EFFECTS OF MARS’ ROTATION ON ORBITER DYNAMICS

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ABSTRACT. Of first importance in the theoretical study of a spacecraft motion, in view of
a cosmic flight, is the modelling of as much perturbing effects as possible; this contributes
essentially to a better knowledge of the trajectory.

This paper deals with the influence of Mars’ rotation on the dynamics of an orbiter via
rotation and oblateness of its atmosphere (considered separately from the general atmospheric
drag). For the density distribution, the nominal density profile proposed by Sehnal is adopted.
As regards the orbiter dynamics, one considers only initially circular trajectories lying entirely
in the altitude range 100-1000 km.

Using a method proposed by Zhongolovich, that uses the argument of latitude as independent
variable (allowing in this way the study of circular orbits), the first order difference between the
nodal period of the orbiter and the corresponding Keplerian period is analytically determined.
The first order changes of five independent osculating orbital elements (the classical set of six
without the dynamical one) along one nodal period are determined, too. All analytic results
are translated in terms of physical motion. The evolution of the radius vector and the related
change of the nodal period prove to be closely related to the initial orbital inclination.

1. INTRODUCTION

While studying theoretically the motion of a spacecraft, in view of a cosmic flight, of first
importance is the modelling of as much perturbing effects as possible. Better known the influence
of the various perturbing factors is, and better known the trajectory of the cosmic vehicle will be.

Among the effects that act on the motion of a planetary orbiter, we dwelt upon the planet’s
rotation. Its influence can be tackled from many standpoints, considering for instance: the
noncentrality of the gravitational field given by the second zonal harmonic, the relativistic
effect of the quadrupole momentum, the Lense-Thirring effect, the atmospheric rotation and/or
oblateness (due, obviously, to the planet’s rotation).

In the sequel we approach the last situation for the concrete case of Mars. The perturbed
motion of an orbiter in the Martian atmosphere was first studied analytically by Sehnal and
Pospिšilová (1988). Using the data provided by Moroz et al. (1988), they modelled the density
distribution by the law

\[ \rho = \exp(a_{1j} + a_{2j}/h), \quad j \in \{1, 2, 3\}, \]

where the numerical value of the density \( \rho \) results in kg/m\(^3\) for the altitude \( h \) above Mars’
surface expressed in km. The constants $a_{1j}$, $a_{2j}$ are separately determined for the minimal ($j = 1$), nominal ($j = 2$), and maximal ($j = 3$) density profiles. In the nominal model used by us for numerical estimates, Sehnal (1990) gave $a_{12} = -37.936$, $a_{22} = 2376.1$. Expression (1) is valid for the altitude range $100 \text{ km} \leq h \leq 1000 \text{ km}$.

The quoted papers did not consider the atmospheric rotation and oblateness. Further analytic approaches took into account these effects (e.g. Mioc et al. 1991), but imbedded in the general atmospheric drag effect, and only for orbital elements, or went deeper in the Martian atmospheric drag problem, but without considering rotation and oblateness (Mioc and Radu 1991b). The subject that interests us here is the way in which the separate influence of the rotation and oblateness of Mars’ atmosphere could affect the motion of an orbiter.

We shall study the respective perturbations in both the nodal period

$$T_\Omega = \int_0^{2\pi} (dt/du) du,$$

and the orbital elements

$$y \in Y = \{p, \Omega, i, q = e \cos \omega, k = e \sin \omega\}.$$

Here $u$ = argument of latitude, $p$ = semilatus rectum, $\Omega$ = longitude of ascending node, $i$ = inclination, $e$ = eccentricity, $\omega$ = argument of periastron (all with respect to a frame originated in Mars’ mass centre, whose fundamental plane is the Martian equatorial plane.

We study the orbiter motion within the following physical and dynamical framework:

(i) the atmosphere rotates with the same angular velocity ($\omega_M = 6.124 \text{ rad/day}$) as Mars, and is oblate (the surfaces of equal density having the same oblateness $\eta = 0.0065$ as the planet);

(ii) the initial orbit lies entirely in the range of heights 100-1000 km above Mars’ surface;

(iii) the initial orbit is circular ($e_0 = e(t_0) = 0$, which implies $q_0 = 0$, $k_0 = 0$).

2. BASIC EQUATIONS

Under the influence of any perturbing factor depending on a small parameter $\varepsilon$, the orbiter motion undergoes a perturbing acceleration (of radial, transverse, and binormal components denoted by $R$, $T$, $N$, respectively).

We have chosen the nodal period as basic time interval. Accordingly, let us describe the motion via the Newton–Euler equations written with respect to the argument of latitude (cf. Mioc and Stavinschi 1995, 1997):

$$
\begin{align*}
  dp/du &= 2(\gamma/\mu)r^3T, \\
  d\Omega/du &= (\gamma/\mu)r^3[\sin u/(p \sin i)]N, \\
  di/du &= (\gamma/\mu)r^3(\cos u/p)N, \\
  dq/du &= (\gamma/\mu)\{r^2\sin u\}R + r^2[r\cos u/(p \cos i)]T + r^3[k \cos u/(p \sin i)]N, \\
  dk/du &= (\gamma/\mu)\{-r^2\cos u\}R + r^2[r\sin u/(p \sin i)]T + r^3[q \cos u/(p \sin i)]N, \\
  dt/du &= (\gamma/\sqrt{\mu p}) r^2,
\end{align*}
$$

where $\gamma = [1 - r^2 \cos i / \sqrt{\mu p}]^{-1}$, $\mu$ = Mars' gravitational parameter ($\mu = 0.426503 \times 10^{14}$ \text{ m}^3\text{ s}^{-2}$), and $r$ = planetocentric radius vector.

Using the method proposed by Zhongolovich (1960) for a wholly different model, the variation of the nodal period can be expressed as $\Delta T_\Omega = \sum_{i=1}^4 J_i$, with

$$J_1 = (3/2) \sqrt{p_0/\mu} \int_0^{2\pi} (1 + q_0 \cos u + k_0 \sin u)^{-2} \Delta p du,$$
\[ J_2 = -2p_0\sqrt{p_0/\mu} \int_0^{2\pi} (1 + q_0 \cos u + k_0 \sin u)^{-3} \Delta q \cos \, u \, du, \]
\[ J_3 = -2p_0\sqrt{p_0/\mu} \int_0^{2\pi} (1 + q_0 \cos u + k_0 \sin u)^{-3} \Delta k \sin \, u \, du, \tag{5} \]
\[ J_4 = \int_0^{2\pi} \{ \partial[p^4 \Omega / \partial p] / \partial \varepsilon \} \varepsilon \, du, \]

in which the subscript "0" means (as above) the initial values, whereas the changes \( \Delta y = y - y_0 \), \( y \in Y \), after the interval \([u_0, u]\) are determined from

\[ \Delta y = \int_{u_0}^{u} (dy/du) \, du, \quad y \in Y, \tag{6} \]

under the condition (natural and not restrictive) that the orbital parameters experience small perturbations over one revolution.

Since our intention is to consider here only initially circular orbits \( (q_0 = 0, k_0 = 0, p_0 = r_0) \), expressions (5) reduce to

\[ J_1 = (3/2)\sqrt{p_0/\mu} \int_0^{2\pi} \Delta p \, du, \]
\[ J_2 = -2p_0\sqrt{p_0/\mu} \int_0^{2\pi} \Delta q \cos \, u \, du, \]
\[ J_3 = -2p_0\sqrt{p_0/\mu} \int_0^{2\pi} \Delta k \sin \, u \, du, \tag{7} \]
\[ J_4 = (p_0^3/\mu) \cos i_0 \int_0^{2\pi} (\partial \Omega / \partial \varepsilon) \varepsilon \, du. \]

For sake of simplicity, in the sequel we shall drop the subscript "0" for the initial values of \( \{3\} \) and functions of them. Every further subscript "0" (except \( u_0 \)) is a simple notation and does not refer to initial values. Actually, every quantity in the right-hand side of (4) or (7) that does not depend on \( u \) will be considered constant over one revolution, and equal to its value for \( u = u_0 \).

3. EQUATIONS OF MOTION

Let us consider only the influence of Mars' atmospheric rotation (oblateness entailed) on the orbiter motion. In this case, the components of the perturbing acceleration, for an initially circular trajectory, reduce to (cf. Micoc and Radu 1991a):

\[ R = 0, \]
\[ T = \rho \delta \sqrt{\mu R_M} \cos i, \]
\[ N = -\rho \delta \sqrt{\mu R_M} \sin i \cos u, \tag{8} \]

where \( \delta \) stands for the drag parameter (ballistic coefficient) of the orbiter.

Now we have to express the density as a function only of \( u \). By \( h = r - R_M(1 - \eta \sin^2 \varphi) \), where \( R_M = 3380 \) km is the mean equatorial radius of Mars, \( \varphi = \) latitude, and \( \sin \varphi = \sin i \sin u \), we get \( h = p - R_M(1 - \eta \sin^2 i + \eta \sin^2 i \sin^2 u) \), hence the density expression (1) becomes

\[ \rho = \exp \left[ a_{12} + \frac{a_{22}}{(p - R_M + \eta R_M \sin^2 i)(1 - W \cos^2 u)} \right], \tag{9} \]
where \( W = \eta R_M \sin^2 i / (p - R_M + \eta R_M \sin^2 i) \). Condition (ii) easily leads to \( W_{\text{max}} = 0.15 \), therefore we have to first order in \( W \):

\[
\rho = \alpha \exp(\beta \cos^2 u),
\]

with \( \alpha = \exp(a_{12} + a_{22} / (p - R_M + \eta R_M \sin^2 i)) \), \( \beta = a_{22} W / (p - R_M + \eta R_M \sin^2 i) \). A straightforward computation shows that \( \beta \) does not exceed 3; consequently, we shall expand (10) as

\[
\rho = \alpha \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n} u.
\]

Having in view the fact that the perturbations of \( y \in Y \) are small over one revolution, and taking into account Zhongolovich’s (1960) method, we easily get the expressions of the first five equations (4) for our case:

\[
\begin{align*}
\frac{dp}{du} &= 2\alpha p^3 \sqrt{p/\mu \delta \omega_M} \sin i \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n+1} u \\
\frac{d\Omega}{du} &= -\alpha p^2 \sqrt{p/\mu \delta \omega_M} \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n+1} u \\
\frac{di}{du} &= -\alpha p^2 \sqrt{p/\mu \delta \omega_M} \sin i \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n+2} u \\
\frac{dq}{du} &= 2\alpha p^2 \sqrt{p/\mu \delta \omega_M} \cos i \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n+1} u \\
\frac{dk}{du} &= 2\alpha p^2 \sqrt{p/\mu \delta \omega_M} \cos i \sum_{n=0}^{\infty} (\beta^n / n!) \cos^{2n} u \sin u.
\end{align*}
\]

**4. RESULTS**

By (7), the integrals (6) must be performed only for \( p, q, \) and \( k \). Performing them with the integrands provided by the corresponding equations (12), we get

\[
\begin{align*}
\Delta p &= -Zp \left[ u + \sum_{n=0}^{\infty} (\beta^n / n!) \left( P_n u + \frac{1}{2n} \sum_{j=0}^{n-1} P_{nj} \cos^{2n-2j-1} u \sin u \right) - F_1^0 \right], \\
\Delta q &= Z \left[ \sum_{n=0}^{8} \frac{\beta^n}{n!(2n+1)} \sum_{j=-1}^{n-1} Q_{nj} \cos^{2n-2j-2} u \sin u - F_2^0 \right], \\
\Delta k &= -Z \left[ \sum_{n=0}^{8} \frac{\beta^n}{n!(2n+1)} \cos^{2n+1} u - F_3^0 \right],
\end{align*}
\]

where we abridged \( Z = 2\alpha p^2 \sqrt{p/\mu \delta \omega_M} \cos i \), and

\[
\begin{align*}
P_n &= \frac{(2n-1)!!}{2^n n!}, & n = 1, 2, \ldots; \\
P_{n0} &= 1, & P_{nj} = \frac{(2n-1)(2n-3)(2n-2j+1)}{2j(n-1)(n-2)\ldots(n-j)}, & j = 1, n-1; \\
Q_{n-1} &= 1, & Q_{nj} = \frac{2^{j+1} n(n-1)(n-j)}{(2n-1)(2n-3)\ldots(2n-2j-1)}, & j = 0, n-1.
\end{align*}
\]
while $T_{12}^n$, $s = 1, 3$, stands for the value of the whole expression (function of $u$) preceding it inside the square brackets, for $u = u_0$.

Substituting (13) in (7), and performing the integrations, we find $J_1 \neq 0$ (see its expression below), $J_2 = 0$, $J_3 = 0$. As regards $J_4$, using the last equation (4) and the second equation (12), then performing the last integral (7), in which $\alpha$ plays the role of the small parameter, we get $J_4 = 0$. So, we obtain the first order difference between the nodal period and the corresponding Keplerian period:

$$\Delta T_{12} = 6\pi \alpha (p^4/\mu) \delta \omega_M \cos i \left[ \pi \sum_{n=0}^{8} (\beta^n/n!) P_n - R_i^0 \right]. \quad (15)$$

To end, we give the variations of the orbital elements (3) over one nodal period. Replacing (12) in (6), then integrating between 0 and $2\pi$, and keeping, by abuse, the same notation $\Delta y$, $y \in Y$, for these changes, we obtain

$$\Delta p = 4\pi \alpha p^3 \sqrt{\mu/p} \delta \omega_M \cos i \sum_{n=1}^{8} (\beta^n/n!) P_n,$$

$$\Delta \Omega = 0, \quad \Delta q = 0, \quad \Delta k = 0, \quad (16)$$

$$\Delta i = -2\pi \alpha p^2 \sqrt{\mu/p} \delta \omega_M \sin i \sum_{n=1}^{8} (\beta^n/n!) P_{n+1}.$$

5. CONCLUSIONS

Examining formulae (15) and (16), we can draw some conclusions concerning the physical behavior of the orbiter:

5.1. Since $\Delta q = \Delta k = 0$, the initially circular orbit returns to a circular shape after one nodal period.

5.2. Since $\Delta p$ contains $\cos i$, multiplied by a positive quantity, after one nodal period, the new orbital radius, compared to the initial one, will be greater if the initial orbit is direct ($i < \pi/2$), equal for a polar orbit ($i = \pi/2$), and smaller for a retrograde orbit ($i > \pi/2$).

5.3. Since $\Delta \Omega = 0$, the node comes back to its initial position after one nodal period.

5.4. Since $\Delta i$ contains $\sin i$, multiplied by a negative quantity, after one nodal period the inclination becomes smaller: a direct initial orbit will be shifted towards an equatorial one, whereas a retrograde initial orbit will be shifted towards a polar one.

5.5. By 5.2, the nodal period, compared to the corresponding Keplerian period, is greater for direct initial orbit, equal for the polar case, and smaller for retrograde motion.

Two general conclusions, as regards the techniques we used, can also be formulated:

5.6. The use of the argument of latitude as independent variable allows the study of the perturbations in the period for very low eccentric orbits (even circular, as we considered). Such a study is impossible if one of the anomalies is taken as independent variable.

5.7. Our results are valid only under the hypotheses (i)-(iii) formulated in Section 1, and only in a first order approximation with respect to the small parameter $\alpha$. However, they constitute a sufficiently real departure point for further refinements.

6. REFERENCES