HIGH FREQUENCY NUTATIONS

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ABSTRACT. Expressions for the torques exerted by components of the tidal potential of any degree and order (n, m) are given and used to obtain the high frequency nutations excited, and the corresponding polar motions, as functions of frequency. Special features relating to different types of excitations are discussed, and a few examples of numerical results are shown.

1. INTRODUCTION

High frequency (HF) nutations are those with frequencies exceeding 0.5 cycles per sidereal day (cpsd). They appear in both prograde and retrograde diurnal, semidiurnal, $\cdots$ bands, which are bands of width 1 cpsd centered at ± 1, ± 2, $\cdots$, cpsd. They arise from the action of the tidal potential on geopotential coefficients $C_{l,k}, S_{l,k}$, with $k > 0$. Recent rigid Earth nutation series (e.g., Bretagnon et al. 1997, Roosbeek and Dehant, 1998, Souchay et al. 1999) include many diurnal and subdiurnal terms with coefficients up to ~ 15 μas in $\Delta\psi \sin\epsilon$ and $\Delta\epsilon$. Such nutations may no longer be ignored, since realistic uncertainties as low as 5 μas have already been attained in the estimation of many (low frequency) nutations: see Herring et al. (2001). A number of works focusing specifically on HF nutations have appeared more recently, e.g., Bretagnon (1999), and Roosbeek (1999) for the rigid Earth, Bizouard et al. (2000), Getino et al. (2001) and Brzeziński (2001) for the nonrigid Earth, and references therein. The Bizouard et al. (2000) results were obtained by applying (an early version of) the transfer function of Mathews et al. (2001) to the rigid Earth numbers from earlier works. This transfer function, constructed for the low frequency nutations, is not quite correct for HF nutations, but the differences are small in most cases, and the amplitudes of high frequency circular nutations are expected to be less than 20 μas. The numbers obtained by Getino et al. (2001) for semidiurnal nutations of the nonrigid Earth are consistent with the transfer function being essentially constant across the semidiurnal band; but the elementary physical reason for such a behaviour stays hidden behind the complexities of their Hamiltonian formalism. The treatment by Brzeziński (2001), while approximate, does offer useful insights.

We outline here a general treatment of HF nutations based on a generalization of the torque equations of Sasao et al. (1980) for the whole Earth and the fluid core. (Since amplitudes beyond about 20 μas are not expected, inner core effects and certain features of the more recent nutation theories, e.g., electromagnetic couplings, are ignorable when seeking an accuracy of 0.1 μas on the HF nutations.) Their expression for the torque is replaced by one appropriate to the potential that drives the nutations in the band of interest. Solution of the pair of coupled equations is elementary, once the torque is given. One sees almost trivially that the transfer function is essentially equal to the ratio of the moments of inertia of the whole Earth and the mantle for all HF nutations except for the prograde diurnals. The Chandler resonance is important in this exceptional case, which includes a secular wobble / polar motion term, as will be seen below.

2. WOBBLE, NUTATION, AND POLAR MOTION: KINEMATICAL RELATIONS

The complex nutation variable $\tilde{\eta}(t)$ defined by

$$\tilde{\eta}(t) \equiv \Delta\psi(t)\sin\epsilon + i\Delta\epsilon(t), \tag{1}$$

is related to the mantle wobble variable $\tilde{m}(t)$:

$$i\frac{d\tilde{\eta}(t)}{dt} = \Omega_0\,\tilde{m}(t)\,e^{i\Omega_0 t}, \tag{2}$$

with $\tilde{m}(t) = m_1(t) + i\,m_2(t)$, where $\Omega_0\,m_1$ and $\Omega_0\,m_2$ are the equatorial components of the Earth's angular velocity vector $\mathbf{\Omega}$ in the terrestrial frame, Ω_0 being the mean angular velocity. The *kinematical relation* (2), which holds independently of the Earth's structure and deformability properties, implies the correspondence

$$\tilde{m}(t) = \tilde{m}(\sigma)e^{i\sigma\Omega_0 t} \quad \leftrightarrow \quad \tilde{\eta}(t) = \tilde{\eta}(\nu)e^{i\nu\Omega_0 t} = -\tilde{m}(\sigma)e^{i\nu\Omega_0 t}/(1+\sigma), \qquad (\nu = 1+\sigma), \tag{3}$$

between a wobble of frequency $\sigma\Omega_0$ (σ cpsd) and its associated nutation. The case $\sigma = -1$ is an exception: it leads to a secular variation of $\tilde{\eta}(t)$, namely, precession:

$$\tilde{\eta}(t) = -i\tilde{m}(-1)\,\Omega_0 t. \tag{4}$$

An unfamiliar situation wherein $\tilde{m}(t)$ is proportional to t will be encountered in this work. This behaviour occurs when the tidal potential (and hence the torque which drives the wobble) is of zero frequency, as will be seen later. In this case, one finds from equation (2) that

$$\tilde{m}(t) = i\Omega_0 K t \quad \leftrightarrow \quad \tilde{\eta}(t) = K(1 - i\Omega_0 t)e^{i\Omega_0 t}. \tag{5}$$

In the representation adopted by the IAU for the transformation between terrestrial and celestial reference frames, only the low frequency part of the motion of the terrestrial pole in space is considered as nutation, while the high frequency part is viewed as "polar motion", namely, the motion of the celestial pole in the terrestrial frame. Polar motion is represented by

$$\tilde{p}(t) \equiv x_p(t) - iy_p(t) = -\tilde{\eta}(t)e^{-i\Omega_0 t} \tag{6}.$$

3. DYNAMICS: THE TORQUE AND EQUATIONS OF MOTION

The Cartwright and Tayler (1971) tidal potential of type (n, m) with frequency ω is

$$V_\omega^{(n,m)}(\mathbf{r};t) = g_e H_\omega^{(n,m)}\,(r/a_e)^n\,\mathrm{Re}\,[\,\zeta_{n,m}Y_n^m(\theta,\varphi)\,e^{i\omega t}\,], \tag{7a}$$

$$Y_n^m(\theta,\varphi) = N_{n,m}\, P_n^m(\cos\theta)\, e^{im\lambda}, \qquad N_{n,m} = \left(\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}\right)^{1/2}, \tag{7b}$$

where a_e is the Earth's equatorial radius, $g_e = GM_E/a_e^2$ (M_E being the Earth's mass), and $H_\omega^{(n,m)}$ is the tide height. (An initial phase has been ignored; it can be easily restored.) The frequency ω is confined, for given m, to the interval

$$0 \le \omega < (1/2)\Omega_0, \quad (m=0); \quad \text{and} \qquad (m-1/2)\,\Omega_0 < \omega < (m+1/2)\,\Omega_0, \quad (m>0). \tag{8}$$

The factor $\zeta_{n,m}$ is so defined that the real part in (7a) yields a cosine or a sine as appropriate:

$$\zeta_{n,m} = 1, \quad (n-m) \quad \text{even}; \qquad \zeta_{n,m} = -i, \quad (n-m) \quad \text{odd}. \tag{9}$$

The *torque* exerted by $V_\omega^{(n,m)}$ on the Earth has both prograde and retrograde frequencies,

$$\omega_p = \sigma_p\Omega_0 \equiv \omega \qquad \text{and} \qquad \omega_r = \sigma_r\Omega_0 \equiv -\omega, \tag{10}$$

respectively. The orgin of the torques and of the wobbles and nutations that they produce, as displayed in Table 1, may be seen from the expressions given below for the complex combination $\tilde\Gamma_\omega^{(n,m)} \equiv (\Gamma_{\omega,1}^{(n,m)} + i\Gamma_{\omega,2}^{(n,m)})$ of the equatorial components of the torque.

Table 1. Origin of high- and low-frequency nutations

TGP of type	Acting on	Produces Wobbles	Produces Nutations	For
$(n,0)$	$(C_{n,1}, S_{n,1})$	Low Frequency	Pro Diurnal	$n>2$
$(n,1)$	$C_{n,0}$	Ret Diurnal	Low Frequency	$n\ge 2$
$(n,1)$	$(C_{n,2}, S_{n,2})$	Pro Diurnal	Pro Semidiurnal	$n\ge 2$
$(n,2)$	$(C_{n,1}, S_{n,1})$	Ret Semidiurnal	Ret Diurnal	$n>2$
$(n,2)$	$(C_{n,3}, S_{n,3})$	Pro Semidiurnal	Pro Terdiurnal	$n\ge 3$
$(n,3)$	$(C_{n,2}, S_{n,2})$	Ret Terdiurnal	Ret Semidiurnal	$n\ge 3$

$$\tilde\Gamma_\omega^{(n,m)}(t) = \tilde\Gamma^{(n,m)}(\sigma_p)\; e^{i\sigma_p\Omega_0 t} + \tilde\Gamma^{(n,m)}(\sigma_r)\; e^{i\sigma_r\Omega_0 t}, \tag{11}$$

$$\tilde\Gamma^{(n,m)}(\sigma_p) = (i\Omega_0^2\bar A)\;(-1)^{m+1}\, G_{n,m}^{(+)}\, H_\omega^{(n,m)}\,[\,\zeta_{n,m}\,(C_{n,m+1} + iS_{n,m+1})\,], \quad (0\le m<n), \tag{12a}$$

$$\tilde\Gamma^{(n,m)}(\sigma_r) = (i\Omega_0^2\bar A)\;(-1)^{m}\, G_{n,m}^{(-)}\, H_\omega^{(n,m)}\,[\,\zeta_{n,m}^*\,(C_{n,m-1} - iS_{n,m-1})\,], \quad (1\le m\le n), \tag{12b}$$

$$\tilde\Gamma^{(n,0)}(\sigma_r) = -(i\Omega_0^2\bar A)\, G_{n,0}^{(+)}\, H_\omega^{(n,0)}\, \zeta_{n,0}^*\,(C_{n,1} + iS_{n,1}), \tag{12c}$$

$$G_{n,m}^{(-)} = \left(\frac{2n+1}{4\pi}\frac{(n+m)!}{(n-m)!}\right)^{1/2} \frac{g_e M_E}{2(2-\delta_{m,1})\Omega_0^2\bar A}, \qquad G_{n,m}^{(+)} = (n-m)(n+m+1)G_{n,m}^{(-)}. \tag{13}$$

$\bar A \equiv (A+B)/2$ is the mean equatorial moment of inertia. The range of values of σ_p and σ_r for each (n,m) may be found from (10) and (8).

As in Sasao et al. (1980), the dynamical equations consist of the coupled equations for the wobble $\tilde m$ of the mantle and the differential wobble $\tilde m_f$ of the fluid core relative to the mantle:

$$\left(\frac{d}{dt} - ie\Omega_0\right)\tilde m(t) + \frac{1}{\bar A}\left(\frac{d}{dt} + i\Omega_0\right)[\tilde c_3(t) + \bar A_f\tilde m_f(t)] = \frac{\tilde\Gamma(t)}{\bar A\Omega_0}, \tag{14a}$$

$$\left(\frac{d}{dt} + i\Omega_0\,(1+e_f)\right)\tilde{m}_f + \frac{d}{dt}\left(\tilde{m} + \frac{\tilde{c}_3^f(t)}{\bar{A}_f}\right) = 0, \tag{14b}$$

wherein $\bar{A}_f$ is the mean equatorial moment of inertia of the fluid core, e, e_f are the dynamical ellipticities of the whole Earth and of the fluid core (e.g., $e = C/\bar{A} - 1$), and $\tilde{c}_3(t), \tilde{c}_3^f(t)$ are complex combinations, $\tilde{c}_3 \equiv c_{1,3} + ic_{2,3}$ and $\tilde{c}_3^f \equiv c_{1,3}^f + ic_{2,3}^f$ of the indicated elements of their respective inertia tensors. Triaxiality terms on the left hand side of eqs. (14) have been suppressed but will be considered in the context of prograde semidiurnal nutations excited by the (2,1) potential, to which they make marginal contributions.

These equations may be solved for prograde wobbles by taking $\tilde{\Gamma}(t)$ to be the $e^{i\sigma_p\Omega_0 t}$ term of (11) together with (12a), and for retrograde wobbles, using the other term of (11) with (12b) or (12c) depending on the value of m.

In going over to the frequency domain, we generalize the expressions of Sasao et al. (1980) for $\tilde{c}_3$ and $\tilde{c}_3^f$ to potentials of arbitrary types (n, m):

$$\tilde{c}_3 = \bar{A}\,[\kappa(\tilde{m} - \tilde{\phi}\delta_{n,2}\delta_{m,1}) + \xi\tilde{m}_f], \qquad \tilde{c}_3^f = \bar{A}_f\,[\,\gamma(\tilde{m} - \tilde{\phi}\delta_{n,2}\delta_{m,1}) + \beta\tilde{m}_f\,], \tag{15}$$

wherein $\tilde{\phi}$, which is a dimensionless equivalent of H_ω, and $\tilde{m}, \tilde{m}_f$, are all viewed as functions of σ. The $\tilde{\phi}$ terms represent the contributions to $\tilde{c}_3$ and $\tilde{c}_3^f$ from the deformation due to the *direct* action of the tidal potential; they vanish for prograde frequencies (there being no prograde tidal waves), and for all $(n, m) \neq (2, 1)$ excitations (because of the spherical harmonic structure of the displacement fields produced). The other terms are due to centrifugal perturbations which are of the $(2, 1)$ type. Each of the compliance (deformability) parameters κ, ξ, γ, and β has a small complex and frequency dependent part when the effects of mantle anelasticity and ocean tides on deformations of the solid Earth are taken into account (see Mathews et al. (2001)). The frequency domain equations obtained from eqs. (14) are then, for all wobbles except the retrograde ones due to $(2, 1)$ potentials,

$$[\,(\sigma - e) + (1+\sigma)\kappa\,]\,\tilde{m}(\sigma) + (1+\sigma)(\xi + \bar{A}_f/\bar{A})\,\tilde{m}_f(\sigma) = \tilde{\Gamma}(\sigma)/(i\bar{A}\Omega_0^2), \tag{16a}$$

$$(1+\gamma)\,\sigma\,\tilde{m}(\sigma) + [\,1 + e_f + (1+\beta)\sigma\,]\,\tilde{m}_f = 0, \tag{16b}$$

where $\tilde{\Gamma}(\sigma)$ is $\tilde{\Gamma}^{(n,m)}(\sigma_p)$ of eq. (12a) or $\tilde{\Gamma}^{(n,m)}(\sigma_r)$ of eq. (12b) or (12c) according as σ is chosen as σ_p or σ_r. The absence of the $\tilde{\phi}$ terms causes the transfer function corresponding to eqs. (16) to differ from the $(2, 1)$ case.

4. WOBBLES DUE TO POTENTIALS OF TYPE (n, m)

For $(n, m) \neq (2, 1)$

Solution of eqs. (16) is almost trivially done. One obtains

$$\tilde{m}(\sigma) = \frac{X(\sigma)}{D(\sigma)}\,\frac{\tilde{\Gamma}(\sigma)}{i\Omega_0^2 A}, \tag{17}$$

where $D(\sigma)$ is the determinant 2×2 matrix of coefficients in (16), and $X(\sigma)$ is the coefficient $[1 + e_f + (1+\beta)\sigma]$ of $\tilde{m}_f(\sigma)$ in the second equation. As is well known,

$$D(\sigma) = (\bar{A}_m/\bar{A})(\sigma - \sigma_1)(\sigma - \sigma_2), \tag{18}$$

where $\bar{A}_m$ is the mean equatorial moment of inertia of the mantle, and σ_1 and σ_2 are, respectively, the Chandler wobble (CW) and free core nutation (FCN) resonance frequencies in cpsd:

$$\sigma_1 = \frac{A}{A_m}(e - \kappa) \quad \text{and} \quad \sigma_2 = -1 - \frac{A}{A_m}(e_f - \beta) \tag{19}$$

Now, both σ_1 and $|\sigma_2+1|$ are about 2×10^{-3}. It is evident then from (17) and (18) that $\tilde{m}(\sigma)$ will have a Chandler resonance for excitation by zonal ($m=0$) potentials for which $-1/2<\sigma<1/2$. The FCN resonance occurs if σ is in the *retrograde* diurnal band ($-3/2<\sigma<-1/2$), but the corresponding nutations, being of *low* frequencies, are not considered here. In all but these two bands, σ is close to $+1$ or ±2 or $\pm3,\cdots$, and hence,

$$\sigma-\sigma_1\approx\sigma,\quad \sigma-\sigma_2\approx\sigma+1,\quad X(\sigma)\approx\sigma+1,\quad D(\sigma)\approx(\bar{A}_m/\bar{A})\,\sigma(\sigma+1). \tag{20}$$

The approximation obtained by substituting these in (17) differs from $\tilde{m}_R(\sigma)$ of the rigid Earth only by the factor (A/A_m); thus the *transfer function* $T(\sigma)$ is very close to the constant (A/A_m).

In the case of the *low frequency wobbles* and the associated *prograde diurnal nutations* excited by zonal potentials $(n,0)$ for $n>2$, the value of κ becomes important because of its role in the Chandler resonance. (The $(2,0)$ and $(2,2)$ potentials do not produce any equatorial torque.) The complex anelasticity contribution to κ, which is strongly frequency dependent in the low frequency tidal band, is very relevant here. Its evaluation for frequencies down to about 1/7000 cpsd may be done as in Mathews et al. (2001) where certain subtle points involved are discussed in detail. Here we focus on the special case of excitation by the *zero frequency* term present in $(n,0)$ potentials with n even. The secular (or "fluid limit") values of the compliances have to be used for this constant term; for κ, the value may be shown to be e. The frequency domain equations do not apply then because the time dependence of $\tilde{m}(t)$ and $\tilde{m}_f(t)$ is secular, not periodic. Going back to the time domain equations (14), one finds, with $\kappa=e$, that

$$\tilde{m}(t)=i\Omega_0\,Kt,\quad \tilde{m}_f(t)=-K,\quad \text{with}\quad K=(A/A_m)K_R,\quad K_R=\tilde{\Gamma}(0)/(i\Omega_0^2A). \tag{21}$$

The solution $\tilde{m}_R$ for a *rigid* Earth is quite different, being just constant: $\tilde{m}_R=(K_R/e)$.

For prograde diurnal wobbles due to potentials of type $(2,1)$

Triaxiality terms which enter through the angular momenta $\mathbf{H}$ and $\mathbf{H}_f$ of the Earth and its fluid core are relevant here. They contribute the additional terms $-Z(d/dt+i\Omega_0)\tilde{m}^*$ to the left hand member of eq. (14a) and $-Z_f\,d(\tilde{m}^*+\tilde{m}_f^*)/dt$ to that of eq. (14b), where

$$Z=(B-A)e^{2i\alpha}/(2\bar{A})=(2M_Ea_e^2/\bar{A})(C_{2,2}+iS_{2,2})\quad\text{and}\quad Z_f=(B_f-A_f)e^{2i\alpha_f}/(2\bar{A}_f), \tag{22}$$

where $\alpha\approx-15°$ is the longitude of the minor equatorial principal axis, and quantities with subscripts f pertain to the core. In the absence of any reliable estimate for the triaxiality of the core, we assume its order of magnitude to be no larger than that of the whole Earth, and take $\alpha_f=\alpha$. The frequency domain equations for the prograde wobbles may then be seen to be

$$[\,(\sigma_p-e)+(1+\sigma_p)\kappa\,]\,\tilde{m}(\sigma_p)+(1+\sigma_p)(\xi+\bar{A}_f/\bar{A})\,\tilde{m}_f(\sigma_p)=\tilde{\Gamma}^p/(i\bar{A}\Omega_0^2)+Z\tilde{m}^*(\sigma_r), \tag{23a}$$

$$(1+\gamma)\,\sigma_p\,\tilde{m}(\sigma_p)+[\,1+e_f+(1+\beta)\sigma_p\,]\,\tilde{m}_f(\sigma_p)=Z_f\sigma_p\tilde{m}^*(\sigma_r)+Z_f\sigma_p\tilde{m}^*(\sigma_r), \tag{23b}$$

where $\tilde{\Gamma}^p$ stands for (12a) with $(n,m)=(2,1)$: $\tilde{\Gamma}^p=(15/8\pi)^{1/2}(g_e\bar{A}/a_e^2)ZH_\omega^{(2,1)}$. Both $\tilde{m}^*(\sigma_r)$ and $\tilde{m}_f^*(\sigma_r)$ (with $\sigma_r=-\sigma_p$), which are the frequency domain versions of $\tilde{m}^*(t)$ and $\tilde{m}_f^*(t)$, are known from the well-studied theory of low frequency nutations associated with the *retrograde* diurnal wobbles (frequency σ_r); their contributions may be shown to be marginal in any case. Solution of eqs. (23) is straightforward.

5. NUTATIONS

Once the wobbles $\tilde{m}(\sigma)$ are determined for the desired frequency band, prograde or retrograde, the associated nutations are readily computed from eqs. (3) (or from (5) with K from

(21), in the special case $\sigma = 0$); and the polar motions may be obtained from (6). One finds from (1) and (6) that these alternative pictures are linked by the following relations among the coefficients of the sine and cosine parts (identified by superscripts s and c, respectively) of $\Delta\psi(t)$, $\Delta\epsilon(t)$, $x_p(t)$, and $y_p(t)$:

$$x_p^s = y_p^c = -\Delta\psi^s \sin\epsilon = \Delta\epsilon^c = \eta_I(\nu) \qquad -x_p^c = y_p^s = \Delta\psi^c \sin\epsilon = \Delta\epsilon^s = \eta_R(\nu), \tag{24}$$

where $\eta_R = \mathrm{Re}\ \eta$, $\eta_I = \mathrm{Im}\ \eta$. These relations hold for (prograde/retrograde) circular nutations, but not for the elliptical nutations obtained by combining the pair of nutations with frequencies ν and $-\nu$. For example, we find $(\Delta\psi^s, \Delta\psi^c, \Delta\epsilon^s, \Delta\epsilon^c)$ to be $(-34.2, -4.2, -1.7, 13.6)$ and $(0.6, -0.1, -0.0, -0.2)$ μas respectively, for the prograde and retrograde rigid Earth nutations (due to (3,0) and (3,2) potentials, respectively), of period 1.03505 day. They combine to give $(-34.8, -4, 3, -1.6, 13.4)$ μas for the elliptical nutation, in full agreement with the SMART series of Bretagnon et al. (1997). Our computations were based on the tidal amplitudes of Roosbeek (1996) and geopotential coefficients $C_{k,l}$ and $S_{k,l}$ listed in Bretagnon et al. (1997). The effect of the Chandler resonance is reflected in the amplitudes found for the pair of polar motion terms due to a single term of the (3,0) potential: For the +3231 day polar motion, $(x_p^s, y_p^s) = (-16.2, 1.8)$ μas for the nonrigid Earth and $(-9.5, 1.2)$ for the rigid; and for -3231 days, $(12.3, -1.6)$ and $(7.9, -1.0)$, respectively. The secular polar motion due to the constant term of the (4,0) tide is $(dx_p/dt, dy_p/dt) = (-3.8, -4.3)$ μas/yr.

Tables of nutations/polar motions, and details of the derivation of the expressions for the torque and of the treatment of low frequency polar motions will be presented elsewhere.

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