FREE AND FORCED RESPONSE OF A NON-RIGID MARS WITH AN INNER-CORE. NUMERICAL APPROACH.

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ABSTRACT.
If the planet Mars contains a liquid core and a solid inner core, there is, in addition to the Free Core Nutation (FCN, existing only if the core is liquid) the Free Inner Core Nutation (FICN). The FCN is known to have a frequency in the retrograde band of the forced nutation frequencies. The FICN is in the prograde nutation frequency band. Inside the planet Mars, it is presently not known if there exists an inner core, and what would be the dimension of this inner core. Consequently, it is necessary to examine the impact of the presence of an inner core on the nutations of Mars. We use a numerical integration method, integrating from the center up to the surface. We have implemented our numerical code in order to compute the FICN as well as the forced nutations that may feel the resonance due to their proximity to the mode.

1. INTRODUCTION
The present knowledge of the interior of Mars is sparse and relies mainly on a few observational constraints. This is about to change in the future due to the NEtlander Ionospheric and Geodesic Experiment (NEIGE) (Barriot et al., 2001). The NEIGE experiment will be part of the NEtlander mission planned for 2007. Four identical landers will be sent to Mars, all of them will contain a coherent transponder which will be in communication with an orbiter connected via radio-links to the Earth. The Doppler shifts associated with the variations of Mars’ orientation in space will be measured, and will provide us, for the first time, with measurements of the variations of length-of-day, of polar motion, and of precession and nutations (see Dehant et al., 2002, this issue). The determination of these rotation parameters will improve our knowledge of the interior structure of Mars, and in particular the knowledge about the state of the core. If the core is fluid there exists a normal mode called FCN which induces a resonance in the nutations, that will be visible in the NEIGE measurements. Moreover, a inner-core induces another normal mode called FICN with a corresponding resonance in the nutations.

The present models of Mars’ interior, such as those of Sohl and Spohn (1997), are obtained from extrapolation of the Earth’s internal structure, from chemical composition derived from the SNC meteorites and constraints such as the chondritic Fe/Si ratio or the precession rate (and
consequently the polar moment of inertia) determined from the Viking and Pathfinder missions.

Nevertheless those models cannot resolve the uncertainties related to the core’s dimension the
core’s state and the possible existence of an inner core. Both the liquid and the solid configuration
be consistent with the observational constraint that the Martian dipole magnetic field is very
weak or non-existent compared to that of the Earth. Also the thermodynamical state of the
iron alloy, with a composition incorporating iron and a light element (14 wt%) sulfur, favors a
liquid core and no inner-core, but the percentage of light elements is controversial and a slightly
smaller value would allow for inner-core formation.

In this paper we compute the free and forced response of a non-rigid Mars as function of the
inner-core size. The inner-core size is varied from 0 km (without inner-core) to approximately
1470 km (fully solidified core).

The paper is organized as follows. In Section 2, the basics of nutation and normal mode
theory are reviewed, specifically the resonances induced by the free core nutation (FCN) and
the free inner-core nutation (FICN), which both exist only if the core is liquid and the second
only exists if the inner-core is present. In Section 3 the equations and the interior structure
involved in our numerical modeling are discussed. In Section 4 we display the results, namely
the influence of the inner-core size on the amplitude of the transfer function. Then we compare
the transfer function for a planet with a 900 km and a 1100 km inner-core size to a configuration
without inner-core.

Finally the last section is devoted to discussion and concluding remarks.

2. NUTATIONS OF MARS.

Nutations of a planet are periodic changes of the orientation of the rotation axis with respect
to an inertial frame. These motions are due to the gravitational torque acting on the equatorial
bulge of the flattened planet. This torque can be computed from the tidal potential which
essentially depends on the relative position of the planet with respect to the Sun, the moon(s)
and the other planets.

The computation of the nutational motions of the rotation axis is done in two steps. First
the nutation series for a rigid planet is derived (Roosbeek, 2000), then the non-rigid effects on
nutation are calculated with the help of a transfer function, for each frequency of this series.
The transfer function is defined as the ratio of the nutation amplitudes for a non-rigid planet to
the nutation amplitude for a rigid planet considered at the same frequency (Dehant et al, 2000).

The most important normal modes for nutations are (see Figure 1):

- the Chandler Wobble related to a rotation of the instantaneous rotation axis around the
  axis of greatest polar principal moment of inertia; it has long period and is prograde in a
  frame tied to the planet

- the free core nutation (FCN) related to a relative rotation of the liquid core instantaneous
  rotation axis with respect to that of the mantle; it is diurnal in a frame tied to the planet
  and has a retrograde long period in space.

- the free inner-core nutation (FICN) related to a relative motion of the instantaneous
  rotation axis of the inner-core with respect to that of the liquid core and the mantle; it is
  diurnal in a frame tied to the planet and has a prograde long period in space.

The influence of Mars’ interior (mantle and core) on the CW and on the FCN have been
investigated by Van Hoolst et al (2000 a,b) and Defraigne et al (2001). The present paper is
more concerned with the effect of a possible inner core on the FCN and on the FICN.
3. NUMERICAL MODELING.

In our approach we consider the linearized equations of motion of a small displacement for a rotating elliptical planet

\[
\rho_0 \frac{d^2 \vec{u}}{dt^2} + 2\rho_0 \Omega_0 \wedge \frac{d\vec{u}}{dt} = \rho_0 \vec{F}_e - \rho_0 \nabla W_1 + \nabla \cdot \vec{T} + \rho_0 (\nabla \cdot \vec{u})(\nabla W_0) - \rho_0 \nabla(\vec{u} \cdot \nabla W_0) - \rho_0 (\Omega_0 \wedge (\Omega_0 \wedge \vec{r})).
\]

They contain the tidal forcing \( \vec{F}_e \), the initial and the Eulerian gravitational potential \( (W_0, W_1) \), the rotational contribution like the Coriolis and centrifugal potential \( (\Omega \wedge (\Omega \wedge \vec{r})) \), the Cauchy stress-strain tensor \( \overline{\text{T}} \) and the initial density \( \rho_0 \). Those parameters are related by the Poisson equation,

\[
\Delta (W_1 + \phi_l^{\text{CJ}}) = -4\pi G \nabla \cdot (\rho_0 \vec{u}),
\]

which relates the change of the Eulerian gravitational \( W_1 \) and the Eulerian centripetal \( \phi_l^{\text{CJ}} \), potentials to the local density change and the stress-strain relation,

\[
\overline{T} = \lambda(\nabla \cdot \vec{u}) \overline{I} + \mu(\nabla \vec{u} + (\nabla \vec{u})^T),
\]

where \( \lambda \) and \( \mu \) Lamé are the parameters.

This system of coupled partial differential equations is first shifted into phase space and then expanded in generalized spherical harmonics to give a system of 10 linear ordinary differential equations of first order. Numerical integration and application of the boundary conditions, supplemented by a density profile and the values of the rheological properties of the martian interior give for each generalized spherical harmonic a set of frequency dependent solutions, consisting of the values of the displacement, of the potentials and of the stress-strain tensor for each radius from the center to the surface. Due to an apparent singularity at the origin, the numerical integration is done from the surface of a small initial sphere to the surface of the planet. An analytical solution is used for the initial sphere (Gilbert et al., 1960).

The martian core modeling is done essentially in four steps (see figure 2). At a given temperature and at fixed concentrations of iron and a light element, the core is completely liquid. The core will remain in this state with decreasing temperature until the liquidus curve is reached somewhere in the core. This happens first at the center, and liquid iron starts to precipitate to the center forming a growing solid inner core. The concentration of iron in the liquid core decreases with the iron precipitation going on until the eutectic point is reached. At this point the alloy of iron and a light element starts to freeze and build a layer around the iron kernel. With decreasing temperature this layer grows until the core is fully solidified.
4. RESULTS

The transfer function as a function of inner-core radius and period, for Mars, is shown in figure 3. The inner-core size varies from a small 100 km to 1400 km. The FCN is in the retrograde frequency-band and its period stays at a more or less constant value of 270 days for an inner-core size up to 1000 km, the precipitation regime. After a 1000 km inner-core size, when freezing of light elements around the inner-core occurs, the FCN period starts decreasing until it reaches a value around 188 days for an almost completely solid core.

During the precipitation regime, the FICN period, which is in the prograde frequency band, is increasing from 360 days at 100 km with growing inner-core size to 600 days at 1000 km. After precipitation, in the freezing regime a further increase in kernel size makes the period of the FICN decrease until it reaches a value of 200 days for a kernel size of 1400 km.

Figure 4 and figure 5 show the influence of an inner-core of 900 km and 1100 km on the transfer function. As the FICN is prograde, mainly the prograde nutations will be influenced. For a 900 km inner-core the FICN is located close to the prograde annual nutation frequency. Increasing the inner-core size to 1100 km decreases the FICN frequency to a value near the semi-annual nutation frequency.
Figure 3: B-Ratio of Mars as a function of inner-core radius

Figure 4: Influence of a 900 km inner-core on the transfer function

Figure 5: Influence of a 1100 km inner-core on the transfer function
5. CONCLUSIONS

The Free Core Nutation FCN and the Free Inner Core Nutation FICN periods depend on the inner core dimension. For an iron-rich alloy composition with 14 wt% of light element sulfur, they can vary from short periods to 280 days for the FCN and to 600 days for the FICN. The FCN and the FICN may be observed from resonances in nutations excited by the Sun (and to a smaller extend by Phobos and Deimos). Their observation by the NELander Ionosphere and Geodesy Experiment (NEIGE), in the framework of the NELander mission, will allow to infer properties of the mantle, the core, and of a possible inner core. In particular, the geophysical properties such as the dimensions and the physical state of the core will be determined from the resonances.

6. REFERENCES


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