IS THE DIFFERENTIAL ROTATION DETECTABLE FROM EARTH NUTATION ?

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ABSTRACT. In the past half decade, the possibility of a differential rotation between the inner core (IC) and the mantle (M) have been thoroughly studied from both geodynamo and seismological observations. In this paper, the effect on the Earth nutation and rotational modes of the possible differential rotation between the solid inner core, the fluid outer core and the mantle are calculated by numerical integration of the scalar equations of infinitesimal elastic-gravitational motion for a rotating, slightly elliptical Earth. We show that the effect of large differential rotation, compatible with the observation, according to several authors, can not be ignored in the nutation theory. The possibility to detect these differential rotations from the nutation observation is also discussed and shown to be at the present precision limit of the observations, even for large differential rotations . In the rotational modes, the motion equations in the Earth are used also to solve the eigenfrequencies of three eigen modes of the PREM Earth, i.e. the tilt-over mode (TOM), the Free Core Nutation (FCN) and the Free Inner Core Nutation (FICN), as well as their respective eigenfunctions in the particular case of differential rotation. For each eigen mode, we have obtained the displacement field, and in particular, the global rotational displacement and we have shown the differences in the amplitudes for the cases examined.

1. INTRODUCTION

It is usually considered that the three parts of the Earth, the solid inner core (IC), the fluid outer core (OC) and the mantle (M), rotate with the same mean angular velocity, $\Omega_0 = 7.292115\,10^{-5}$ rad/s i.e., 1 cycle per sidereal day.

Several years ago, Glatzmaier and Roberts (1995, 1996) suggested that, from their three-dimensional self-consistent geo dynamo, a super-rotation of the inner core relative to the mantle can be maintained by magnetic coupling between the inner core and an eastward thermal current in the fluid outer core, the differential rotation velocity of the inner core relative to the mantle being about 1 cycle/500 years, i.e. about 0.7°/year (eastward).

Less than one year later, several works reported, that differential rotation between the inner core and the mantle was indeed compatible with the observations (mainly seismological). For

example, Song and Richards (1996) reported a faster rotation of the inner core with respect to the mantle with 1 cycle per 300 years; Su et al. (1996) pointed out that the super-rotation velocity could be as fast as 3°/year and so on... These reports from both geodynamo and seismological observations are sometimes controversial. Souriau and Poupinet (2000) have shown that the interpretation of seismic data in terms of differential rotation needs further caution. More and more people think that, e ven though the differential rotation exists, the velocity may not be so large, for example, may be less than $+0.2^o$/yr and may vary with time (Tromp, 2001). But none of them could definitely show the non-existence of a differential rotation between the inner core and the outer core or mantle. Using seismic normal modes, Vidale et al. (2000) found the differential rotation velocity to be about 0.1^o/year, Laske and Master (1999) and Laske et al. (2000) have found another value 0.0^o/year. A theoretical study of the gravitational interaction between the inner core and the mantle has lead Buffet and Glatzmaier (2000) also to a very small angular velocity of 0.02^o/year. So, the question of whether the differential rotations among the Earth's three parts are real, still remains an open question, while the present trend is that this differential rotation, if it exist, would be very small.

This controversial subject is nevertheless very interesting because the study of differential rotations would provide us with new insights to the Earth interior. For example, constraints in the dynamo motions underlying the geomagnetic field would be provided, and the flow models that can reproduce secular variation of the geomagnetic field and its reversals would be refined. The viscosity in the outer core may be obtained with a new value that, in turn, may provide new information on melting conditions and temperature in the outer core, and may refine the estimates of the heat flux across the core-mantle-boundary. Moreover, due to the new distribution of the angular momentum of the three parts of the Earth, the rotation axis of inner core may not track the spin axis in the precession of the equinox, and it may also influence the gravity field (Song and Richards, 1996). These are examples, but the existence of differential rotation opens new questions, one of them being its influence on the Earth's orientation.

Here, we are interested in an important sub-question, i.e., if there is a differential rotation in the Earth, doe s it result in observable changes in the Earth nutation? Or say, is it detectable from the nutation observations? This is an important question in the frame of nutation modeling because nutation observations are interpreted in terms of physics of the Earth's interior and the parameters involved in MHB2000 model (Mathews et al. 2001), which has been adopted by the IAU (International Astronomical Union), do not incorporate any differential rotation between the inner core and the rest of the Earth.

In the next section, the principle of the Earth nutation and its calculation is briefly reviewed; several cases of the differential rotations are hypothesized in Section 3; in Section 4, the results of the contribution to the main Earth nutation terms and the four Earth modes are presented, and the possibility to detect these differential rotations from the nutation observation is also discussed; finally, the displacement field associated with each case is computed and compared with the nominal case.

2. EARTH NUTATION AND ITS CALCULATIONS

Besides the polar motion and the length-of-day variation, the precession and nutation are another set of important parameters describing the orientation of the Earth rotation axis in space. They are caused by the gravitational attraction of the Sun, the Moon and the planets on the Earth's equatorial bulge. Precession is the secular part, while the nutation is the collection of the short-period components (in fact, the precession does also involve periods, but very long periods which can be considered as trends around J2000). In the nutation series, there is a discrete number of terms of periods from 2 days to 300 years, and all of these periods can be

expressed as linear combinations of fundamental arguments related to the orbits of the Sun, the Moon and the planets.

The rigid Earth nutation theory can nowadays be very accurate at the level of a few tens of micro-arc-second (μas), because it relays on celestial mechanics considerations and ephemer edes (Souchay et al., 1999; Bretagnon et al., 1998; Roosbeek and Dehant, 1998). The nutation theory for a more realistic Earth model (usually called non-rigid Earth) is not precise at that level. Residuals with respect to Very Long Baseline Interferometry (VLBI) observations are at the level of a tenth of milli-arc-second (μas) or even less (20 μas) for nutations of which the periods are less than a few year (see Dehant et al., 1999, for a complete review). There are several approaches for computing the non-rigid Earth nutations. For example, Shirai and Fukushima (2001) obtained a pure numerical nutation model fitted from VLBI observation ; Mathews et al. (1991, 2001) presented a semi-analytical model, which is the basis of the IAU2000A nutation model, in which several geophysical parameters are fitted from VLBI observation ; Getino and Ferrandiz (1999) apply the Hamilton method from the nutation study of rigid Earth. In the numerical integration approach used by Wahr (1981), Dehant and Defraigne (1997), Schastok (1997) and Huang et al. (2001), the so-called Earth Transfer Function (ETF) is computed for each nutation frequency. It is the response of the Earth to the gravitational force, integrated from a set of scalar equations of the infinitesimal elastic-gravitational motion for a rotating, slightly elliptical Earth, with appropriate boundary conditions as well as an appropriate a priori Earth model (density profile and rheological property profiles) (Dahlen, 1972; Smith, 1974). The non-rigid Earth nutation can then be obtained from the convolution of the ETF and a certain rigid Earth nutation model. In this numerical integration approach, some parameters of geophysical characters of the Earth interior are included, therefore geophysical informations may be obtained from comparing the nutation results with the modern precise observations (mainly VLBI), and then, these informations can be used, in return, to improve the Earth model and the nutation model. This numerical integration approach is also used in this work.

3. HYPOTHESES AND SIMULATIONS

When no differential rotation is considered, the mean velocities of all the three parts of Earth are considered to be equal to:

$$\Omega(r) = \Omega_0 = 2\pi/sid.day = 7.292115\ 10^{-5} radin/s$$

We suppose the rotation velocity throughout the Earth is a stepwise function as:

$$\Omega = \Omega(r) = \begin{cases} \Omega_0 + \delta\Omega_{IC} & (0 < r \leq r_{ICB}) \\ \Omega_0 + \delta\Omega_{OC} & (r_{ICB} < r \leq r_{CMB}) \\ \Omega_0 + \delta\Omega_M & (r_{CMB} < r \leq r_M) \end{cases} \quad (1)$$

where r_{ICB}, r_{CMB} and r_M are the radii of the inner core (of the Inner Core Boundary, ICB), the outer core (of the Core Mantle Boundary, CMB), and the mantle respectively.

So, in addition to the nominal case,

$$\delta\Omega_{IC} = \delta\Omega_{OC} = \delta\Omega_M = 0$$

we discuss the following three cases:

1. Only the inner core rotates differentially , so, $\delta\Omega_{OC} = \delta\Omega_M = 0$.

Case 1A: the inner core rotates faster than the other parts with 1°/year eastward, which is the case of Song and Richards (1996);

Case 1B: the super-rotation velocity of the inner core is 5°/year eastward, which can be considered as the maximum value found in the literature, deduced from present seismic observations;

2. Only the fluid outer core is hypothesized to rotate differentially, so, $\delta\Omega_{IC} = \delta\Omega_M = 0$.
Case 2A: if the change of the angular momentum of the inner core due to the differential rotation of the inner core of $\delta\Omega_{IC} = 5^o$/year is transferred to the outer core, then we can obtain by angular momentum conservation, from $\delta\Omega_{OC}C_{outer\ core} = \delta\Omega_{IC}C_{inner\ core}$, that $\delta\Omega_{OC}$ = 0.003^o/yr , where $C_{outer\ core}$ and $C_{inner\ core}$ are the principal polar moments of inertia of the outer core and inner core respe ctively.
Case 2B: the super-rotation velocity of the outer core is hypothesized to be $\delta\Omega_{OC}$ =+0.1 ^{o}/year (eastward).
3. There is no differential rotation among the three parts ($\delta\Omega_{IC} = \delta\Omega_{OC} = \delta\Omega_M = 0$). But in the history, mainly due to the dissipation by tidal friction, there is a secular deceleration, $\Delta\Omega_0/\Delta t = -5.5\ 10^{-22}$ (rad/s^2). So
Case 3A: 0.1 Million years ago, $\Omega = \Omega_0 + 3.1^o/year = (1 + 2.38\ 10^{-5})\Omega_0$;
Case 3B: 1 Million years ago, $\Omega = \Omega_0 + 31^o/year = (1 + 23.8\ 10^{-5})\Omega_0$. The simulation results are presented in the following section.

4. RESULTS AND DISCUSSIONS

4.1. Contribution to the main nutation terms

Table 1 gives the largest contribution of the differential rotation to the main nutation terms, and the unit is mas.

Table 1: The contribution of the differential rotation to main nutations (mas)

Case	$\delta\Omega_{IC}$	$\delta\Omega_{0C}$	$\delta\Omega_M$	$\Delta\psi^{ip}_{18.6yr}$	$\Delta\psi^{op}_{18.6yr}$	$\Delta\epsilon^{op}_{18.6yr}$	$\Delta\epsilon^{ip}_{18.6yr}$
1A	1	0	0	-0.01	0.	0.	+0.01
1B	5	0	0	-0.02	-0.01	0.	+0.01
2A	0	0.003	0	-0.03	0.	0.	+0.02
2B	0	0.1	0	-1.03	-0.02	0.	+0.53
3A	3.1	3.1	3.1	-0.24	-0.02	0.	+0.12
3B	31	31	31	-2.83	-0.23	-0.07	+1.49

Notes:
3A) $\Delta\psi^{ip}_{0.5yr}$ = -0.02, $\Delta\epsilon^{ip}_{0.5yr}$ = +0.01
3B) $\Delta\psi^{ip}_{0.5yr}$ = -0.17, $\Delta\psi^{op}_{0.5yr}$ = -0.02, $\Delta\epsilon^{ip}_{0.5yr}$ = +0.07, $\Delta\psi^{ip}_{1yr}$ = +0.02, $\Delta\psi^{op}_{13.66d}$ = -0.03, $\Delta\epsilon^{ip}_{13.66d}$ = +0.01

F rom these numbers, we can find the following considerations:
1) The differential rotations mainly influence the 18.6 year in-phase (ip) components of the nutations in longitude ($\Delta\psi$) and in obliquity ($\Delta\epsilon$); this provides a large contribution on the retrograde in-phase 18.6 year nutation;
2) The super-rotation of the inner core contributes to nutation, for a super-rotation velocity of 5°/year, but the effect is very small: even for this large differential velocity, it contributes only to the $\Delta\psi^{ip}_{18.6yr}$ with 0.02 mas and $\Delta\epsilon^{ip}_{18.6yr}$ with 0.01 mas;
3) On the other hand, all results show that the importance of the outer core is obviously large: if the differential rotation velocity of the outer core is 0.1^o/year relative to the inner core and the mantle, it can influence $\Delta\psi^{ip}_{18.6yr}$ and $\Delta\epsilon^{ip}_{18.6yr}$ at the mas level and 0.5 mas level respectively. Again, the largest contribution is on the 18.6 year retrograde nutation. For the case of 2A, the effects of the differential velocity of 0.003^o/yr is similar to the effects of the case of 1B, i.e., $\delta\Omega_{IC}$

$= 5^o$/year, which was expected.
4) If a relative change in Ω_0 of 2 10^{-5} is considered, which was the case 0.1 million years ago, the associated 18.6 year nutation changes are $\Delta\psi^{ip}_{18.6yr} = 0.24$ mas and $\Delta\epsilon^{ip}_{18.6yr} = 0.12$ mas; while for the case of a relative change of 2 10^{-4}, which was the case of 1 million years ago, the changes of $\Delta\psi^{ip}_{18.6yr}$ and $\Delta\epsilon^{ip}_{18.6yr}$ can reach about 2.8 and 1.5 mas respectively. Again, this situation will correspond to a maximum change in the retrograde 18.6 year nutation. It must be noted that the changes in the amplitudes of the nutation do not incorporate the changes in the flattening associated with these changes in rotation.

As general conclusion, the values obtained in all simulations show im portant contributions to nutations, some times well above the actual precision of the observation (20 to 40 μas in the frequency domain). They show that for the level of differential rotation that we have chosen, the contributions can not be ignored in the nutation theory. The exageration done on purpose has pushed us to evaluate the actual possibilities for the differential rotations and the results are presented in Huang and Dehant (2001).

4.2. Influence on the normal mode periods

In all simulations, the influence of the differential rotation on the normal mode periods is also obtained (see Table 2). We have examined the Tilt-Over-Mode (TOM), which represents a steady rigid rotation of the body around an axis that does not coincide with the z-axis of the uniformly rotating reference frame. We have also examined the Chandler Wobble (CW), associated to the equatorial bulge of the Earth and related to the non-coincidence of the rotation axes and the mean polar moment of inertia l axes, the Free Core Nutation (FCN) and Free Inner Core Nutation (FICN), related mainly to gravitational and fluid pressure interactions at the CMB and ICB respectively.

Table 2: The influence of the differential rotation to normal mode periods (sidereal day)

Case	$\delta\Omega_{IC}$	$\delta\Omega_{0C}$	$\delta\Omega_M$	TOM	CW	FCN	FICN
normal	0	0	0	1-3.47 10^{-8}	-408.38	-457.67	+470.61
1A	1	0	0	1-2.90 10^{-8}	-408.38	-457.67	+472.40
1B	5	0	0	1-0.66 10^{-8}	-408.38	-457.66	+479.10
2A	0	0.003	0	1-3.21 10^{-8}	-408.38	-457.67	+470.61
2B	0	0.1	0	1+5.10 10^{-8}	-408.38	-457.52	+470.61
3A	3.1	3.1	3.1	1+2.361 10^{-5}	-408.39	-452.73	+475.92
3B	31	31	31	1+2.364 10^{-4}	-408.48	-412.67	+529.76

Notes of units:
1) for the $\delta\Omega_{IC}$, $\delta\Omega_{0C}$ and $\delta\Omega_M$, the unit is degree/year;
2) the unit of TOM and CW is sidereal day observed from the frame tied with the rotating Earth body, while the unit of FCN and FICN is sidereal day observed from the inertial space;
3) for the cases 3A and 3B, the unit is sidereal day.

In Table 2, the unit is degree/year for $\delta\Omega_{IC}$, $\delta\Omega_{0C}$ and $\delta\Omega_M$; the unit of TOM and CW periods is sidereal day observed in a frame tied with the rotating Earth, while the unit of FCN and FICN periods is sidereal day observed from the inertial space. It must be noted that the periods listed here are for an Earth in hydrostatic equilibrium (for that reason, there is about 30 days difference with the observed period for FCN). This does not prevent us to obtain the induced changes in the periods, which was the objective of this paper. For the cases 3A and 3B, the unit is sidereal day.

The period of the TOM is the spin period of the Earth and should accurately be one sidereal

day. This mode is not related to the internal constitution of the body. The result of this period from integration can serve as a check of the deformation theory and the numerical calculation (Smith, 1977; Rogister, 2001).

From Table 2, it can be seen that:

1) Due to the differential rotations in the interior, the period of the TOM can change from case to case but the changes are very small. In case 2B, its difference with 1.0 changes sign from minus to plus. The effects remain very small except when the global rotation is changing as in cases 3A and 3B.

2) The period of Chandler Wobble is not changed for the cases 1 and 2. This is because the CW represents the rotation of the instantaneous rotation axis about the axis of the polar principal moment of inertia, and the cases 1 and 2 do not change either the instantaneous rotation axis and have a negligible effect on the polar principal moment of iner tia. However, for the case 3, ignoring tidal friction on the polar principal moment of inertia, the rotation speed around the instantaneous rotation axis changes with 2.38 10^{-5} in case 3A and 23.8 10^{-5} in case 3B. This results in a change of the CW period with similar ratio.

3) The FCN period is quasi-diurnal (but larger than 1 day) in the terrestrial frame or about -460 (retrograde) day (for an Earth in hydrostatic equilibrium) in the inertial space. It is usually thought to depend mainly on the ellipticity of the core-mantle-boundary. For case 2B, in which the differential rotation speed between the outer core and the mantle reaches 0.1^o/year, the FCN period changes about 0.15 day.

4) Similar to the FCN, the FICN has a quasi-diurnal period but less than 1 day in the terrestrial frame. It is a prograde mode in the inertial space. Cases 1A and 1B indicate that the effects of the differential rotation of the inner core are dominant in the FICN. However, even though the possible dif ferential rotation (1°/year, case 1A) of the inner core can change the period of FICN with about 2 days, it is still difficult to be seen from nutation because the FICN contributes to a very small amount to nutation (either the total nutation or the prograde annual nutation), and the free mode itself, contrary to the FCN, is not observed at the present-day precision of the observation.

4.3. The TOM, FCN and FICN eigenfunctions for the hypothesized differential rotations.

Here, we present the results about the eigenfunctions of the three modes: TOM, FCN and FICN.

We note $W_\ell m$ the toroidal displacement of degree ℓ and order m, while $U_\ell m$ and $V_\ell m$ are the spheroidal radial and spheroidal tangential displacements. Due to the Earth rotation and flattening, the displacement field in response of the Earth to a tidal potential of degree l and order m, is at the first order, a sum of spheroidal displacement (ℓ, m) $(\ell+2, m)$, $(\ell-2, m)$ and toroidal displacements $(\ell-1, m)$, $(\ell+1, m)$.

For the FCN, W_1^1 is very large in the outer core, decreases linearly there, jumps over the zero mean line at the CMB and increases linearly in the mantle. Although not changing linearly in the inner core, it decreases monotonously with similar trend as in outer core. It reflects that the FCN is the relative rotation between outer core and mantle. We think that the non-constant increase in the inner core is related to the starting solution at the center and is not real. The slops, i.e. the linear trends, S_{OC} and S_M, can be obtained from least square fit in outer core and mantle separately. Their ratio S_{OC}/S_M is about -7.8. U_2^1, V_2^1 and the non-linear part of W_1^1 are only 10^{-3} of the linear part of W_1^1 in the outer core.

For the FICN, the trends of W_1^1 in inner core and outer core are obviously reversed with a jump over the zero mean line at ICB, and the value in the inner core is about two-order of m agnitude larger than in the outer core. It shows that the FICN happens at ICB (where the displacement are maximum) and is mainly related to the inner core. The slop near the ICB

(representing the real slop due the initial solution problem), S_{IC}, can be compared with the slop in the outer core, S_{OC}; their ratio S_{IC}/S_{OC} is about +0.6.

Again, U_2^1, V_2^1 and the non-linear part of W_1^1 are only 10^{-3} of the linear part of W_1^1 in the inner core.

5. CONCLUSIONS

In the above sections, the contributions of the possible differential rotation between the three parts of the Earth's interior to the Earth nutation have been examined and the associated changes in the periods of four modes are calculated by using a numerical approach. The cases considered are rather extreme cases in order to see the order of magnitude of the effects. For the hypothesized cases presented, we have shown that the differential rotation, if large, can not be ignored in the modern nutation theory and observation. We have shown that, if they are large, it is possible to detect them from the nutation observation. We nevertheless believe that, considering the recent studies of the differential rotation of the inner core, the effects on nutations are at the limit of VLBI precision (20 mas) or less. In this paper, we have also shown that the amplitudes of the principal nutations (18.6 year nutations) may have changed as the global mean rotation has changed.

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