4. Longitude.

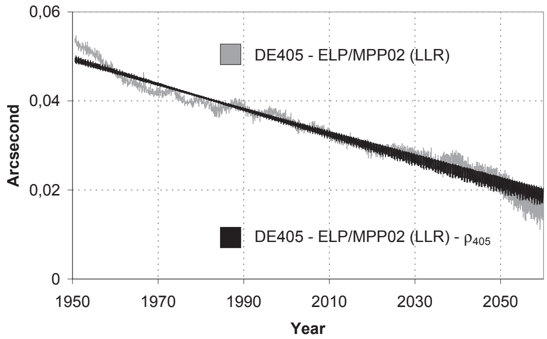


Fig 5. Latitude.

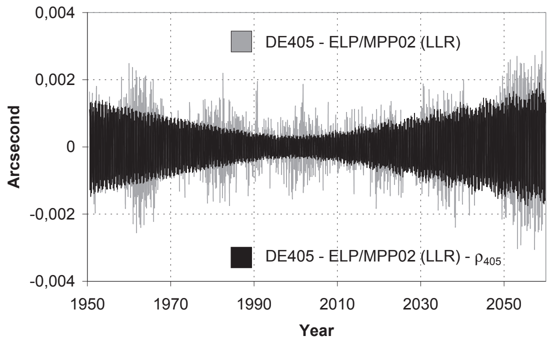
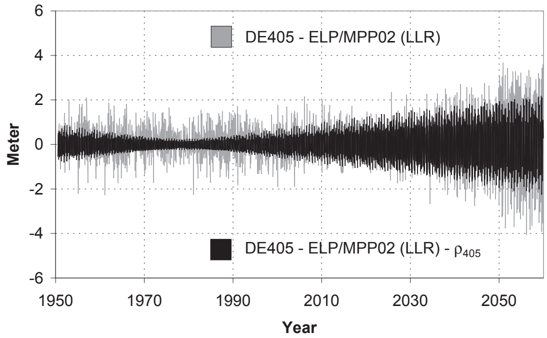


Fig 6. Distance.



Following the same method as we have applied earlier with DE245 (Chapront et al. 1999) we build numerical complements with DE405 that we call ρ_{405} in such a manner that: DE405 = ELP/MPP02(405) + ρ_{405}, over the time span $[+1950; +2060]$. The notation ELP/MPP02(405) means than the constants are derived from the fit to DE405. This solution which is nothing else than DE405 is now compared to LLR observations. A new set of constants is provided with this comparison. We finally substitute in our analytical solution this new set, and we obtain the solution ELP/MPP02(LLR) which results from our fit to LLR observations. Adding the numerical complements ρ_{405} which are insensible to the change of constants at the millimeter level, the so-completed solution ELP/MPP02(LLR) + ρ_{405} keeps the precision of a numerical integration. Figs. 4, 5 and 6 show the comparison of these 2 solutions to DE405.
We see on Fig.4 an offset in the longitudes of about $0.''036$ which is due to a difference between the two reference frames (ELP/MPP02 and DE405). The slope which amounts to $0.025''$/century is within the estimated error on the mean motion in longitude. (Shelus et al. 2001) give an estimate of $0.015''$/century for this error. The discrepancies as large as 30 cm in distance in the mid period of laser observation show that, in spite of post-fit residuals of about 2 to 3 cm, part of the difference arises from the models and the determinations of various parameters (lunar and solar parameters, libration, positions of reflectors and stations, tidal coefficients,...).

4. A COMPARISON WITH DE406 OVER 6 MILLENNIA

In order to estimate the precision of the solution on a very long range we have made a comparison between ELP/MPP02(405) and DE406 over the long interval $[-3000; +2500]$. Figs 7 to 9 illustrate the crude differences (in light). If one wants to keep closer to DE406, we can compute a new solution ELP/MPP02* which is the same as ELP/MPP02 but with secular variations of the lunar arguments w_i (mean longitude, perigee and node) fitted on DE406, i.e.: $w_i^{(2)}t^2 + w_i^{(3)}t^3 + w_i^{(4)}t^4 + w_i^{(5)}t^5$. The determination of the polynomial coefficients $w_i^{(k)}$ is realized by a mean square fit.

Fig 7. Longitude.

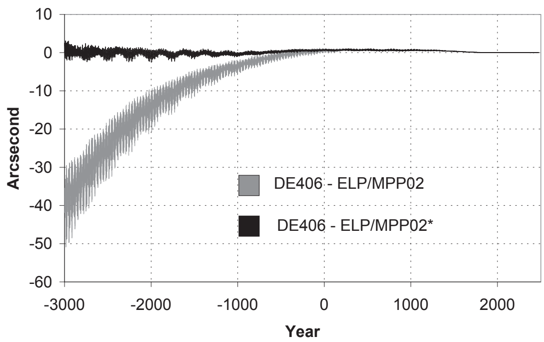


Fig 8. Latitude.

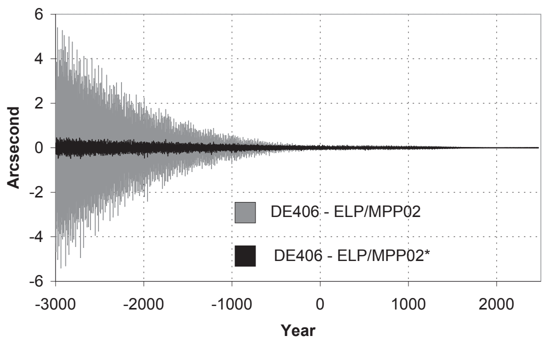
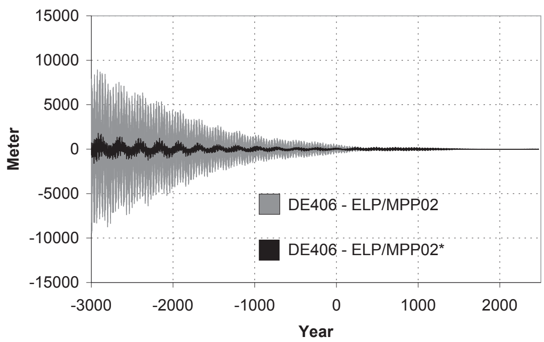


Fig 9. Distance.



We obtain a noticeable reduction of the differences with DE406 (in dark). Indeed, these 'empirical corrections' are a fit rather than an improvement of the secular variations of the angles in the theory. Although DE406 and ELP/MPP02 are very close one should be aware of the uncertainty on secular acceleration in longitude which is very much larger than these empirical corrections. Besides such a fit absorbs the numerical drift of the numerical integration.

5. CONCLUSION

ELP/MPP02(LLR) whose constants are fit to LLR data is our new analytical version of ELP that replaces ELP2000-82B. Practically ELP/MPP02 is presented in form of Fourier series for the Main Problem and its partials plus Poisson series for perturbations to the Main Problem. Two sets of constants are provided as well as literal expressions for the arguments (Moon, Sun and Planets):

- Constants fit to LLR data for ELP/MPP02 (LLR);
- Constants fit to DE405 (one century around J2000) for ELP/MPP02(405).

When it is coupled to a solution for the lunar libration and other physical models, the new ephemeris ELP/MPP02(LLR) + ρ_{405} is used for the comparisons to LLR and the determination of various parameters: lunar and solar parameters, but also tidal acceleration, position of the dynamical reference frame, correction to the precession constant, libration parameters, position of the stations ...

A solution with secular correction to the lunar angles (ELP/MPP02*) 'reproduces' DE406 on the long range (6 millenia) within a few arcsecond.

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