ACCURATE FORMULATION FOR THE TRANSFORMATION BETWEEN THE TERRESTRIAL AND CELESTIAL SYSTEMS

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ABSTRACT. The use of the International Celestial Reference System (ICRS) since 1998 together with the adoption of IAU 2000 Resolutions and the achieved accuracy in VLBI observations of the celestial pole have brought fundamental changes regarding reference systems and Earth rotation. This allows for an highly accurate formulation of the transformation between the International Terrestrial Reference System (ITRS) and the Geocentric Celestial Reference System (GCRS). This formulation is based on the IAU 2000A precession-nutation and on the IAU refined definition of the Celestial Intermediate Pole (CIP). The IAU recommended paradigm for the terrestrial-to-celestial transformation involves ITRS and GCRS coordinates of the CIP and the use of the Terrestrial and Celestial Ephemeris Origins, TEO and CEO (i.e. "non rotating origin" in the ITRS and GCRS, respectively) to define the Earth Rotation Angle. IAU Resolutions have moreover recommended that the IERS continue to provide users with data and algorithms for the conventional transformation. The implementation of the IAU 2000 resolutions thus requires microarcsecond expressions for both classical and new quantities that must be compliant with the IAU 2000 precession-nutation model. This paper reports on this implementation.

1. INTRODUCTION

There have been fundamental changes regarding reference systems and Earth rotation, namely the use of the International Celestial Reference System (ICRS) since 1998, the adoption of IAU 2000 Resolutions and the unprecedented precision, that modify the expressions to be used in the transformation between the terrestrial and celestial systems.

This paper first emphasizes the consequences of these recent changes and then reports on the implementation of the new formulation and the IAU 2000 expressions for Earth Rotation.

2. RECENT FUNDAMENTAL CHANGES

2.1 Consequences of the adoption of the ICRS

The ICRS, adopted by the IAU as the International Celestial Reference System since the 1st January 1998, is based on barycentric directions of distant extragalactic objects (Ma *et al* 1998) and its definition is independent of the models used for precession and nutation and of the Earth's orbital motion as well. IAU 2000 Resolution B1.3 has clarified the definition of the system of space-time coordinates within the framework of General Relativity for the Earth or Solar System, respectively. It has also specified the metric tensor to be used in both systems and the 4-dimensional space-time transformation between BCRS and GCRS.

Consequently, the celestial reference system for the Earth Orientation Parameters (EOP) has been changed from the FK5 to the GCRS.

2.2 Consequences of the submilliarcsecond determination of the nutation offsets

VLBI observations being sensitive to the actual orientation of the equator with respect to the GCRS, currently provide, with a submillarcsecond accuracy, the "nutation offsets" that include both the inaccuracies in the precession-nutation model and the frame bias of the mean equator at the reference epoch of the model in the GCRS.

Series of these VLBI observables during a 20-year period, have been used for determining the Basic Earth Parameters (BEP) of the geophysically-based MHB theory (Mathews *et al.* 2002) that has been adopted as the IAU 2000A precession-nutation.

2.3 Consequences of the adoption of the IAU 2000 Resolutions

IAU 2000 Resolutions B1.6 to B1.8 have recommended refined definitions, models and formulations for the celestial to terrestrial transformation.

The new definition of the pole (Resolution B1.7) to which the Earth's rotation refers (i.e. the Celestial Intermediate Pole, CIP) explicitly considers the high frequency variations (for more detail, see Capitaine 2000a and 2000b). According to this definition of an intermediate pole between the ITRS and the GCRS, the new precession-nutation model has to provide nutations with periods greater than two days; in contrast, nutations with periods lower than two days in the GCRS are considered as being variations of polar motion.

The IAU 2000 precession-nutation model adopted by Resolution B1.6 is based on the rigid Earth nutation model (Souchay *et al.* 1999) and on the transfer function of Mathews *et al.* (2002). This model includes two versions with associated precession and obliquity rates and celestial pole offsets at epoch. The most accurate version, denoted IAU 2000A, provides the direction of the celestial pole in the GCRS with a 0.2 mas accuracy (Mathews *et al.* 2002), whereas it shorter version, denoted IAU 2000 B, provides a 1 mas accuracy (McCarthy & Luzum 2003). The IAU 2000A model includes more than 1300 nutations with in-phase and out-of-phase periodic components in longitude and obliquity.

The new paradigm for the transformation between the ITRS and GCRS, which is recommended by Resolution B1.8, is based on the use of the non-rotating origin (Guinot 1979) both in the GCRS (i.e. the Celestial Ephemeris Origin (CEO)) and in the ITRS (i.e. the Terrestrial Ephemeris Origin, TEO). The CEO replaces the equinox as the origin for the Earth Rotation Angle (ERA) and the TEO provides an exact definition of the "instantaneous origin" of longitudes in the ITRS. Moreover, the classical precession and nutation quantities are replaced by the coordinates X and Y of the CIP in the GCRS that include combined precession, nutation and frame biases, together with their coupling effects. In addition to providing an explicit separation between precession-nutation of the equator from Earth rotation, this new paradigm is more simple, compact and direct than the classical one, which is a non-negligible advantage for achieving accuracies at the level of a few microarcseconds.

3. IMPLEMENTATION OF THE NEW FORMULATION

3.1 Implementing the IAU Resolutions

Two equivalent ways of implementing the IAU Resolutions in the transformation from ITRS to GCRS can be used, namely (a) the new paradigm based on the direct use of the CEO and the ERA and (b) the classical paradigm based on the equinox and GST, but using the CEO and the ERA indirectly (for more detail, see Capitaine *et al.* 2000). They are called respectively "CEO-based" and "equinox-based" transformations in the following.

Common to both paradigms is the polar-motion matrix, which requires, in addition to the coordinates x_p, y_p of the CIP in the ITRS, the use of the quantity s' for providing the position of the TEO in the ITRS, which was neglected in the classical form prior to 1 January 2003. Implementation of the IAU 2000A precession-nutation model using the new paradigm requires expressions for the positions of the CIP and the CEO in the GCRS computed to an accuracy of a few microarcseconds over a time span of a few hundred years, in order to meet the requirements of high-accuracy applications. Implementation using the classical paradigm requires expressions for the various precession and nutation angles and Greenwich Sidereal Time (GST) as well.

3.2 Position of the CIP in the ITRS

In order to realize the CIP as recommended by Resolution B1.7, nutations with periods less than two days are to be considered using a model for the corresponding motion of the pole in the ITRS. The prograde diurnal nutations correspond to prograde and retrograde long periodic variations in polar motion, and the prograde semidiurnal nutations correspond to prograde diurnal variations in polar motion.

Models have been developed to provide such variations in polar motion that are due to tidal gravitation (see Brzeziński & Mathews 2003, this Volume) and are to be considered together with the variations of polar motion due to oceanic tides that appear at the same periods.

3.3 Position of the CIP in the GCRS

The x and y coordinates of the CIP unit vector in the GCRS, denoted X and Y, include (see Capitaine 1990) (i) precession and nutation referred to a fixed conventional ecliptic, (ii) coupling between precession and nutation giving rise to Poisson terms, (iii) celestial offsets ξ_0, η_0 of the CIP at J2000.0 w.r.t. the GCRS (associated with the precession-nutation model) and (iv) the coupling between offset in right ascension, $\mathrm{d}\alpha_0$ and precession-nutation.

Whereas the offsets ξ_0, η_0 are derived from VLBI observations, the determination of the equinox offset requires the use of observations which are dependent on the position of the ecliptic. The numerical value that has been used for the implementation of the IAU 2000 precession-nutation model is the GCRS right ascension of the mean dynamical equinox at J2000 (-14.6 ± 0.5 mas) as provided by Chapront *et al.* (2002) from a fit to LLR observations based jointly on the use of a dynamical theory for the Moon and of VLBI Earth Orientation parameters.

3.4 Implementation of the new definition of UT1

The implementation of the new definition of UT1 with the CEO-based transformation (Resolution B1.8) uses the conventional relationship between ERA and UT1 together with the quantity s that positions the CEO on the equator of the CIP.

The expression for GST to be used in the equinox-based transformation is derived from the relationship between ERA and UT1 and the expression for the accumulated precession and nutation (*i.e.* the equinox-based right ascension of the CEO; see Capitaine & Gontier 1993 for more detail) based on the IAU 2000A model.

4. IAU 2000 EXPRESSIONS FOR EARTH ROTATION

Expressions, numerical Tables and software for implementing the IAU 2000 system with either the classical or the new transformation have been made available during the year 2002.

Provisional expressions have been discussed during the IERS Workshop 2002 in Paris and the final versions have been provided in Chapter 5 of the IERS Conventions 2000.

4.1 Expressions for the variations in polar motion

Model for the variations $(\Delta x, \Delta y)_{nutation}$ in polar motion corresponding to diurnal and subdiurnal celestial nutations has been adopted by an *ad hoc* Working Group (Brzeziński, 2002, Brzeziński & Mathews 2003, this Volume)). The model including all components with amplitudes greater than 0.5 μas, is based on nonrigid Earth models and developments of the tidal potential (Brzeziński, 2001, Brzeziński and Capitaine, 2003, Mathews and Bretagnon, 2003). The amplitudes of the diurnal terms are in very good agreement with those estimated by Escapa *et al.* (2003). A Table for operational use is provided in the IERS Conventions 2000.

The diurnal components of these variations should be considered similarly to the diurnal and semidiurnal variations due to ocean tides. They are not part of the polar motion values reported to the IERS and distributed by the IERS and should therefore be added after interpolation. The long-periodic terms, as well as the secular variation, are already contained in the observed polar motion and need not be added to the reported values.

4.2 Expression for the position of the TEO in the ITRS

The quantity s' is only sensitive to the largest variations in polar motion. The expression to be used has been derived from the current mean amplitudes for the Chandlerian and annual wobbles (Lambert & Bizouard 2002):

$$s' = -47\ \mu\text{as}\ t. \tag{12}$$

4.3 Expression to implement the IAU 2000 definition of UT1

The Earth Rotation Angle, θ, is obtained by the use of its conventional relationship with UT1 as given by Capitaine *et al.* (2000),

$$\theta(T_u) = 2\pi(0.7790572732640 + 1.00273781191135448 T_u), \tag{13}$$

where T_u = (Julian UT1 date − 2451545.0), and UT1 = UTC + (UT1−UTC),

This definition of UT1 based on the CEO is insensitive at the microarcsecond level to the precession-nutation model and to the observed celestial pole offsets.

4.4 IAU 2000 expression for the position of the CIP in the GCRS

IAU 2000 expressions have been developed for the coordinates X and Y of the CIP in the GCRS, valid at the microarcsecond level (Capitaine *et al.*, 2003a); they are based on the IAU 2000A or IAU 2000B model for precession-nutation and on their corresponding pole offset at J2000.0 with respect to the pole of GCRS. The complete series are provided in the IERS Conventions 2000.

4.5 IAU 2000 expression for the position of the CEO in the GCRS

The position of the CEO in the GCRS is provided by the expression for the quantity s using the developments of X and Y as functions of time (Capitaine *et al.*, 2003a). The numerical development is in fact provided for the quantity $s + XY/2$, which requires less terms to reach the same accuracy than a direct development for s.

The constant term for s, which was previously chosen so that $s(J2000) = 0$, has now been fit (Capitaine *et al.*, 2003b) in order to ensure continuity of UT1 at the date of change (1 January 2003) consistent with the Earth Rotation Angle (ERA) relationship and the current VLBI procedure for estimating UT1.

The complete series for $s+XY/2$ with all terms larger than 0.1 μas is available electronically on the IERS Convention Center website.

4.6 IAU 2000 precession-nutation expressions for the classical paradigm

The IAU 2000 expressions for the precession quantities ψ_A, ω_A, ϵ_A, compatible with the IAU 2000 precession-nutation model can be provided by using the developments of Lieske *et al.* (1977) to which the MHB estimated values for the precession rates, $\delta\psi_A$ and $\delta\omega_A$ have to be added. These expressions together with the Lieske *et al.*'s one (1977) for the planetary precession parameter χ_A have been considered as being the "canonical" 4-rotation series.

Then, expressions have been developed (Capitaine *et al.*, 2003c) for the more usual equatorial precession quantities ζ_A, θ_A, z_A in order to match the canonical 4-rotation series to sub-microarcsecond accuracy over 4 centuries.

Numerical comparisons have shown that the equivalence between the two paradigms requires that the classical one take account rigorously of the corrections to precession using the "canonical" four rotations (Wallace 2002), and frame bias, through appropriate rotation matrices.

4.7 IAU 2000 expression for Greenwich Sidereal Time

The IAU 2000 numerical expressions (Capitaine *et al.* 20003b) linking GST and ERA and locating the CEO have been developed such that they produce no discontinuity in UT1 on 1 January 2003 when changing from the current VLBI procedure to the new IAU 2000 system in which ERA(UT1) is a conventional relationship. This takes into account (i) the change from the former IAU relationship between GMST and UT1, (ii) the change from the IAU 1994 equation of the equinoxes to a more accurate expression and (iii) the systematic error of the order of 100 μas due to the incorrect use of UT (instead of TT) for computing precession in RA in the old expression for GST.

The polynomial part of GST is conventionally defined as GMST, whereas the non-polynomial part is the "complete equation of the equinoxes". This is the sum of the *classical* part and the *complementary terms*. The latter (*i.e.* the right ascension of the CEO in the mean frame at J2000) is very similar to the non-polynomial part of the expression for $s + XY/2$.

The expected discontinuity in UT1 rate, shown to be unavoidable due to the improved models and the fixed relationship between ERA and UT1, will have an effect on the determination of UT1 that is less than a few hundreds of microarcseconds over the next century.

5. CONCLUDING REMARKS

The expressions to be used to implement the IAU 2000 formulation based on the IAU 2000 precession-nutation model, either in the new (CEO-based) or classical (equinox-based) transformations between the ITRS and GCRS have been developed and are provided in the IERS Conventions 2000.

Tables and Fortran subroutines implementing the IAU 2000 terrestrial-to-celestial transformations are provided through the IERS Conventions website, and additional software have been released through the SOFA website (Wallace 2000).

Various comparisons and numerical checks have been performed between the IAU 2000 formulations of the classical and the new transformations. These comparisons revealed the need for improvements to the classical form of the transformation in order to achieve the required level of accuracy. Once these improvements are applied, the consistency between (i) the positions of the CIP in the GCRS and (ii) the rotation about the CIP axis, in the CEO-based and equinox-based transformations, when using the IAU 2000 expressions, are at a level of a few microarcseconds after one century (See Figures 1 and 2).

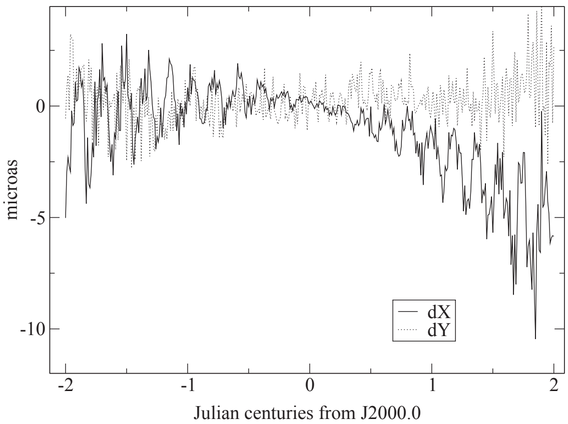

Figure 1: Differences in the GCRS CIP coordinates between the new and classical paradigms using the IAU 2000 expressions (Capitaine et al. 2003a)

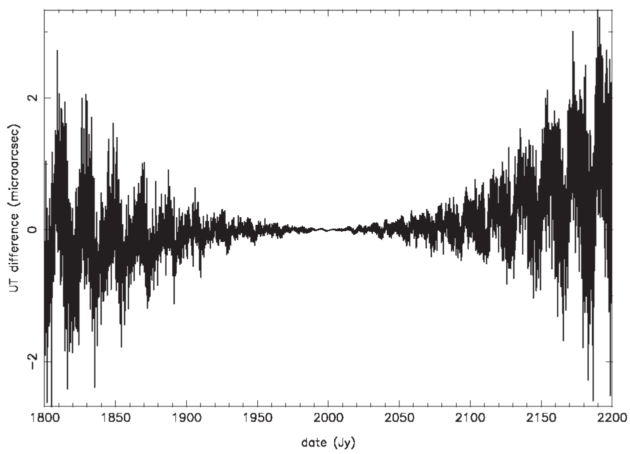

Figure 2: Differences in UT1 between the new and classical paradigms using the IAU 2000 expressions (Capitaine et al. 2003b)

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