VARIATIONS IN THE ORBITAL ELEMENTS OF ASTEROIDS INDUCTED BY THE COMPARISON HIPPARCOS-FK5

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ABSTRACT. In the last years the old reference system, based in the FK5 fundamental catalogue, has been replaced by the new fundamental system, initially based in the Hipparcos catalogue. The FK5 catalogue is included in Hipparcos, but some bias between them are to be expected. The bias may be summed up by the existence of infinitesimal rotation. They induct corrections to the angular elements of the asteroids referred to these catalogues.

1. CORRECTION MODEL

The 1st of January 1998 the old reference system, materialized by the old FK5, was replaced by the new reference system, initially materialized by the Hipparcos Catalogue for optical wavelength. The resulting reference system is consistent with the FK5 except for the existence of infinitesimal rotations. The correction model from with we compute the system rotations is based in the minimization of the O-C residuals. Basically, we consider the differences $\Delta\alpha_C = \alpha_C - \alpha_F$ and $\Delta\delta_C = \delta_C - \delta_F$, where with the C subindex we denote the coordinates obtained from a catalog which we suppose that contains systematic errors and with the subindex F we denote the coordinates obtained after removing the systematics errors of C and after applying an infinitesimal transformation given by a matrix R:

$$R = \begin{bmatrix} 1 & \Delta\xi & \Delta\eta \\ -\Delta\xi & 1 & \Delta\varepsilon \\ -\Delta\eta & -\Delta\varepsilon & 1 \end{bmatrix} \tag{1}$$

so the corrections model results:

$$\begin{aligned} \Delta\alpha_C \cos\delta &= \Delta\xi\cos\delta + \Delta\eta\sin\alpha\sin\delta - \Delta\varepsilon\cos\alpha\sin\delta \\ \Delta\delta_C &= \Delta\eta\cos\alpha + \Delta\varepsilon\sin\alpha \end{aligned} \tag{2}$$

Applying to this model a mean squares method we obtain a normal system of equations given

by:

$$A \begin{bmatrix} \Delta\xi \\ \Delta\eta \\ \Delta\varepsilon \end{bmatrix} = \overrightarrow{b} \tag{3}$$

with

$$A = \begin{bmatrix} \cos^2\delta & \frac{1}{2}\sin\alpha\sin(2\delta) & -\frac{1}{2}\cos\alpha\cos(2\delta) \\ \frac{1}{2}\sin\alpha\sin(2\delta) & \sin^2\alpha\sin^2\delta+\cos^2\delta & \frac{1}{2}\sin(2\alpha)\cos^2\delta \\ -\frac{1}{2}\cos\alpha\cos(2\delta) & \frac{1}{2}\sin(2\alpha)\cos^2\delta & \cos^2\alpha\sin^2\delta+\sin^2\delta \end{bmatrix} \tag{4}$$

and the independent term is the vector:

$$\overrightarrow{b} = \left(\Delta\alpha\cos^2\delta, \frac{1}{2}\Delta\alpha\sin\alpha\sin(2\delta)+\Delta\delta\cos\alpha, -\frac{1}{2}\Delta\alpha\cos\alpha\sin(2\delta)+\Delta\delta\sin\alpha\right)^t \tag{5}$$

2. VARIATIONS IN THE ANGULAR ELEMENTS OF ASTEROIDS INDUCED BY THE SYSTEM ROTATIONS

Let us consider the vectors h and p, which are, respectively, the normal to the orbital plane of the asteroid vector and the vector in the direction of the periaster and let Δh and Δp the corresponding incremental vectors. The composition of the matrix (1) with the transformation P into equatorial coordinates provides the equation:

$$\begin{aligned} R^t P(-\varepsilon)\, h &= P(-\varepsilon)\,[h+\Delta h] \\ R^t P(-\varepsilon)\, p &= P(-\varepsilon)\,[p+\Delta p] \end{aligned} \tag{6}$$

The application of the previous formulas to obtain variations of the angular elements of the asteroids gives the following relationships:

$$\begin{aligned} \Delta i &= \Delta\xi\sin\Omega\sin\varepsilon - \Delta\eta\sin\Omega\cos\varepsilon + \Delta\varepsilon\cos\Omega \\ \Delta\Omega &= \Delta\xi(\cos\varepsilon+\cos\Omega\cot i\cos\varepsilon) + \Delta\eta(\sin\varepsilon-\cos\Omega\cot i\cos\varepsilon) \\ &\quad -\Delta\varepsilon\sin\Omega\cot i \\ \Delta w &= [-\Delta\xi\cos\Omega\sin\varepsilon + \Delta\eta\cos\Omega\cos\varepsilon + \Delta\varepsilon\sin\Omega]\csc i \end{aligned} \tag{7}$$

Given the numerical values of the rotational parameters relating both systems from [2] and applying the formulas (7) the corrections to the angular elements of the asteroids give the following results (in arc seconds, No represents the number of the asteroid):

Table 1:

N^o	1	2	3	4	6	7	11	18	25	39	40
Δi	-.003	.020	.020	.005	.015	.003	.012	.018	.016	.019	.002
$\Delta\Omega$	.130	.027	.037	.178	.072	.009	.220	.076	-.005	.064	.290
Δw	-.003	.002	.003	.004	.009	-.004	.009	.008	-.010	.007	.001

Addittionally, we have calculated the values of partial derivatives of the elements with respect the initial ones following the perturbed elements method explained in [1] and the same derivatives with respect to the initial elements corrected according with Table1.

3. REFERENCES

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