ANALYTICAL SOLUTION OF THE MARS MOTION

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ABSTRACT. Taking into account the progress of the observations, in particular the distances Earth-planet, we undertook the construction of new analytical solutions of the motion of the planets. It is desirable to reach precisions, for the distance Earth-Mars for example, of about a few tens of meters. One of the difficulties of the study of the motion of Mars is the determination of the perturbations by the asteroids, the masses of which are badly known. We examine the most significant perturbations of the longitude of Mars by the asteroids in analytical form and in numerical form.

1. INTRODUCTION

The analytical theories VSOP of the motion of the planets were built at the Bureau des longitudes in the 1980's. The improvement of the techniques of observation and the needs for the spacecraft navigation make necessary the construction of new solutions of high accuracy. For example, the maximum uncertainty on the geocentric distance of Mars is expected to be a few tens of meters over short intervals of time about ten years.

Recently, the construction of such solutions was undertaken : VSOP2000 by Moisson (2000) and a new solution is under development by an iterative method. The integration constants of this last solution will be determined by adjustment with DE403 or DE405 (1995) and also by direct comparison with the observations.

In the VSOP2000 solution, the mutual Newtonian perturbations of the eight main planets were computed by an iterative method. The perturbations of the Earth-Moon barycenter by the Moon, the perturbations at the first order of the masses by the five asteroids Vesta, Iris, Bamberga, Ceres and Pallas, the perturbations due to Pluto (Simon, 2000) and the relativistic complements were added to the Newtonian solution.

In the new solution, we took care simultaneously of integrating all the effects by an iterative method.

2. COMPARISON WITH NUMERICAL INTEGRATIONS

To evaluate the accuracy of the solutions, we carried out comparisons of VSOP82 (1982) with DE200 and VSOP2000 with DE403. The table 1 gives the maximum differences over one century between VSOP2000 and DE403 for the semimajor axis a, the mean longitude λ, the

variables k and h related on the eccentricity and the longitude of the perihelion, the variables q and p related on the inclination and the longitude of the node. This comparison shows that VSOP2000 is 10 times more precise than VSOP82 for Venus and the Earth-Moon barycenter, 50 times for Jupiter, Saturn, Uranus and Neptune, 4 times for Mars. Actually, this difference of 3 mas in the longitude of Mars is due to the introduction into DE403 of the perturbations by 300 asteroids whereas currently, we disturbed the main planets only by five asteroids.

Table 2 gives the differences between DE403 and the new solution. This solution is in progress and the current lack of perturbations by Pluto gives errors on Uranus and Neptune of 3 mas and 5 mas. The maximum differences for the planets Jupiter and Saturn are 3 times smaller than for the VSOP2000 solution like for Mercury, Venus and the Earth. For Mars, the difference of 3 mas comes from the asteroids introduced into DE403.

Table 1: Maximum difference between the VSOP2000 solution and DE403 over one century

Planet	$\Delta a/a$	λ		k	h	q	p
	10^{-10}	10^{-10}	mas	10^{-10}	10^{-10}	10^{-10}	10^{-10}
Mercury	1.0	13	0.27	8	4	0.5	0.8
Venus	1.0	14	0.29	10	3	1.1	1.5
EMB	1.4	17	0.35	3	17	3.4	4.2
Mars	5.9	140	2.89	12	30	2.6	7.2
Jupiter	11.7	23	0.47	15	16	5.0	4.3
Saturn	47.4	85	1.75	19	35	7.8	8.1
Uranus	82.1	72	1.49	66	57	38.2	11.0
Neptune	104.4	90	1.86	61	68	4.1	11.3

Table 2: Maximum difference between the analytical solution and DE403 over one century

Planet	$\Delta a/a$	λ		k	h	q	p
	10^{-10}	10^{-10}	mas	10^{-10}	10^{-10}	10^{-10}	10^{-10}
Mercury	0.52	1.62	0.034	0.69	0.69	0.29	0.09
Venus	0.41	4.95	0.102	0.95	0.83	0.36	0.09
EMB	0.54	4.80	0.099	1.28	1.90	1.17	2.49
Mars	4.62	143.66	2.963	8.54	12.67	0.61	5.56
Jupiter	16.52	8.39	0.173	7.01	8.30	1.07	0.60
Saturn	258.18	30.71	0.633	11.13	13.25	2.60	1.30
Uranus	365.70	128.72	2.655	54.15	136.26	18.85	7.93
Neptune	295.94	258.10	5.324	159.12	149.85	1.88	11.61

Figure 1 gives the differences for the longitudes of Mercury, Venus, the Earth, and Mars. The unit is 10^{-12} radian. The maximum differences are, over one century, 12 meters for Mercury, 55 meters for Venus, 75 meters for the Earth and 3.5 kilometers for Mars.

The differences between the analytical solution of Mars and DE403 come mainly from the introduction of the perturbations by a great number of asteroids into DE403. As a test of the quality of the resolution of the equations, we compared the new solution with a numerical integration corresponding to the same physical model used in the analytical solution. Table 3 gives the heliocentric differences between the analytical solution and the numerical integration

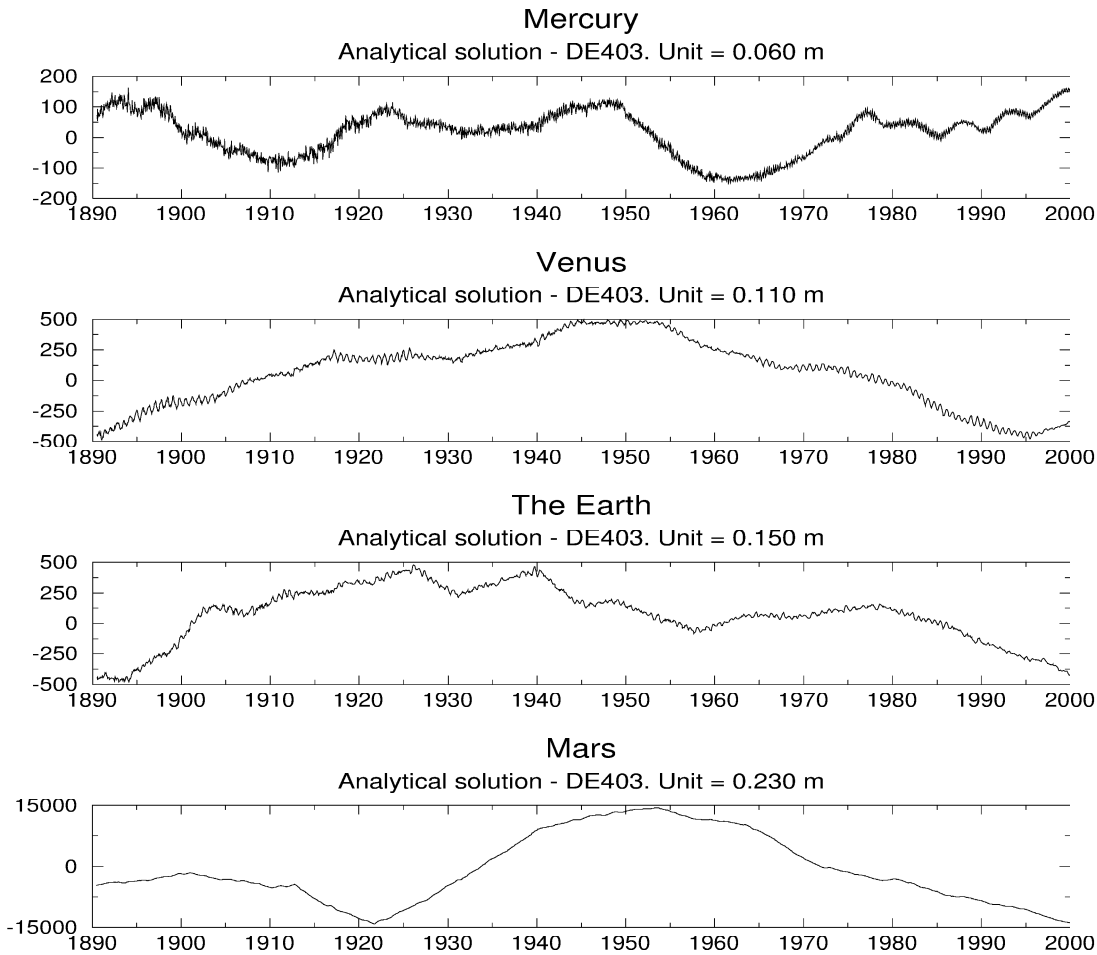

Fig. 1. Difference between the analytical solution and DE403 in 10^{-12} radian over [1891-2000].

over one century and over 10 years in mas and the corresponding values in meters. The impact of the asteroids on the motion of the Earth being not very significant, the residuals are similar to the previous ones : 63 meters instead of 75 meters in the comparison to DE403. For Mars, the residuals come from 3 mas to 0.4 mas. For the chosen model, the distance Earth-Mars can be given over 10 years with an accuracy of a few tens of meters. Figures 2 and 3 plot the differences between the analytical solution and the numerical integration over (1891-2000) (smaller than 500 meters) and over (1989-2000) (smaller than 17 meters).

3. PERTURBATIONS OF MARS BY THE ASTEROIDS

The accuracy of the position of Mars essentially depends on the computation of the perturbations by the asteroids. Table 4 gives, for the longitude of Mars, the number of terms of the perturbations by the five asteroids computed at the first order of the masses with an amplitude bigger than one meter. Many terms are produced at the higher orders mainly due to the resonance between Pallas and Jupiter. Tables 5 and 6 give the most important perturbations of the longitudes of the Earth and Mars with the amplitude in meters, the period in years and the corresponding argument. In the case of Mars, several terms have a period closed to 50 years. The perturbations of the longitude of Mars by Vesta, Iris, Bamberga, Ceres, and Pallas are plotted over one century in figure 4.

Table 3: Maximum difference of the longitude between the analytical solution and the numerical solution

Planet	over one century			over 10 years		
	10^{-10}	mas	meters	10^{-10}	mas	meters
EMB	4.20	0.087	63	0.50	0.010	8
Mars	20.92	0.431	477	0.75	0.015	17

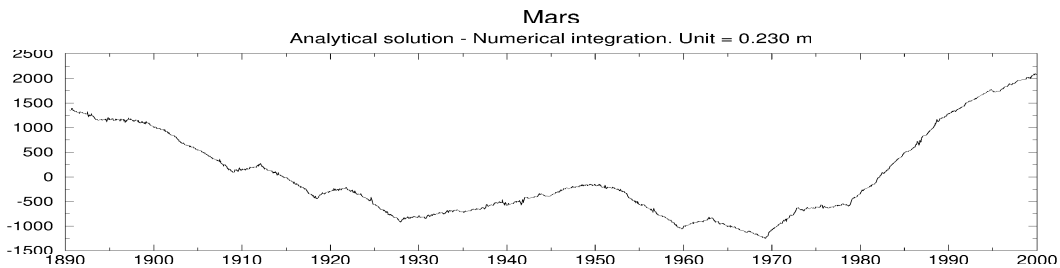

Fig. 2. Difference between the analytical solution and the numerical integration in 10^{-12} radian over [1891-2000].

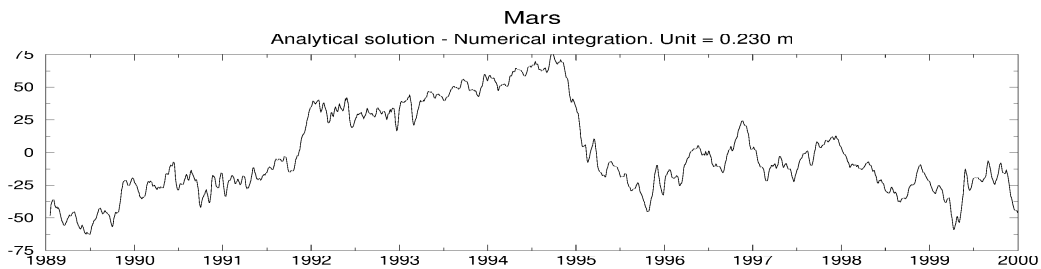

Fig. 3. Difference between the analytical solution and the numerical integration in 10^{-12} radian over [1989-2000].

Table 4: Number of terms due to the asteroids with an amplitude greater than 1 meter in the longitude

	Vesta	Iris	Bamberga	Ceres	Pallas
Mars first order solution	43	31	42	50	47
complete solution	67	44	56	83	500
Earth complete solution	12	2	7	8	32

4. MASS OF THE ASTEROIDS

We will now estimate the precision of this perturbation considering our present knowledge of the asteroid masses. The comparison between the asteroid masses determined by Fienga and Standish (2001) and by Krasinsky *et al.* (2001) shows important differences. Uncertainties on only the five used asteroids involve errors of about one kilometer over one century. For the 24

other asteroids with a mass higher than 5×10^{-12} solar mass, one can note that the sum of the differences of masses represents 75×10^{-12} solar mass and can produce, according to whether one uses one or the other set of masses, differences of several kilometers on the position of Mars over one century. Standish and Fienga (2001) showed that, over a shorter interval, about 10 years, uncertainties on the position of Mars could be limited to one kilometer.

Table 5: Most important perturbations of the Earth longitude by the asteroids

Origin	amplitude (in meters)	period (in years)	argument
Bamberga	33.6	292	5T − 22B
Ceres	13.1	1	2T − 2C
Vesta	6.8	1	2T − 2V
Bamberga	6.3	21	2T − 9B
Pallas	5.9	8	T − 4P
Pallas	5.8	12	T − 12P + 18J
Ceres	5.8	1	T − C
Pallas	5.8	12	T + 2P − 18J
Pallas	5.3	12	T − 5P
Vesta	5.1	6	T − 3V

Table 6: Most important perturbations of the Mars longitude by the asteroids

Origin	amplitude (in meters)	period (in years)	argument
Bamberga	12939	327	3M − 7B
Vesta	4683	52	M − 2V
Ceres	1968	44	2M − 5C
Pallas	1577	49	2M − 12P + 18J
Pallas	1540	49	2M + 2P − 18J
Bamberga	1258	164	6M − 14B
Pallas	1250	49	2M − 5P
Vesta	722	26	2M − 4V
Pallas	663	50	2M − 19P + 36J
Pallas	632	48	2M + 9P − 36J
Ceres	630	10	M − 2C
Iris	604	92	M − 2I
Pallas	558	119	2M − 3P − 5J
Pallas	540	492	M − 11P + 22J

5. CONCLUSION

The iterative process of integration of the equations will be continued for the model which we indicated at the introduction until reaching a precision of the solution of about one meter over 10 years.

The following step will consist in introducing the perturbations of Mars by the 180 asteroids

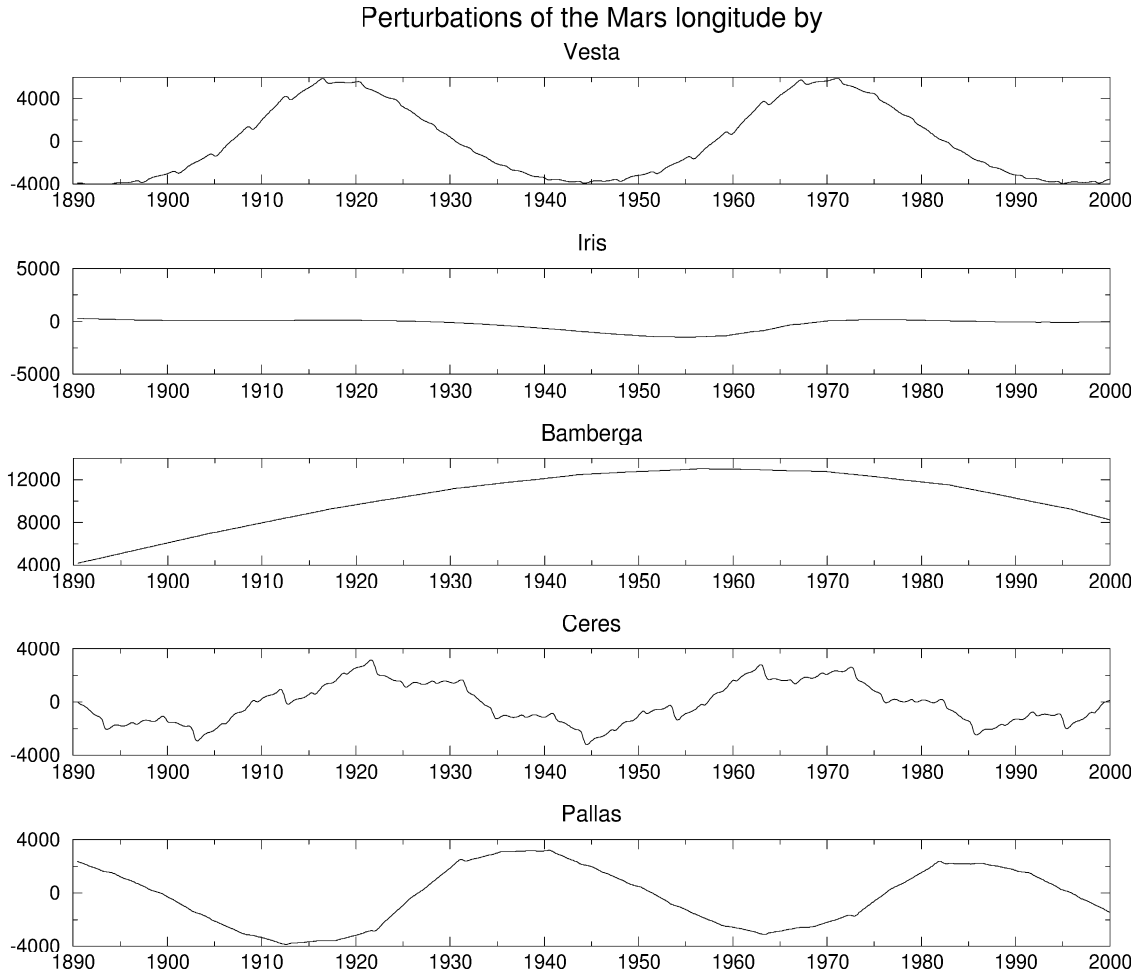

Fig. 4. Perturbations of Mars by the asteroids in meters over [1891-2000].

with a mass bigger than 10^{-12} solar mass. The precision of the solution will have to be about a few meters for the selected value of the masses but of course the accuracy of the solution could not be better than only a few hundred meters because of uncertainties on the mass of the asteroids. An internal precision about a few meters is however required to be certain that the difference between the observation and the analytical solution comes only from uncertainties on the mass of the asteroids.

Lastly, the integration constants will be determined by adjustment to the observations with the help of the methods developed by Fienga (1999).

6. REFERENCES

Bretagnon, P., 1982, *Astron. Astrophys.* 114, 278

Fienga, A., 1999, Thèse, Paris

Fienga, A., Standish, E. M., 2001, JPLIOM 312.F - 01 - 017

Krasinsky, G. A., Pitjeva, E. V., Vasilyev, M. V., Yagudina, E. I., 2001, Russian Academy of Sciences, Institute of Applied Astronomy, 139

Moisson, X., 2000, Thèse, Paris

Standish, E. M., 1998, JPLIOM 312.F - 98 - 048

Standish, E. M., Fienga, A., 1998, *Astron. Astrophys.* to be published