THEORETICAL CONSIDERATIONS ON PRECESSION AND
NUTATION REFERRED TO THE GCRS

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ABSTRACT. IAU 2000 Resolutions bring significant improvement for precession and nutation. This concern definitions, theoretical models and parameters to be used. The celestial reference system is clearly specified in the framework of General Relativity, the definition of the pole to which the motion refers is extended to the high frequency domain (Celestial Intermediate Pole, CIP), the new theory (IAU 2000) for representing the motion is accurate at the level of 0.1 mas and the parameters to be used (celestial coordinates of the CIP) are close to the parameters to which the observations are sensitive. There is moreover a clear separation between Earth rotation and precession-nutation thanks to the use of the non-rotating origin on the moving equator (Celestial and Terrestrial Ephemeris Origins). This paper describes the consequences of these resolutions and show their corresponding improvements for precession-nutation.

1. INTRODUCTION
After the adoption of the International Celestial Reference System, ICRS, by the IAU in 1997, new IAU Resolutions have been adopted in 2000 that specify the new models and algorithms to be used in order to take advantage of the accuracy and the fundamental property of non-rotation of the ICRS. These resolutions bring significant improvements concerning definitions, theoretical models and parameters to be used and several of them concern more specifically precession and nutation. The first improvement is that the definition of the celestial reference system is now clearly specified in the framework of General Relativity and that the metric tensors are explicitely written both in the BCRS and in the GCRS in a compact and self-consistent form. Second, the definition of the pole to which the motion refers (Celestial Intermediate Pole, CIP) is sharpened in order to consider the non-negligible high frequency variations at the microsecond level. Third, a new geophysically-based theory (IAU 2000) for representing precession and nutation is adopted that is accurate at the level of 0.1 mas. Fourth, the Earth Orientation Parameters to be used (celestial coordinates of the CIP and Earth Rotation Angle) are close to the parameters to which the observations are sensitive and the use of the non-rotating origin on the moving equator (Celestial and Terrestrial Ephemeris Origins) is recommended so that UT1 is directly proportional to the Earth Angle of Rotation resulting in a clear separation between Earth rotation and precession-nutation.

The following sections explain the consequences of the implementation (on 1 January 2003) of the IAU resolutions concerning precession and nutation and the corresponding improvements.
2. CONSEQUENCES OF THE IAU RESOLUTIONS OF THE XXIVth GA 2000 FOR PRECESSION AND NUTATION

2.1 IAU 2000 Resolutions concerning precession and nutation

The IAU resolutions of the XXIVth GA 2000 that have consequences for precession-nutation are the following: Resolution B1.3 on *Definition of BCRS and GCRS*, Resolution B1.6 on *IAU 2000 Precession-Nutation Model*, Resolution B1.7 on *Definition of Celestial Intermediate Pole* and Resolution B1.8 on *Definition and use of Celestial Ephemeris Origins and Terrestrial Ephemeris Origins*.

2.2 Consequences of IAU Resolution B1.3 for precession and nutation

Resolution B1.3 provides clarification of IAU’s 1991 definition of the coordinate systems in the framework of GR for distinction between the celestial system for Solar System or for the Earth. It specifies that the system of space-time coordinates within the framework of General Relativity be called the Barycentric Celestial Reference System (BCRS) for the Solar system and the Geocentric Celestial Reference System (GCRS) for the Earth and specify the metric to be used in these systems. The celestial reference system to refer precession nutation parameters and Earth’s angle of rotation is thus changed from the FK5 to the GCRS.

2.3 Consequences of IAU Resolution B1.6 for precession and nutation

Resolution B1.6 describes the new IAU 2000 precession-nutation model which replaces the IAU 1976 Precession Model (Lieske *et al.* 1977) and IAU 1980 Theory of nutation (Seidelmann 1982). The IAU 2000 model (see IERS Conventions 2000) is based on (i) the rigid Earth nutation model (Souchay *et al.* 1999), (ii) the transfer function of Mathews *et al.* (2002) which is explicitly expressed as a function of basic Earth parameters that have been estimated from VLBI nutation offsets.

The model is IAU 2000A for 0.2 mas level (Mathews *et al.* 2002) or IAU 2000 B (its shorter version) for 1 mas level (McCarthy & Luzum 2003), with in both cases, associated precession and obliquity rates and associated celestial pole offsets. The 106 nutations of the IAU 1980 model is thus replaced by more than 1300 nutations with in-phase and out-of-phase components and the secular terms in longitude and obliquity of the IAU 1976 precession are revised.

2.4 Consequences of IAU Resolution B1.7 for precession and nutation

Resolution B1.7 recommends that the CIP be the pole, the motion of which is specified in the GCRS by the motion of the Tisserand mean axis of the Earth with periods greater than two days, and that it be realized by the IAU 2000A model for precession and forced nutation for periods greater than two days plus additional time-dependent corrections provided by the IERS through appropriate astro-geodetic observations. It also specifies that the motion of the CIP in the ITRS be provided by the IERS through appropriate astro-geodetic observations and models including high-frequency variations.

2.5 Consequences of IAU Resolution B1.8 for precession and nutation

Resolution B1.8 recommends that:

(i) UT1 be linearly proportional to the Earth Rotation Angle defined as the angle measured along the equator of the CIP between the unit vectors directed toward the Celestial Ephemeris Origin (CEO) and the Terrestrial Ephemeris Origin (TEO) (i.e. “non-rotating origin” as defined by Guinot, 1979);

(ii) the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS.

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This Resolution leads to abandon the current parameters in the FK5 System and especially the use of the true equinox to reckon the Earth Rotation Angle, and adopt instead the NRO (Guinot 1979) as the origin on the moving equator (CEO and TEO). It also leads to abandon the current formulation combining the motions of the equator and the ecliptic with respect to the ICRS and use instead parameters that include both precession and nutation.

3. IMPROVEMENTS OF THE IAU 2000 PRECESSION-NUTATION

3.1 Improvement in the reference system for precession-nutation

The FK5 was based on positions and proper motions of bright stars and was oriented so that at the “epoch”, the positions are referred to the best estimate of the location of the mean pole and mean equinox. The proper motions of stars were evaluated so that, for the adopted model of precession, they provide the best access to the mean pole and mean equinox of epoch, at any other date. The direction of the mean pole and mean equinox of date, which had a major role in the realization of the FK5, were the natural directions of reference for the EOP. The FK5 System was composed of the FK5 catalog of positions and proper motions of fundamental stars and of the associated models: IAU1976 precession, IAU 1980 nutation and the conventional relationship between Greenwich sidereal time and UT1 (Aoki et al. 1981).

The ICRS, adopted by the IAU as the International Celestial Reference System since the 1st January 1998, is based on barycentric directions of distant extragalactic objects (Ma et al 1998) and its definition is independent of the models used for precession and nutation and of the relation between Earth Rotation Angle. Its fundamental property is that the barycentric directions of the extragalactic objects show no global rotation with respect to them. The GCRS follows the kinematical condition of absence of global rotation of geocentric directions w.r.t. the objects defining the ICRS. The ICRS was aligned with the FK5 at J2000.0 so that its x-axis is approximately directed to the J2000 dynamical mean equinox and its z-axis is approximately directed towards the J2000 mean pole. However, this was in order that the ICRS be consistent with the FK5, but no attempt has been made to refer the positions of the sources to the mean pole and mean equinox at J2000.0 and it was decided that further improvements of the ICRS will be accomplished without introducing any global rotation. This corresponds to a conceptual change, namely the abandonment of the link of the conventional celestial reference system with the precession-nutation model. The GCRS is independent from such models. Such a change requires to adopt a definition of the equatorial system without any relation to the orbital motion of the Earth and consequently to abandon the equinox as the origin on the equator. Moreover, as the mean pole is no more a fundamental point of the celestial frame of reference, precession has no reason to be considered separately from nutation, these two motions being observationally not separable.

3.2 Improvement in the precession-nutation theory

The new precession-nutation model (MHB2000) corresponds to an improvement both in the accuracy of the rigid Earth nutation (Bretagnon et al. 1998, Souchay et al. 1999, Roosbeck & Dehant 1999) and in the theoretical expression of the transfer function (Mathews et al. 2002) between rigid and non-rigid nutations. This transfer function is based on Basic Earth Parameters (BEP) which have been estimated from VLBI observations providing corrections to a number of prograde and retrograde nutations and to the precession rates in longitude and obliquity.

In addition to the MHB model for nutations of periods greater than two days, there are also recent models for polar motion which take account of the diurnal and sub-diurnal nutations for non-rigid Earth models (Bizouard et al. 2000, Brzeziński & Capitaine 2003, Mathews & Bretagnon 2003). Diurnal and sub-diurnal tidal variations in polar motion and Earth’s rotation have also to be taken into account (Herring & Dong 1994).
3.3 Improvement of the pole of reference (CIP) for precession-nutation

The definition of the CIP sharpens the definition of the CEP by taking into account the high frequency variations of polar motion, that need to be considered in accurate theory and estimates (Brzeziński & Capitaine, 1993, Capitaine 2000a and 2000b). According to Resolution B1.7, the CIP is an intermediate pole separating, by convention, the motion of the pole of the TRS in the CRS into two parts (see the schematic representation below):

- the celestial motion of the CIP (precession/nutation), including all the terms with period greater than 2 days in the CRS (i.e. frequencies between $-0.5$ cpsd and $+0.5$ cpsd), plus the celestial pole offsets; the forced nutations with periods less than two days are, in contrast, included in the model for the motion of the CIP in the ITRS.

- the terrestrial motion of the CIP (polar motion), including all the terms outside the retrograde diurnal band in the TRS (i.e. frequencies lower that $-1.5$ cpsd or greater than $+0.5$ cpsd); the motion of the CIP in the ITRS is provided by observations taking into account a predictable part specified by a model including high frequency variations.

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<tr>
<th>frequency in the TRS</th>
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The pole is thus defined such that it can be realized by a model as accurately as necessary and be not dependent on the technique of observation. Note that the corrections to the model for the motion of the CIP in the ITRF may be estimated by extracting the high frequency signal in the pole coordinates. Note also that the forced nutations with periods less than two days (prograde diurnal and prograde semi-diurnal nutations) are not included in the nutation model, but is included in the model for the motion of the CIP in the TRS as follows:

- prograde diurnal terms in nutation are considered as long periodic prograde and retrograde variations of polar motion $27\ d, \ldots$ (amplitude of the order of $15\ \mu$as),

- prograde semi-diurnal terms in nutation are considered as prograde diurnal variations of polar motion (polar libration)(amplitude of the order of $15\ \mu$as).

3.4 Improvement in the parameters for precession-nutation

Resolution B1.8 introduces a change of parameters from the classical angles for precession ($\epsilon_A, \zeta_A, \gamma_A, \theta_A, \chi_A$) and nutation ($\Delta \psi, \Delta \epsilon$) and the angle of Earth’s rotation (GST) referred to the equinox of date $\gamma$ in the FK5 to new parameters $X, Y$ for positioning the CIP in the GCRS and the Earth Rotation Angle (ERA) as the angle from the CEO to the TEO on the equator of the CIP.

The new $X, Y$ parameters for positioning the CIP are the $x, y$ coordinates of the CIP unit vector in the GCRS (Capitaine 1990, McCarthy 1996). The change of origin for the Earth’s angle of rotation from the classical equinox to the non-rotating origin (CEO) provides a clear separation between precession-nutation and Earth’s rotation and a definition of UT1 as directly proportional to the ERA.

The use of an intermediate frame referred to the CIP and the new parameters clearly separate high frequency and low frequency motions and reduce to five the parameters for transformation between ITRF and GCRF:

- position of the CIP in the GCRS: $E, d$,
- position of the CIP in the ITRS : $l', g$,
- Earth’s angle of rotation : $\theta$. 

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There is thus a symmetric representation for the celestial and terrestrial parts of the motion:
\[ PN(t) = R_3(-E).R_2(d).R_3(E) \quad \text{and:} \quad W(t) = R_3(-F).R_2(g).R_3(F) \quad (2). \]

The coordinates of the CIP in the GCRS \((X = \sin \cos E \text{ and } Y = \sin \sin E, E \text{ and } d \text{ being the GCRS polar distance and azimuth of the CIP})\) provide the celestial position of the CIP; they include precession, nutation, the coupling effects between precession and nutation and the offsets \(\xi, \eta, \delta, \omega, \alpha, \rho\) of the precession-nutation models at J2000 with respect to the pole and equatorial origin of the GCRS. They fulfill all the requirements for EOP in the GCRS and can be derived from the precession and nutation model and developed as function of time (Capitaine et al. 2000). The position of the CEO in the GCRS is given by the quantity \(s\), which can be given by a development as function of time (Capitaine et al. 2000).

4. FURTHER IMPROVEMENTS NEEDED

The future solutions of Earth rotation theory should be referred to the GCRS and not to the mean ecliptic of date as they are provided in the current expressions.

The use of the celestial coordinates \(X, Y\) of the CIP, which can be expressed as function of the precession and nutation angles referred to a fixed ecliptic, can provide solutions which would be closer to the observables and would therefore be more adequate than the current precession and nutation angles for estimating theoretical parameters from the observations.

This would contribute in the future improvement of the estimated amplitudes of the "geophysical" nutations from VLBI observations.

5. REFERENCES


Brzeziński & Capitaine, 2003, this Volume, p 53


Capitaine, N., 2000 b, in “Towards Models and Constants for Sub-microarcsecond Astrometry”.


