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PREFACE

The Journées 2000 "Systèmes de référence spatio-temporels", which have been held from 18 to 20 September 2000 at Paris Observatory, have been the twelfth conference in this series which have been organized, on the topic of the reference systems, in Paris each year from 1988 to 1992 and alternately, since 1994, in Paris and other European centres. The first "Journées" out of Paris have been organized in Warsaw, the second ones in Prague and the third ones in Dresden, whereas, the Journées 1996 and Journées 1998 have been held in Paris.

The Journées 2000 have been supported, for their organization as well as the publication of the Proceedings, by the Scientific Council of Paris Observatory and CNRS. We are grateful for such a support and also to the Institut d'Astrophysique de Paris for having made the IAP Amphitheatre available for the sessions.

The sub-title "J2000, a fundamental epoch for origins of reference systems and astronomical models", has been chosen in connection with this year 2000, but also in connection with the fundamental epoch for the new models and reference systems. The subjects discussed during these Journées have been related to the celestial and terrestrial reference systems and the astronomical models at the highest level of accuracy consistent with the current observations of Earth rotation and the newly adopted International Celestial Reference System (ICRS). The topics include a special emphasis on the role of the origins (in space and time) and on the practical application to fundamental astronomy, time and celestial mechanics, of the Resolutions which have been adopted by the XXIVth General Assembly of the IAU in August 2000. A special session has been devoted to nutation including an historical part, the year 2000 being about 250 years after the discovery of the principal term of nutation by Bradley.

The scientific program of the meeting was composed of the five following sessions : I) Reference systems : the realization, the role of the origins of ICRS and ITRS, II) Astronomical models and conventions referred to the ICRS and J2000, III) Nutation from discovery to nowadays, IV) Time for the ephemerides and the observations and V) Irregular variations in Earth rotation and new methods of estimation. This program has been elaborated by the Scientific Organizing Committee that is thanked here for its efficient contribution to the success of the meeting. Each session included introductory papers and oral contributions and there have been two poster sessions. There were 70 attendants to this meeting from 16 different countries; 45 oral presentations have been given and there were 17 posters. The SOC thanks here all the colleagues from all countries who have attended our Journées and participated in the scientific discussions.

These Proceedings are divided into five sections corresponding to the five sessions of the meeting including, for each session, the invited talks, as well as oral and poster contributions which have been presented during the meeting. The list of participants is given on pages vii and viii, the scientific program on pages ix to xi (see Table of Contents). The Postface gives the announcement for the next "Journées" which will be organized in September 2001 in Brussels.

I thank very much all the authors of the papers who have sent their contribution in the required form and within the required deadline in order that these Proceedings can be published within a reasonable delay after the meeting. I also thank the local organizing Committee for its important effort and very efficient work before and during the meeting and especially S. Débarbat, L. Garin, O. Becker and J.B. N'Guyen for the practical organisation.

Nicole CAPITAINE

Chair of the SOC

July 2001

Liste des participants / List of Participants

ABGRALL Michel	Paris, France	michel.abgrall@obspm.fr
ARIAS Elisa Felicitas	Sèvres, France	farias@bipm.fr
BACCHUS Pierre	La Grande Paroisse, France	pierre.bacchus@worldonline.fr
BARILLET Roland	Paris, France	roland.barillet@obspm.fr
BARLIER François	Grasse, France	Francois.Barlier@obs-azur.fr
BIZOUARD Christian	Paris, France	bizouard@.obspm.fr
BOLOTIN Sergei	Kiev, Ukraine	bolotin@mao.kiev.ua
BOLOTINA Olga	Kiev, Ukraine	olga@mao.kiev.ua
BRETAGNON Pierre	Paris, France	pierre@bdl.fr
BRZEZIŃSKI Aleksander	Warsaw, Poland	alek@cbk.waw.pl
CAPITAINE Nicole	Paris, France	capitain@.obspm.fr
CHAPRONT Jean	Paris, France	jean.chapront@obspm.fr
CHAPRONT-TOUZÉ Michelle	Paris, France	michelle.chapront@obspm.fr
COLL Bartolome	Paris, France	coll@.obspm.fr
DAMLJANOVIC Goran	Belgrade, Yugoslavia	gdamljanovic@aob.bg.ac.yu
DÉBARBAT Suzanne	Paris, France	debarbat@.obspm.fr
DE BIASI Maria Silvina	La Plata, Argentina	debiasi@fcaglp.fcaglp.unlp.edu.ar
DEHANT Véronique	Bruxelles, Belgique	v.dehant@oma.be
DICK Wolfgang R.	Frankfurt, Germany	wdi@gp4.potsdam.ifag.de
DUMONT Simone	Paris, France	
DVORAK Rudolf	Vienne, Austria	dvorak@astro.univie.ac.at
EISENSTAEDT Jean	Jussieu, France	jee@ccr.jussieu.fr
ENGELHARDT Gerald	Leipzig, Germany	engelhardt@leipzig.ifag.de
FOLGEIRA Marta	Madrid, Spain	martafl@iagmat1.mat.ucm.es
FRANCOU Gérard	Paris, France	gerard.francou@obspm.fr
FUKUSHIMA Toshio	Tokyo, Japan	Toshio.Fukushima@nao.ac.jp
GAMBIS Daniel	Paris, France	daniel.gambis@obspm.fr
GONTIER Anne-Marie	Paris, France	Anne-Marie.Gontier@obspm.fr
GREFF-LEFFTZ Marianne	Paris, France	greff@ipgp.jussieu.fr
GUINOT Bernard	Paris, France	guinot.bernard@wanadoo.fr
HEFTY Jan	Bratislava, Slovak Rep.	hefty@cvt.stuba.sk
KABELAC Joseph	Praha, Czech Rep.	kabelac@inet.tpc.cz
KHARIN Arkadiy	Kiev, Ukraine	kharin@mao.kiev.ua
KOLACZEK Barbara	Warsaw, Poland	kolaczek@cbk.waw.pl
KOLESNIK Yuri B.	Moscow, Russia	kolecnick@osiris.inasan.rssi.ru
KONDO Tetsur	Tokyo, Japan	kondo@crl.go.jp
KORSUN Alla	Kiev, Ukraine	akorsun@mao.kiev.ua
KOVALEVSKY Jean	Grasse, France	kovalevsky@obs-azur.fr
KUDLAY Oleksandr	Kiev, Ukraine	kudlay@mao.kiev.ua
KUDRYAVTSEV Sergey	Moscow, Russia	ksm@sai.msu.ru
LAMBERT Sebastien	Paris, France	lambert@format.obspm.fr
LE BAIL Karine	Paris, France	lebail.@format.obspm.fr
MA Chopo	Greenbelt, USA	cma@virgo.gsfc.nasa.gov
MATSUZAKA Shigeru	Ibaraki, Japan	shigeru@gaos.gsi-mc.go.jp
MIGNARD François	Grasse, France	francois.mignard@obs-azur.fr
MIOC Vasile	Bucharest, Romania	vmioc@roastro.astro.ro
MISSANA Marco	Milano, Italia	missana@brera.mi.astro.it
MOLOTAJ Olexandr	Kiev, Ukraine	mo1@aoku.freenet.kiev.ua
NOTHNAGEl Axel	Bonn, Germany	nothnagel@uni-bonn.de
PAKULIAK Ludmila	Kiev, Ukraine	pakuliak@mao.kiev.ua
PASHKEVICH Vladimir	St. Petersburg, Russia	apeks@gao.spb.ru
PETROV Sergei	Bruxelles, Belgique	petrov@oma.be

POMA Angelo	Cagliari, Italie	poma@ca.astro.it
RON Cyril	Praha, Czech Rep.	ron@ig.cas.cz
SAVOIE Denis	Paris, France	denis.savoie@palais-decouverte.fr
SCHLÜTER Wolfgang	Koetzing, Germany	Schlueter@Wettzell.ifag.de
SCHUH Harald	Wien, Austria	hschuh@luna.tuwien.ac.at
SILLARD Patrick	Paris, France	sillard@ensg.ign.fr
SOFFEL Michael	Dresden, Germany	soffel@rcs.uvz
SOUCHAY Jean	Paris, France	jean.souchay@obspm.fr
STAVINSCHI Magda	Bucharest, Romania	magda@roastro.astro.ro
TEYSSANDIER Pierre	Paris, France	Pierre.Teyssandier@obspm.fr
TITOV Oleg	Saint-Petersburg, Russia	olegtitov@mail.ru
TOURDE Raymond	Paris, France	raymond.tourde@obspm.fr
URAS Silvano	Capoterra, Italie	uras@ca.astro.it
VITYAZEV Veniamin	St.Petersburg, Russia	vityazev@venvi.usr.pu.ru
VONDRÁK Jan	Praha, Czech Rep.	vondrak@ig.cas.cz
WEBER Robert	Vienna, Austria	rweber@luna.tuwien.ac.at
YATSKIV Yaroslav	Kiev, Ukraine	yatskiv@mao.kiev.ua
YAYA Philippe	Paris, France	philippe.yaya@obspm.fr

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THE ICRF: CONSTRUCTION, MAINTENANCE AND INTERACTION WITH EOP AND NUTATION

C. MA
Space Geodesy Branch, Goddard Space Flight Center
Greenbelt, MD 20771, USA
e-mail: cma@virgo.gsfc.nasa.gov

1. INTRODUCTION

While the first thoughts of using extragalactic objects to define the celestial reference frame are not recent, the process that led to the eventual replacement of the ancient concept of star catalogs had to await both the discovery of point-like extragalactic objects and a means to measure their positions with sufficiently improved accuracy and precision to justify such a radical step. The discovery of quasars and the recognition that they are extremely distant coupled with the development of radio- frequency VLBI as an astrometric tool led the 1988 IAU General Assembly to adopt a resolution calling for the use of extragalactic objects to define the celestial reference frame when the data and analysis were available. Parallel to the continued acquisition of VLBI data, largely for geodesy, and refinement of astrometric VLBI analysis, the IAU took a series of steps to lay a solid foundation for the International Celestial Reference System (ICRS). The 1990 IAU Colloquium 127, Reference Systems, under the leadership of James Hughes of USNO adopted general relativity as the fundamental theory, confirmed the suitability of extragalactic radio sources, and specified the continuity of the new frame with existing stellar and dynamic realizations. The next two IAU General Assemblies established working groups to identify the set of radio sources and to develop a precise catalog of their positions. The astrometric analysis was done by a subgroup of VLBI and reference system specialists using data freely contributed by many observing programs. The final radio catalog was delivered in 1995 to provide a basis for orienting and fixing the Hipparcos stellar catalog. These positions were adopted by the 1997 IAU General Assembly in Kyoto as the ICRF (International Celestial Reference Frame) with an effective date of 1 January 1998. The Hipparcos catalog was adopted as the realization of the ICRF at optical wavelengths. The responsibility for maintaining the ICRS and ICRF was delegated to the IERS (International Earth Rotation Service) with oversight by an IAU working group. Although formally established for a three-year term, the IAU working group was extended by the 2000 IAU General Assembly and is also responsible for other areas such as extending the ICRF to other wavelengths and codifying all the consequences of the ICRF.

The fundamental principle of the ICRF is that the fiducial objects are stationary points. These radio sources are observed in 24-hr VLBI sessions that determine the relative positions (arclengths) between the sources with high precision and accuracy. A typical session now observes 50–80 sources distributed over all right ascensions and as much declination range as is mutually visible to the network of VLBI stations, typically six stations but sometimes as many as 20 stations. A group of sources observed in common between two sessions links the positions of the other sources in the two sessions and allows the relative positions of these other sources to be determined. Concatenation of many sessions provides multiple links between sources and mul-

Table 1: Temporal distribution of observations

Years	Observations
1979-83	55 000
1984-86	188 000
1987-89	325 000
1990-92	500 000
1993-95.5	580 000

tiple measurements of arclengths, improving the precision of the average values. In practice the right ascensions and declinations of the sources are estimated by an incremental least-squares process but the actual information is the large set of relative positions embodied in the full variance-covariance matrix. The relative positions are reduced to a set of right ascensions and declinations by adopting some right ascension origin and axis directions.

2. CHARACTERISTICS OF THE ICRF

The ICRF is a catalog of extragalactic radio source positions derived from geodetic and astrometric VLBI data consistent with the ICRS maintained by the IERS. The positions and coordinate axes are not conceptually related to the equinox, equator or ecliptic. The numerical values of the positions define the orientation of the coordinate axes in space with respect to the radio sources and thus the origin of right ascension and the direction of the z-axis. Unlike past stellar realizations, which were oriented to the equinox and equator of various reference epochs, the orientation of the ICRF is independent of epoch and will not be changed in the future. Continuity of orientation for future, as yet unplanned, ICRF realizations will be maintained by a no-net-rotation condition between the old and new catalogs using the best common sources. This axis alignment should be stable at the 20 microarcsecond level. The orientation and positions of the ICRF are consistent with FK5 and the most commonly used solar system ephemeris at their levels of uncertainty. Based on a conservative error analysis, the lower limit of accuracy of individual positions is expected to be 250 microarcseconds.

3. DATA USED FOR THE ICRF

The data used for the ICRF consisted of dual-frequency VLBI delays and delay rates observed at 8.4 GHz and 2.3 GHz, the latter for ionosphere calibration. The time span from 1979 to 1995.5 included 1.6 million delay/delay rate pairs from 2549 sessions and 608 sources. The temporal distribution is given in Table 1, which shows increasing data density with time. During this interval there were substantial improvements in instrumentation, sensitivity, observing strategy, and network geometry.

95% of the data set was acquired for geodetic purposes such as measuring contemporary tectonic plate motion and regional deformation and monitoring Earth orientation variation. For such geodynamic programs, ~ 100 sources were observed extensively, those with the strongest fluxes and acceptably small structure. The sources used for geodesy changed over time as fluxes were observed to decrease or increase. The largest number of sources were observed in the 5% of the data acquired for specifically astrometric purposes a few times per year. Consequently the distribution of data over sources is very skewed, as shown in Table 2. The geodetic sources form the core of the ICRF.

Table 2: Distribution of observations over sources

Observations	Sources
$1 - 10$	69
$10 - 10^2$	236
$10^2 - 10^3$	208
$10^3 - 10^4$	50
$10^4 - 10^5$	44
$> 10^5$	1

4. ASPECTS OF ANALYSIS

Because of the nature of the data, the maturity of the astronomical and geophysical models at the time, and the characteristics of the objects, an informed judgment was made by the ICRF subgroup in several areas about the best analysis strategy.

The VLBI observable delay and delay rate are affected by the geometry of the particular baseline of the observation as well as the orientation of the source with respect to the baseline. To reduce the effect of possible terrestrial anomalies on the CRF, the positions of the observing stations were estimated independently for each session instead of as a group of positions at epoch and velocities. While this choice optimizes the analysis for the CRF, it decouples the CRF and TRF, at least conceptually. Further study is needed to see how CRF positions are affected by an integrated CRF/TRF analysis.

It was recognized before the 1988 IAU General Assembly that the IAU 1980 nutation model was inadequate for astrometric analysis of VLBI data because of the much higher precision compared to optical measurements. However, the orientation of the ICRS, as realized from VLBI radio source catalogs submitted annually to the IERS, had been set initially by using the IAU 1980 nutation model as fixed a priori values at an arbitrary epoch in 1980 and by using a right ascension origin from occultation measurements of the radio source 3C273. To free VLBI analysis from the distortions that would be caused by applying the standard precession/nutation model, the practice of estimating offsets in the two nutation directions for each observing session was adopted. The time series of nutation offsets revealed annual and other periods related to the structure of the Earth as well as a drift indicating an error in the precession constant. Since the initial orientation had an unknown error, it was clear (in retrospect) that the nutation offsets would not be zero at the reference epoch J2000.0 as assumed by conventional theory. However, since the ICRS axes had already been decided and used for some time, the ICRF subgroup did not reorient the catalog based on an extrapolation of the nutation times series from 1980–1995.5 to 2000. This difference between the celestial pole at J2000.0 and the z-axis of the ICRF is to be recognized by using the best values of the offsets at J2000.0 applied directly in the next standard IAU nutation model.

The troposphere has been and continues to be the largest source of systematic and random error in VLBI analysis. Over the time span of the ICRF data, the modeling of elevation dependence through a troposphere mapping function and the estimation of tropospheric variation improved in several steps, each entailing a detailed study and reanalysis of the then existing VLBI data set. Changes in mapping function resulted in systematic changes in source declinations and terrestrial scale but tests such as examining the relationship of results from subsets of data with different elevation cutoffs were used to decide whether a new mapping function was more correct than its predecessor. Shortening the time interval of zenith troposphere parameters, at least to some threshold value, decreased the scatter of baseline length time series. The estimation of tropospheric gradients was seen to have a systematic effect on source declinations with a maximum value of 500 microarcseconds near the equator. The final choice of mapping function, troposphere parameter interval, and gradient estimation was determined by what yielded the most consistent geodetic and astrometric results within the modeling and speed capabilities of the software used for the ICRF analysis.

The radio sources themselves were limiting factors for two reasons, position instability (most likely caused by changing source structure) and uneven data distribution. Position stability was studied using time series with a position data point from each session a source was observed. These time series ranged from very sparse to very dense with a range of scatter and drift. The sources were consequently divided into three groups: 1) the best or defining sources with stable positions and sufficient data, 2) candidate sources with usable stability but fewer observations or other minor defects, and 3) "other" sources with clearly unstable or drifting positions. Because of the predominance of geodetic data and observing programs from the norther hemisphere, there are more defining sources there. The candidate, "other" and ensemble of all sources are distributed quite evenly in right ascension and declination. There is some loss near the galactic equator where the lower observing frequency was degraded. To accommodate the instability of the "other" sources and to prevent their affecting the remainder, their positions were estimated as arc parameters, i.e., independently for each session. For the 212 defining sources and the 294 candidate sources, their positions were estimated as global parameters from the entire data set.

The extreme skewing of the data distribution made the formal statistical errors unrepresentative of the accuracy of the positions. A realistic floor for the uncertainties was derived from catalog and software comparisons, test runs with varying models, and subsets of data.

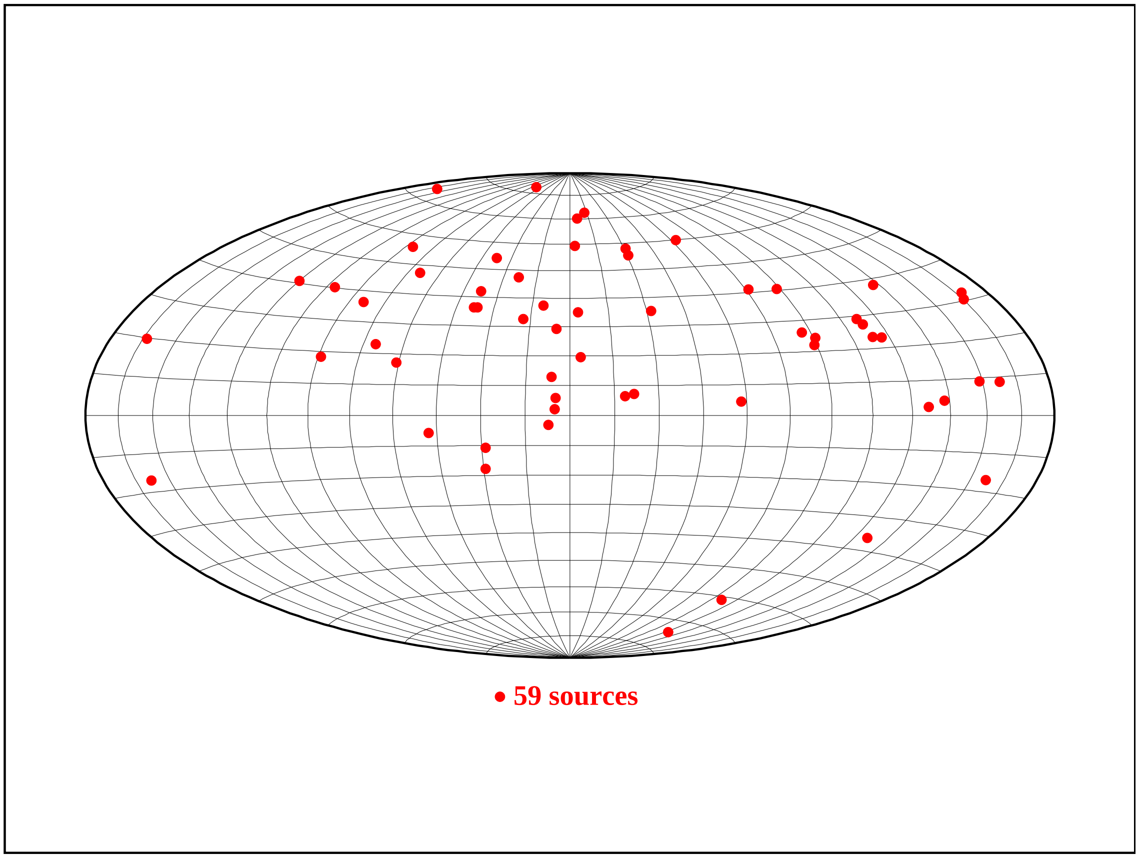
5. ICRF-EXT.1

Since the VLBI observations for both geodesy and astrometry continue, a first extension of the ICRF analysis was done including data from August 1995 – April 1999, adding $\sim$ 600 thousand observations in 461 sessions with 59 new sources, shown in Figure 1. The entire 1979–1999 data set was reanalyzed with some improvements that had effects smaller than the stated error floor of the ICRF. These included further shortening the troposphere parameter interval, increasing the number of high frequency Earth orientation components, adjusting antenna axis offsets as arc rather than global parameters, and changing the troposphere mapping function. Another modeling improvement, the proper use of a priori mean troposphere gradients, was not used because of its systematic effect. The ICRF-Ext.1 catalog contains improved positions and errors for the candidate sources, more data on the "other" sources, but unchanged positions and errors for the defining sources, which constitute the ICRF in a formal sense. The catalog is distributed via the IERS web site. Further extensions are expected at roughly annual intervals.

6. FUTURE DEVELOPMENTS

The worldwide VLBI network for geodesy and astrometry continues to be active. The geodetic observing programs include the weekly NEOS (US National Earth Orientation Service), CORE (Continuous Observation of the Rotation of the Earth) still growing to its full capability, and monthly IRIS. The VLBA is used with up to ten other stations several times a year to support an integrated geodetic/astrometric research effort. A small fraction of the geodetic observing schedule is used to cycle through the ICRF sources. In addition there are a few astrometric sessions per year using the EVN, ATNF, and the geodetic network. A survey of $\sim$ 1700

Figure 1: New sources in ICRF Ext.–1



sources done by the VLBA for phase referencing is now available for astrometric analysis. This would greatly expand the number of sources in the north although the astrometric precision is probably not equal to the ICRF. Of particular interest is an astrometry and source mapping project beginning in the southern hemisphere, where there is a deficit of observations and source maps. Astrometric observations of ~ 150 sources will be made over a five-year interval with Mark III/IV using stations in Australia, South Africa, Japan and Hawaii. In parallel 8.4 GHz maps will be made of ~ 200 sources using S2 recorders.

Astrometric VLBI analysis for the maintenance of the ICRF is the responsibility of the IVS (International VLBI Service for Geodesy and Astrometry), working with the ICRS Product Center of the IERS. The focus of analysis refinement will probably be on the troposphere and data weighting. Considerable work is needed to implement the use of source structure information, which might be useful to improve the stability of the positions. However, source structure information is generally limited to a single epoch and would not be applicable for the entire time span of the data. Several IVS analysis centers have joined in the ICRF work in the past, but future activity will depend heavily on new participants.

There are no current plans for a new realization of the ICRF that would bring the VLBI analysis up to date. The decision for such a revision would rest with the IAU and be carried out by the IERS and IVS.

It is quite possible that the CRF will return to an optical realization using satellite observations of quasars and QSOs from FAME and GAIA. However, this optical CRF would not be accessible with full precision from the ground, and the measurement of Earth orientation parameters and nutation would continue to rely on VLBI and radio sources.

PRELIMINARY ANALYSIS OF THE ITRF2000

Z. ALTAMIMI, P. SILLARD, C. BOUCHER
Institut Géographique National
ENSG/LAREG, 6-8 Avenue Blaise Pascal, 77455 Marne-la-Vallée, FRANCE
e-mail: altamimi@ensg.ign.fr;

ABSTRACT

One of the year 2000 major trends of the International Earth Rotation Service (IERS) is the establishment of the ITRF2000. This Global reference is to be considered as a standard solution for a wide user community (geodesy, geophysics, astronomy, etc.). The ITRF2000 comprises on one hand primary core stations observed by VLBI, LLR, GPS, SLR and DORIS techniques and, on the other hand, significant extension provided by regional GPS networks for densifications as well as other useful geodetic markers tied to the space geodetic ones.

The ITRF2000 combination and implementation strategy will be described in this paper. Important results and quality assessment of the ITRF2000 will be presented. Benefits of such a global frame will be outlined.

1. INTRODUCTION

A new and densified ITRS realization, namely ITRF2000 will be produced in early 2001. New trends and features were implemented to enhance and improve the ITRF combination. One of the major progress is the combination of unconstrained solutions, but for which we add minimum constraints used solely to define the underlying terrestrial reference frame in origin, scale, orientation and time evolution, reflecting so the real quality of space geodesy techniques in providing station positions and velocities.

Moreover, in order to define a datum that is stable at 1 mm/yr level, the ITRF2000 orientation time evolution will be achieved upon a selection of sites with high geodetic quality.

We will focus in this paper on the procedure to implement such features for the ITRF2000. Preliminary analysis and early results of the combination of submitted solutions are presented.

2. ITRF COMBINATION METHODOLOGY

2.1. Analysis Strategy

The current analysis strategy adopted for the generation of ITRF solutions consists in the following steps:

- removing constraints from the constrained solutions and applying minimum constraints (See, Sillard and Boucher, 2000, Altamimi et al, 2000).
- adding minimum constraints to loose solutions.

- leaving as they are, solutions where analysis centers already applied minimum constraints.
- propagating, for each individual solution, station positions at Epochs of Minimal Position Variance (EMPV). In order to assess the relative qualities of the individual solutions used in the ITRF combination, we compute the commonly known Weighted Root Mean Scatter (WRMS) per solution based on the post fit residuals. To be rigorous, station position residuals should be computed at their respective "mean observation epochs". The experience showed that the computed EMPV corresponds approximately to the central epoch of observations of each station. Therefore, propagating station positions at their EMPV's insure a more "realistic" WRMS evaluation, in positions, for each individual solution.
- combining all solutions together with local ties.

2.2. Combination model

Assuming that for each individual solution s, and each point i, we have position X_s^i at epoch t_s^i and velocity $\dot{X}_s^i$, expressed in a given TRF k.

The combination consists in estimating:

- Positions X_{itrf}^i at a given epoch t_0 and velocities $\dot{X}_{itrf}^i$ in ITRS
- Transformation parameters T_k at an epoch t_k and their rates $\dot{T}_k$, from the ITRF to each individual frame k.

The general physical model used is given by the following equation (1):

$$\left\{ \begin{array}{rcl} X_s^i & = & X_{itrf}^i + (t_s^i - t_0)\dot{X}_{itrf}^i + T_k + D_k X_{itrf}^i + R_k X_{itrf}^i \\ & + & (t_s^i - t_k)\left[\dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i\right] \\ & & \\ \dot{X}_s^i & = & \dot{X}_{itrf}^i + \dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i \end{array} \right. \tag{1}$$

where for each individual frame k, D_k is the scale factor, the translation vector T_k and rotation matrix R_k are respectively defined (following IERS conventions) by :

$$T_k = \begin{pmatrix} T1_k \\ T2_k \\ T3_k \end{pmatrix} \quad \text{and} \quad R_k = \begin{pmatrix} 0 & -R3_k & R2_k \\ R3_k & 0 & -R1_k \\ -R2_k & R1_k & 0 \end{pmatrix}$$

The dotted parameters designate their derivatives with respect to time. $T1, T2, T3$ are the 3 origin components, $R1, R2, R3$ are the three small rotations according to the 3 axes, respectively X, Y, Z.

Table 1. ITRF2000: submitted TRF solutions

Technique Analysis Center(AC)	AC SSC	Data Span	Station Number	Constraints
VLBI				
Geodetic Institute of Bonn University	(GIUB) 00 R 01	84-99	51	Loose
Goddard Space Flight Center	(GSFC) 00 R 01	79-99	130	Loose
Shanghai Astronomical Observatory	(SHA) 00 R 01	79-99	127	Loose
LLR				
Forschungseinrichtung Satellitengeodaesie	(FSG) 00 M 01	77-00	3	Loose
SLR				
Australian Surveying and Land Information Group	(AUS) 00 L 01	92-00	55	Loose
Centro Geodesia Spaziale, Matera	(CGS) 00 L 01	84-99	94	Loose
Communications Research Laboratory	(CRL) 00 L 02	90-00	60	Loose
Center for Space Research	(CSR) 00 L 04	76-00	139	Loose
Delft Ins. Earth Oriented Space Research	(DEOS) 00 L 01	83-99	91	Loose
Deutsches GeodM-dtisches Forschungsinstitut	(DGFI) 00 L 01	90-00	43	Removable
Joint Center for Earth System Technology, GSFC	(JCET) 00 L 09	93-00	48	Loose
GPS				
Center for Orbit Determination in Europe	(CODE) 00 P 03	93-00	160	Minimum
GeoForschungsZentrum Potsdam	(GFZ) 00 P 01	93-00	98	Minimum
International GPS Service by Natural Resources Canada	(IGS) 00 P 12	96-00	179	Minimum
Jet Propulsion Laboratory	(JPL) 00 P 01	91-99	112	Minimum
Univ of Newcastle upon Tyne	(NCL) 00 P 01	95-99	90	Minimum
NOAA, National Geodetic Survey	(NOAA) 00 P 01	94-00	165	Removable
DORIS				
Groupe de Recherche de Geodesie Spatiale	(GRGS) 00 D 01	93-00	66	Loose
Institut Géographique National	(IGN) 00 D 09	92-00	80	Minimum
Multi-technique (SLR + DORIS + PRARE)				
GRIM5 project (GRGS+GFZ)	(GRIM) 00 C 01	85-99	183	Loose
CSR: SLR + DORIS on TOPEX	(CSR) 00 C 01	-	147	Loose
GPS Densification				
South America Network by Deutsches GeodM-dtisches Forschungsinstitut	(DGFI) 00 P 01	96-00	31	Loose
IAG Subcommission for Europe (EUREF), by Bundesamt fuer Kartographie und Geodaesie	(EUR) 00 P 02	96-00	81	Minimum
Institut Géographique National	(IGN) 00 P 01	98-00	28	Minimum
Jet Propulsion Laboratory	(JPL) 00 P 02	91-99	28	Minimum
Antartica network, by Institut Géographique National	(IGN) 00 P 02	95-00	17	Minimum
CORS Network by NOAA	(NOAA) 00 P 02	94-99	80	Removable
REGAL Network, France	(REGAL) 00 P 03	96-00	29	Minimum

3. ITRF2000 DATA ANALYSIS

At the time of writing, the individual TRF solutions submitted to the ITRF2000 and included in this analysis are listed in Table 1.

Figure 1 shows the coverage of the approximately 410 sites implied in this analysis, underlying the collocated techniques.

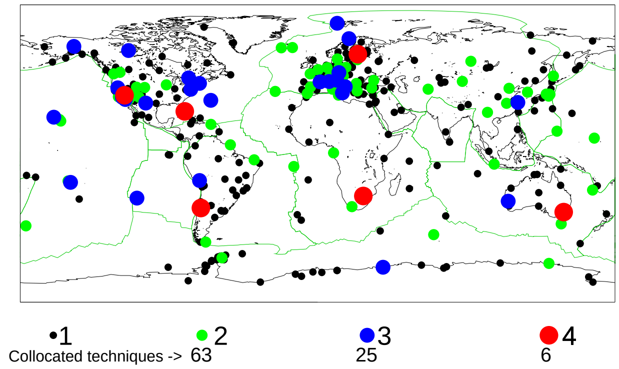


Fig.1 ITRF2000 preliminary network and collocated techniques

3.1. ITRF2000-P datum definition

A preliminary global combination, designated as (ITRF2000-P) was performed incorporating the above selected solutions. The following datum definition was selected for the ITRF2000-P solution:

- the scale and its rate by a weighted average of the VLBI and 5 SLR solutions
- the translations and their rates by a weighted average of 5 consistent SLR solutions
- the orientation by the ITRF97 at 1997.0 epoch and the rotation rates to be such that there is no net rotation with respect to NNR-NUVEL1A upon a selection of ITRF sites with high geodetic quality:
  1. continuously observed during at least 3 years
  2. located far away from plate boundaries and deforming zones
  3. velocity accuracy (as result of the ITRF combination) better than 3 mm/y
  4. velocity residuals less than 3 mm/y for at least 3 different solutions

Based on the ITRF2000-P analysis, sites selection was performed using the above criteria yielding 56 sites shown in Figure 2. Note that there are 41 sites satisfying 1, 3 and 4 selection criteria above, but located near plate boundaries or deforming zones which obviously should not be used in the definition of ITRF2000 time evolution.

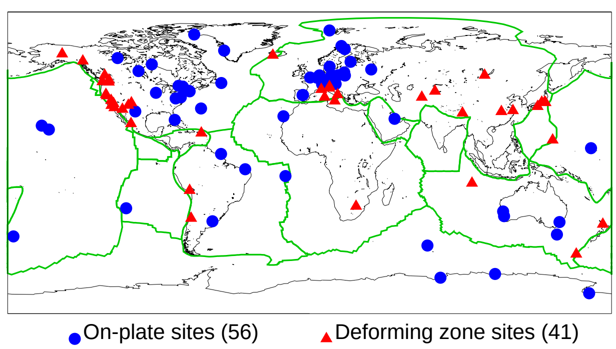


Fig.2 Selected sites to be used in the ITRF2000 datum definition

3.2. Analysis Results

We will focus in this paper on the **relative** scales and origins, the quality and the agreement level of the analyzed solutions.

In terms of the 14 transformation parameters, and given the fact that we combine solutions with minimum constraints, we assume that the rotation parameters and their rates are uninformative. This statement is also valid for the origin of VLBI solutions. Therefore we will focus on the relative scales and origins of satellite-derived solutions as well as the scale of VLBI solutions.

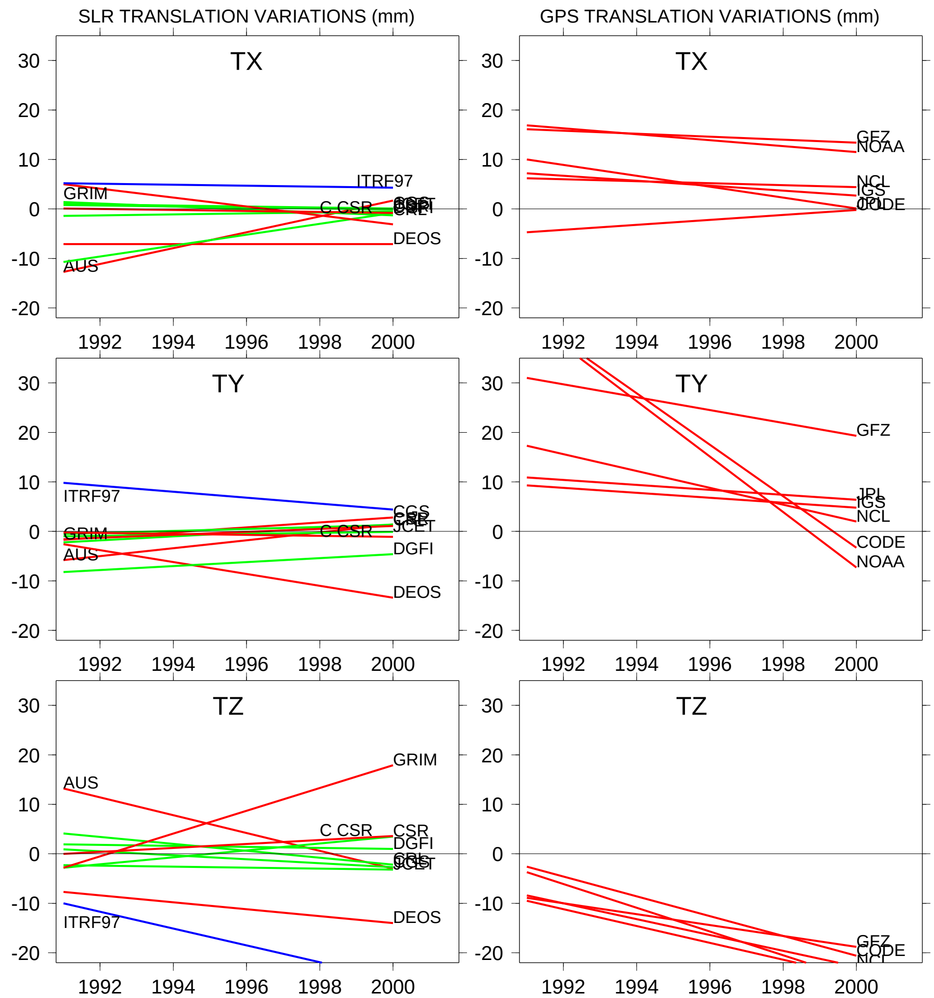


Fig.3 Translation variations (mm) per technique

Selecting the time interval 1991-2000 as the "common" observation period of the analyzed solutions, Figure 3 shows the linear variation of the translations for SLR and GPS selected solutions. Figure 4 illustrates the linear variation of the scale for VLBI, SLR and GPS selected solutions. DORIS solutions are not represented on these figures, their origin differences being in the range 1-10 cm and their scale differences in the range 3-8 ppb. Note however that the Multi-technique GRIM and CSR solutions are shown on SLR plots, since they are dominated by SLR data.

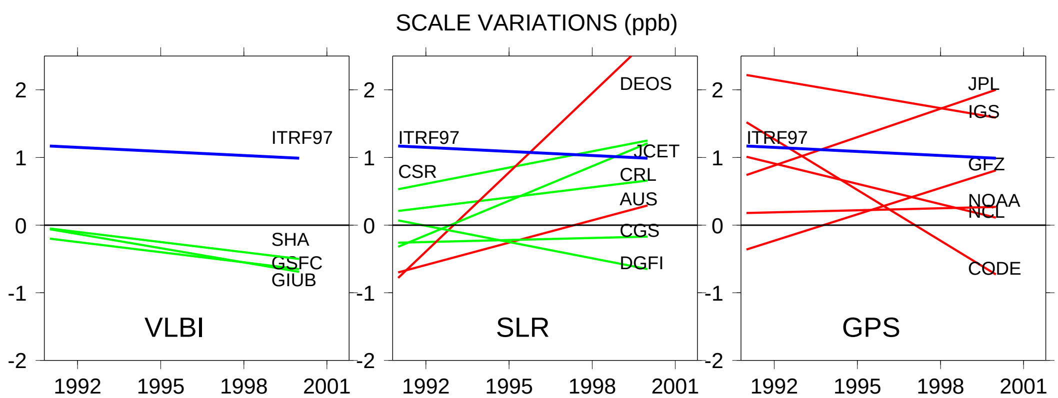


Fig.4 Scale variations (ppb) per technique

Satellite-based techniques such as SLR, GPS and DORIS should theoretically provide geocentric TRF solutions. It is noticeable to see the good agreement in translations for most of SLR solutions within one centimeter level. Meanwhile GPS solutions exhibit large translation discrepancies and in particular around X and Y axis.

Concerning the scale, as shown in Figure 4, one can observe the full agreement between scales of the three VLBI solutions (having the same rate and offset with respect to ITRF2000-P). We can also note the good scale agreement between most SLR solutions, within one ppb (10^{-9}). On the other hand we can see that the scale agreement between VLBI and SLR is within one ppb.

As result of the ITRF2000-P combination, the quality of the individual TRF's was evaluated, by computing the commonly known WRMS per solution. Figures 5 and 6 show the WRMS in positions and velocities, respectively.

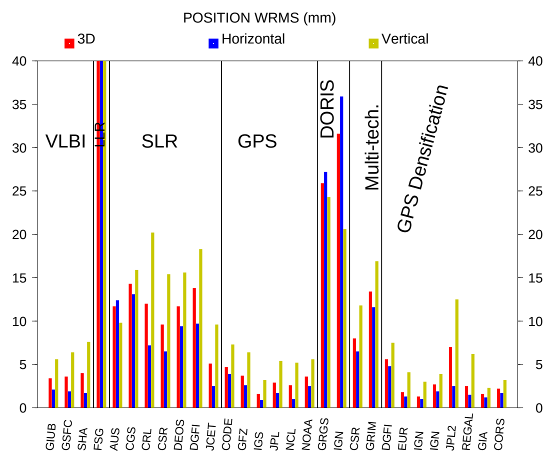


Fig. 5 WRMS in positions (mm)

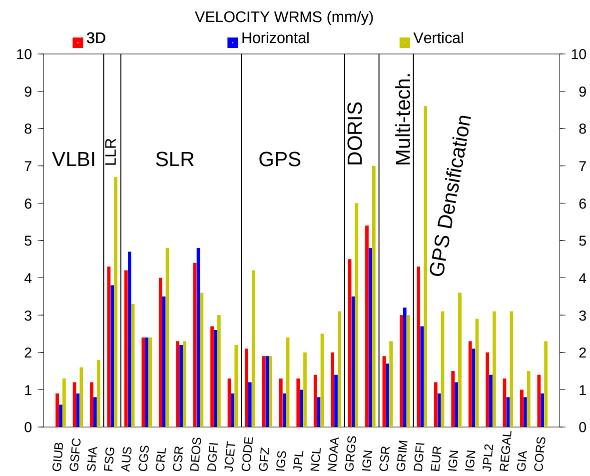


Fig. 6 WRMS in velocities (mm/y)

4. CONCLUSION

A major ITRF combination progress was achieved by using unconstrained individual solutions, providing thus a more realistic ITRF combined solution. Therefore, the assessment of the relative qualities of the contributed solutions become more "exact", reflecting so the actual capability of space geodesy techniques. The relative quality suggested by the preliminary ITRF2000 solution of unconstrained VLBI, LLR, SLR, GPS and DORIS solutions could be summarized in Table 2.

Table 2. Summary of 3-D WRMS of the individual solutions

Technique	Number of Solutions	Position 3D-WRMS mm	Velocity 3D-WRMS mm/y
VLBI	3	3	1
LLR	1	54	4
SLR	7	from 9 to 20	from 2 to 8
GPS	6	from 3 to 5	from 2 to 4
DORIS	2	30	5
Multi-technique	2	10	2

Concerning the TRF datum definition, from the analysis presented in this paper we noticed a good agreement of most SLR solutions in terms of the geocentric origin (at one centimeter level) as well as the scale (at one ppb level). We also noticed a full scale agreement between the three VLBI solutions. GPS solutions seem to have slightly larger discrepancies in translation components as well as in the scale.

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STUDIES OF THE STABILITY OF THE ICRS

A.-M. GONTIER[1], K. LE BAIL[1,2], M. FEISSEL[1,2]
[1] DANOF/UMR8630-CNRS
Observatoire de Paris, 61 avenue de l'Observatoire, 75014 Paris, France
[2] LAREG, Institut Géographique National
8 Av. Blaise Pascal, Champs sur Marne, 77455 Marne la Vallée Cedex 2, France
e-mail: Anne-Marie.Gontier@obspm.fr, lebail@ensg.ign.fr, Feissel@ensg.ign.fr

ABSTRACT. The founding hypothesis in the selection of extragalactic objects for accessing a quasi-inertial reference system is that their directions are fixed in space. Therefore the study of the time variability of the sources is an important step in the process for checking and improving the reliability of the ICRF. The systematic and random behaviours in time series of individual determinations of coordinates for several hundred sources over 1987-1999, are studies. A source selection algorithm to stabilize the celestial reference frame is presented.

1. INTRODUCTION

The IAU recommended in 1997 to use as conventional celestial reference system the International Celestial Reference System (ICRS) (see Feissel and Mignard 1998), materialized by coordinates of compact extragalactic radiosources observed with VLBI, the International Celestial Reference Frame (ICRF). The initial realization of the ICRS was published in 1997 (Ma et al. 1998, see also Ma and Feissel 1997). It includes 608 objects, two thirds of them being quasars and the rest being mainly BL Lac objects and galaxies. The most recent update and extension, ICRF-Ext.1, is now available with 667 objects (IERS 1999). The computation for ICRF-Ext.1 is based on the same analysis options than those for ICRF (Ma 2000).

The radiosource coordinates in ICRF and ICRF-Ext.1 are derived from the complete set of observations over 1979-1999. They are qualified in two ways, 1. by ascribing realistic uncertainties that take into account both the random and systematic errors - a floor of 0.25 mas in $\alpha \cos \delta$ and δ was ascribed to the published position uncertainties to account for residual modelling errors - and 2. by categorizing them, in decreasing order of confidence, as "defining", "candidates", and "other". This complex assessment scheme is made necessary by the existence of variabilities in the apparent directions of the sources.
The above lower limit is considered valid for source positions obtained over many years of observations. Thanks to the availability of the session-per-session coordinates of over 600 sources of ICRF-Ext.1, it is possible to investigate the source stability in a time series approach.

The systematic and random characteristics of several hundred source apparent motions were investigated (Gontier et *al.*, 2001) in order to derive a set of qualifiers that would be helpful in selecting sources to maintain a precise and stable celestial reference frame. In this paper, parts of these studies are presented and a statistical source selection algorithm to stabilize the celestial reference frame is showned.

2. TIME SERIES OF SOURCE COORDINATES

For historical reasons, the numbers of observations per source are extremely uneven. Some bright sources that were used to provide reference directions in the early years of VLBI appeared too variable or too extended after some years and were discarded to the benefit of other, fainter sources that became usable thanks to the progress in technology. Some sources considered as best fitting the astro-geodetic needs are repeatedly observed, while others, considered as less useful for this purpose, are observed less frequently, mainly for astrometry studies.

Our study is based on the computed coordinates of the radiosources in a homogeneous reference frame, with one determination for each of the session in which the source was observed (Eubanks 1999). Figure 1 shows the histograms of rates of observation for the 208 most regularly observed sources over 1988-1999. These sources provide the backbone of the ICRF. About half of them have less than 1500 observations in less than 60 sessions. Twenty-one sources have more than 30 thousands observations in more than 1000 sessions.

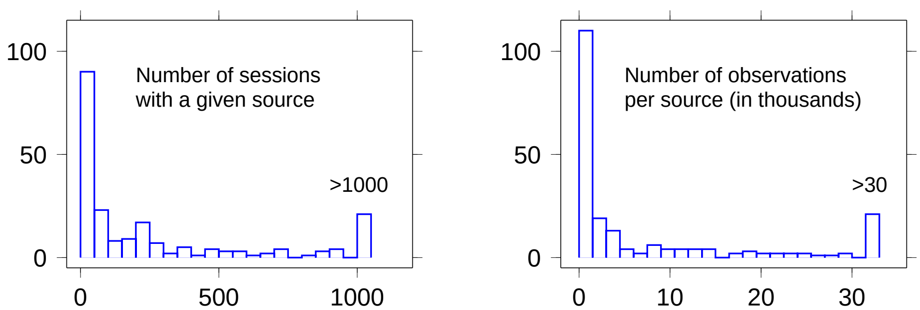

Figure 1: Histograms of rates of observations over 1988-1999 for the 208 best observed ICRF sources

The VLBI results are expressed in the equatorial system of coordinates (right ascensions and declinations), which is linked to the Earth's rotation axis and has no particular relationship with the source geometry. On the other hand, most of the known activity of quasars takes the form of jets, i.e., aligned emissive structures that cause an apparent motion of the observed emission centre relative to a fixed background. In parallel, noise in declination will also simulate an aligned structure, that may or may not hide the real source structure effect. In order to take into account this specific character of the expected apparent variabilities, we define a local system of axes in which the source instability will be studied. These axes have their origin at the mean source coordinates, the x-axis being in the direction of maximum standard deviation of the set of 0.5 year weighted averaged coordinates over 1987.25-1999.25, and the y-axis forming a direct rectangular system with it. Then we define a "variability envelope" as the contour generated in the local frame by vectors centered at the origin and whose length is the standard deviation of the time series of coordinates projected in all directions of the plane.

About 250 sources are found to have a dense enough observational history over 1987-1999 to derive the envelope of their local variability. Figure 2 (resp. 3) shows the local variability envelopes corresponding to 0.5-year averaging intervals, in the equatorial reference frame, for

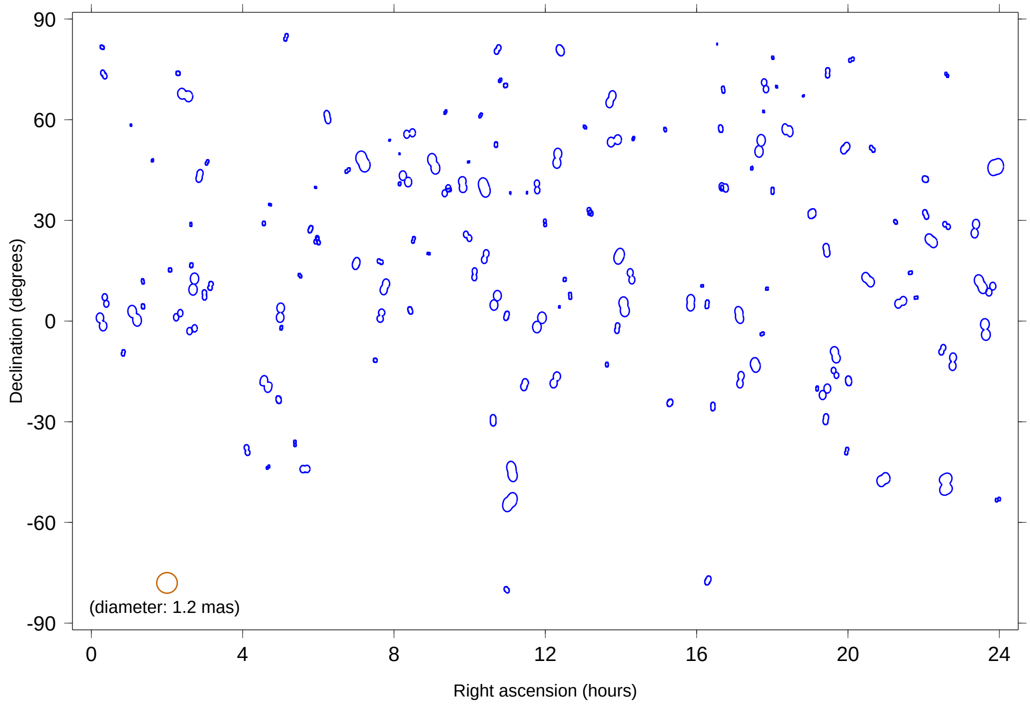

Figure 2: Sources with high or moderate stability. The figure shows the envelopes of the 1987-1999 standard deviation of 0.5 year averages in equatorial coordinates. Abscissae: $\alpha \cos \delta$, ordinates: δ. The scale is given by the circle (lower left).

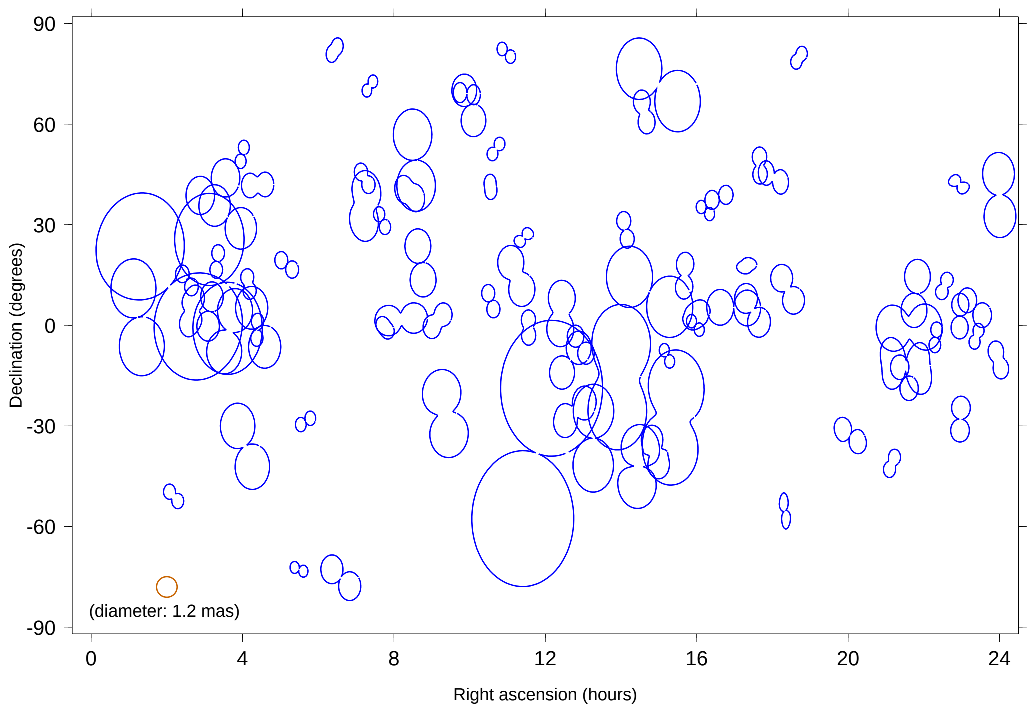

Figure 3: Sources with low stability. The figure shows the envelopes of the 1987-1999 standard deviation of 0.5 year averages in equatorial coordinates. Abscissae: $\alpha \cos \delta$, ordinates: δ. The scale is given by the circle (lower left).

sources with a standard deviation at 0.5-year intervals smaller (resp. larger) than 0.6 mas. Most envelopes are elongated, as could be expected from the active jet structure of the sources. Superimposed to this shape, one can notice a higher occurence of alignment of the local x-axis with the declination direction, particularly visible for the less stable sources (Figure 3).

The set of the most stable sources show a quite even distribution of the local directions of maximum variability in the declination zones north of +20° and south of -20°, suggesting that the observed variability corresponds to real local effects in the sources. However, the results in the declination zone [-20°, +20°] show an excess of the variability in the direction of declinations. This excess is much stronger for the more variable sources, for which an even distribution is reached only north of +20°.

These noise features reflect the medium frequency (0.5-year intervals) effect of the known systematic declination errors due to the troposphere modelling. The global impact of these deformations on a reference frame materialized by the source coordinates can be modelled as a constant offset in the declinations (see Section 3).

3. STABILIZING THE CELESTIAL REFERENCE FRAME

We adopt a time series approach in order to select sources that are stable enough to insure convergence of a global reference frame, based on several years of observations. Using the series of differences of yearly source coordinates with respect to the global mean coordinates for each source, we construct a series of yearly differential frames. We then compute the yearly differential rotation angles $A_1(y)$, $A_2(y)$, $A_3(y)$ around the axes of the equatorial coordinate system for year y, using the two following equations, where α, δ are the the source coordinates and $\Delta\alpha(y)$, $\Delta\delta(y)$ are the differences of the average coordinates for year y with the mean source coordinates. The dz term is introduced to account for any residual equator bias resulting from the systematic errors in declination mentioned in section 2.

$$\Delta\alpha(y) = A_1(y)\tan\delta\cos\alpha + A_2(y)\tan\delta\sin\alpha - A_3(\mathrm{y})$$
$$\Delta\delta(y) = -A_1(y)\sin\alpha + A_2(y)\cos\alpha + dz(y)$$

The estimation of the four unknowns A_1, A_2, A_3 and dz is done at one-year intervals, both by weighted least squares (L2) and by minimizing residuals according to the L1 norm (absolute value of the residuals). In the estimations, the observation equations are weighted according to the variances of the yearly average coordinates.

Among the 283 sources with a long and dense history over 1987-1999 to perform statistical tests, 13 are discarded for the estimation of the rotation angle because of unreasonably large standard errors in the yearly coordinates. Starting with the 270 remaining sources we implement a source selection algorithm based on the stability of the individual source coordinates as measured by the goodness of fit (Bevington, 1969, p. 188)in the local x-axis direction.

For the set of sources under consideration, the time series of unknowns are estimated in both the L1 and L2 norms. The convergence criterion is based on the distance between the two solutions. Sources with the largest goodness of fit are progressively eliminated until the distances at all dates are smaller than the standard error of the L2 estimate (see Gontier et *al.*, 2001).

The algorithm stabilizes when 242 sources out of the original 283 are kept (86%), corresponding to a maximum goodness of fit of 17.5.

Figure 4 shows the time series of the L2 estimates of the angles at the start and end of the stabilizing process. The effective numbers of sources present in each yearly solution are given

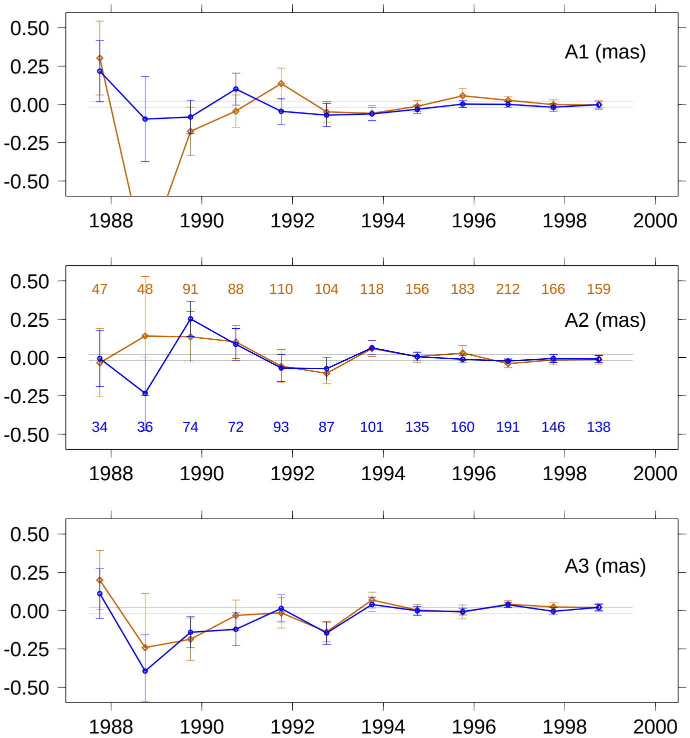

Figure 4: Relative rotation angles of yearly CRFs, before (light) and after (heavy) the implementation of the stabilizing algorithm. The double horizontal line corresponds to the stated uncertainty of the ICRF axes directions ($\pm$ 20 μas)

with the A_2 graph (bottom line for the final selection, upper line for the initial set).

The better stability of the yearly reference frames in the second half of the time span is associated with larger number of sources. The stability is at the level of $\pm$ 20 μas for the first six years, then drops to $\pm$ 3-4 μas over 1993-1998. These estimates of the celestial frame stability are to be compared to the stated uncertainty of the ICRF axes ($\pm$ 20 μas, shown on Figure 4 by a double horizontal line), that was based on the comparison of reference frames obtained from subsets of the observations, with no consideration of time.

The influence of the rejected sources is visible only before 1992. This suggests that, for the sources selected on the basis of continuity of observation, the instability before 1992 is due globally to the actual implementation of the technique rather than to the sources themselves.

4. CONCLUSIONS

This study aimed at describing the time behaviour of extragalactic radio source coordinates measured by VLBI. It is based on the 283 best observed sources over 1987.25-1999.25, extracted from 645 time series of equatorial coordinates computed for over 2000 sessions starting in 1979.

The non circular character of the measurement noise, attributable to troposphere mismodelling and/or to source structure perturbations, was highlighted. Time series of coordinates of sources south of 20° of declination are dominated by noise in declination.

To study the effective stability of the sources, we defined a local reference frame with x-axis in the direction of the maximum variability with time. We then performed a series of statistical tests which are detailed in Gontier *et al.* (2001).

Using an original algorithm, we selected a set of 242 sources that define a time series of yearly reference frames over 1987-1998 realizing a global stability of the axes of 14 μas over the 12 years, a number comparable to the stated 20 μas axis stability of the ICRF. The last six years, however, show a large improvement in stability, with a global axes stability of 4 μas over six years.

The present study also highlights the usefulness of considering the time evolution of radiosource directions, not only for internal purposes such as the ICRF quality control or applications like differential VLBI, but also to enhance the support of VLBI to scientific research, like in the understanding of the physical properties of the non-rigid Earth through analysis of precession and nutation observations (Dehant et al. 2000), or for realistic studies of source structure or microlensing effects.

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THE ICRF SOURCES OBSERVED IN INFRARED

A.S. KHARIN
Main Astronomical Observatory National Academy of Sciences of Ukraine
Golosiiv, 03680 Kyiv-127, Ukraine
e-mail: kharin@mao.kiev.ua

ABSTRACT. The IRAS PSC (Beichman et al.,1988) infrared objects were identified with their radio counterparts from ICRF (Ma et al., 1998) and a list of 31 IRAS PSC/ICRF common sources was compiled. Seven objects from this list are among those 55 that have been obtained early from the ICRF and IRAS 179 QUASARS catalogues (Neugebauer et al., 1986) which were identified by Kharin (2000). If one merges these two files into a full list of the ICRF sources that were observed in infrared that the list would include 79 objects. Both above mentioned identifications are interpreted as preliminary ones. Their verification is planned to be carried out on the base of the 2MASS and DENIS data after lasts will be published.

1. INTRODUCTION

One of the main goal of IR astrometry is building of the IR reference catalogue which is necessary to organize differential position observations and to extend the ICRF coordinate system to the IR range. While precise positional observations in infrared are absent, such a catalogue may be compiled by identification of IR objects with their optical counterparts from precise astrometric catalogues. A method of such an identification has been developed and applied in some researches (Hindsly and Harrington 1994; Kharin 1992).

According to our conception (Kharin, 1997) the IR reference catalogue has to include, besides IR/OPTIC counterparts, also the ICRF sources observed in IR. First of all it is necessary to ensure direct connection between the IR and radio coordinate systems (Babenko et al., 1999).

Therefore it is necessary to find out such ICRF sources which were observed in infrared. To solve this problem two investigations had been undertaken at the Golosiiv Observatory.

Firstly the list of 179 quasars observed by IRAS satellite in 1983 (Neugebauer et al., 1986) was compared with the ICRF list of 608 sources (Ma et al.,1998) and 55 common extragalactic sources were found (Kharin, 2000).

This work is in progress and the next step of this investigation is finished. The same ICRF 608 sources catalogue was compared with the IRAS Point Source Catalog (PSC) and the second list of the IR/ICRF common sources was compiled. The results of this second investigations are presented here.

2. THE SHORT CHARACTERISTIC OF THE ICRF CATALOGUE

The ICRF is realization of the new International Celestial Reference System (ICRS) that had replaced from January 1, 1998 the old coordinate system based on the FK5. The list of the ICRF sources has two published variants. First preliminary ICRF list was prepared by IAU WGRF. It was presented to XXII IAU GA in Hague in Sept 1994 and published in (IAU IB, 1995). It consist from three identical files which contains three different ('d', 'c', 'o') quality group of extragalactic sources where the 'd' stands for the defining sources (441), 'c' stands for additional sources (92), and 'o' stands for optical sources (73). There are 606 objects.

The final ICRF catalogue was prepared by subgroup of IAU WGRF (Ma et al.,1998). It is presented to XXIII IAU GA in Tokio in Sept 1997 and was adopted as the new conventional celestial reference frame instead the FK5. There is many differences between this new and first (606) ICRF list version. To transit from first to final list of 56 objects must excluded and 58 new one - included.

The final version contains 608 sources which are divided into three categories also and the first one has the same name "defining sources" but second and teared ones have some another names - "candidate" and "other" accordingly and they have some other interpretation. "Defining" group include 212 objects with more accurate and reliable positions, better then about 1 mas in both coordinates. These 212 sources practically define orientation of the ICRF axes in relation to the ICRS axes (Arias et al. 1995). "Candidate" category include 294 objects which have not enough VLBI observations to get precise position now. There is proposed that in the future, in other ICRF realization, every sources from this group can be remote into "defining" group also. The "other" group include 102 from 608 sources which have unstable position and they can be used for connection optical and radio coordinate system only.

As one can see there is strong relation between these three files. It is a result of strong criteria which were used here. Because we used only last ISRF realization.

3. EXTRAGALACTIC SOURCES INFRARED OBSERVATIONS

The IR extragalactic sources were fined in two published catalogues. First is IRAS Point Source Catalog (PSC) (Beichman et al.,1988). It was compiled on the base of "All sky survey" programme that had been carried out with IRAS satellite from January to November 1983. The PSC contains the fluxes of 12, 25, 60 and 100 μm for 245889 point IR objects including extragalactic point sources emitted in these four bands. The second catalogue containing 179 observations more bright quasars (Neugebauer et al., 1986) was compiled on the base second IRAS programme named "Point observations". It was carried out apart from the above mentioned "All sky survey" programme. This catalogue includes astrophysical data and instead usual equatorial coordinates contains only identificators formed on B1950 coordinates as HHMM.m,+GG.g or HHMM.m,-GG.g

Comparison of these two catalogues with ISRF 608 sources were carried out by two different ways. The first one (IRAS 179 QUASARS) was crossidentified by above mentioned B1950 identificators which have accuracy 7.5' in R.A. and 3.0' in declination. The second (PSC/ICRF) identification was carried out by comparison of their equatorial coordinates and some criteria to be established from PSC position accuracy.

4. THE IRAS PSC AND ICRF COMPARISON

A method of identification of IR objects with their optical counterparts from astrometrical catalogues developed in Golosiiv has two steps. The first one is preliminary identification based on estimation of coordinate differences of the IR and optic catalogues positions. The second one is practically verification of preliminary identification. It based on the photometrical data which uses to obtain colors for every stars and on their comparisons with color/color diagram which can be built with help of such data.

Unfortunately the ICRF have not necessary photometrical data because the first step only of the above procedure was carried out in relation of both ICRF/PSC and ICRF/IRAS 179 QUASARS identifications. To compare the PSC and ICRF catalogues method proposed by L.Yagudin (1997) was applied and two circles with $\rho = 60''$ and $\rho = 120''$ were used to get synonymous identifications for all sources. The result of this preliminary ICRF/PSC identification presented in table 1.

Table 1. 31 common ICRF/PSC sources obtained by position comparisons

N	*PSC_ID*	*ICRF_ID*	*R.A.*	*DRA*	*DCL*	*DDL*	*FLUX*[12]	*X*	*S*	*H*
1o	00596+5807	0059+581	1 245.7	-0.9	+582411	22	3.171E-01	2		
2o	01348+3254	0134+329	13741.2	-0.8	+33 935	14	2.500E-01			y
3c	01475-0740	0147-076	150 2.6	0.8	- 72548	-4	2.797E-01			
4o	02386-0828	0238-084	241 4.7	-0.3	- 81520	-1	2.949E-01	4	2	
5c	02410+6215	0241+622	24457.6	0.5	+6228 6	1	5.931E-01			y
6d	02483+4302	0248+430	25134.5	1.4	+431515	1	2.500E-01			
7o	03164+4119	0316+413	31948.1	-0.1	+413042	0	9.736E-01			y
8o	04207-0127	0420-014	42315.8	0.3	- 12033	4	4.008E-01	3	1	y
9o	04305+0514	0430+052	43311.0	0.1	+ 52115	2	3.300E-01	4	3	y
10o	05373-4406	0537-441	53850.3	-0.5	-44 5 8	-1	2.694E-01	3	1	y
11d	08291+0439	0829+046	83148.8	-0.8	+ 42939	-2	2.907E-01			
12c	08519+2017	0851+202	85448.8	1.6	+20 630	1	3.471E-01	2	1	y
13o	09514+6918	0951+693	95533.1	0.7	+69 355	4	6.518E-01			
14c	12132-1715	1213-172	121546.7	1.4	-173145	47	2.978E+00			
15o	12265+0219	1226+023	1229 6.6	-0.6	+ 2 3 8	-3	5.175E-01			y
16c	12282+1240	1228+126	123049.4	-1.9	+122328	-18	4.757E-01	3	2	
17c	12373-1120	1237-113	123959.4	-2.3	-113722	5	5.714E-01			
18c	12540+5708	1254+571	125614.2	-0.1	+565225	-1	1.814E+00			y
19c	13225-4245	1322-427	132527.6	-0.9	-43 1 8	-3	1.114E+01			
20c	13451+1232	1345+125	134733.3	0.4	+121724	0	2.500E-01	4	4	
21d	15494-7905	1549-790	155658.8	-0.4	-7914 4	-2	2.500E-01			
22c	16010-4432	1600-445	16 431.0	0.6	-444131	-35	2.977E-01			
23o	16413+3954	1641+399	164258.8	1.5	+394836	6	3.189E-01			y
24c	17326+3859	1732+389	173420.5	-1.6	+385751	-8	2.500E-01			
25c	18036+7827	1803+784	18 045.6	-0.2	+7828 4	-2	6.332E-01	2	1	y
26c	18072+6949	1807+698	18 650.6	-0.5	+694928	-4	2.500E-01			
27d	18456+7943	1845+797	1842 8.9	0.0	+794617	0	2.500E-01			
28c	19440+2251	1943+228	1946 6.2	2.4	+23 0 4	41	1.570E+01			
29c	20232+3332	2023+335	202510.8	1.1	+3343 0	12	4.231E-01	3	3	
30c	22231-0512	2223-052	222547.2	-0.9	- 457 1	12	4.598E-01			
31d	22292+6930	2229+695	223036.4	0.3	+694628	58	2.516E-01			

In table 1 next column are presented:

N - ordinal numbers;

"dco" - code classificators;

PSC_ID - PSC identificators;

ICRF_ID - ICRF B1950 identificators;

R.A. - Right Ascensions;

DRA - The ICRF/PSC Right Ascensions differences in sec;

DCL - Declinations;

DDL - The ICRF/PSC Declinations differences in arcsec;

$FLUX[12]$ - Flux for 12 μm;

X: Structure index at X band;

S: Structure index at S band;

H: "y" indicates that the source serves to link the Hipparcos stellar reference frame to the ICRS.

5. CONCLUSIONS

Fined 79 IR/RADIO objects should be included to the first level IR reference catalogue, compiled in Kiev (Kharin & Molotaj, 1999). Such an add-on permits, firstly, to establish direct connection between the infrared and ICRS coordinate systems and, it can be used, secondly, to improve in the future the connection between radio (ISRF) and optic (HIPPARCOS) coordinate systems (Babenko et al., 1999). To get new prospects we recommend the list of 79 IR/RADIO sources for VLBI (or VLA) observations as well as for infrared precise positional observations.

This work is in progress now and verification of the both preliminary investigation are planned. It can be doing after the ICRF sources are identified with their counterparts from DENIS and 2MASS data. The lasts have to be published in the nearest future (Epchtein 1997). The method of such verification where DENIS data were used is developed now at Golosiiv Observatory (Pakuliak et al. 2000)

The files 31_*tab.dat*, 55_*tab.dat* and *IR_RADIO.dat* contained 31, 55 an 79 sources accordingly can be find at the anonymous $ftp : /ftp2.mao.kiev.ua$

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A CONTRIBUTION TO THE LINK OF THE HIPPARCOS CATALOGUE TO ICRS

M. LATTANZI[1], G. MASSONE[1], A. POMA[2] and S. URAS[3]

[1]Osservatorio Astronomico di Torino
Via dell'Osservatorio 20, 10025 Pino Torinese (TO), Italy
e-mail: lattanzi@to.astro.it, massone@to.astro.it

[2]Osservatorio Astronomico di Cagliari
Poggio dei Pini, Str. 54, 09012 Capoterra (CA), Italy
e-mail: poma@ca.astro.it

[3] CNR - Gruppo Nazionale di Astronomia - Udr di Cagliari
Poggio dei Pini, Str. 54, 09012 Capoterra (CA), Italy
e-mail: uras@ca.astro.it

ABSTRACT. The status and first preliminary results of the program of determination of precise optical position of 80 counterparts of extragalactic radiosources taken from the IERS list are presented. Plate material includes about 350 plates exposed mostly at the Torino astrometric reflector REOSC and photographic refractor Morais. The plates were measured at the Cagliari Observatory on the Tocamm (Torino Cagliari Measuring Machine), an old Ascorecord machine recently converted into automatic and impersonal one. The work of measuring has been completed in the first half of this year. A first analysis indicates that the mean error in the plate co-ordinates is about 0.5 micron, confirming the results of calibration tests. The work of the plate reduction is in progress.

1. INTRODUCTION

We report here the progress in the work on measuring optical counterparts of extragalactic, radio reference frame. This work is the primary goal of the TOCAMM Project, a joint co-operation between the Torino Astronomical Observatory and the Cagliari Astronomical Observatory in the field of astrometry. The TOCAMM program was initiated in 1997 and its first objective was to convert the old measuring machine ASCORECORD into an automatic and impersonal one (completed in 1998). The main scientific goal was to provide precise position of optical counterparts of 80 extragalactic radiosources selected from the IERS list to contribute to the link of the Hipparcos Catalogue to the ICRS. The work of measuring all 350 plates taken during 1986-1994 was recently (May 2000) completed.

Next future plans include contributions to the astrometric accuracy of the Guide Star Catalog (version 1 and 2) and a re-measurement of the old Vatican Carte du Ciel plates (1046 plates covering the zone of declination from $+ 55^o$ to $+ 64^o$) taken in the period 1891-1926 (Bucciarelli et al., 2000). As discussed by Urban et al. (1996) such as re-measurement may be of great value considering the poorly accurate method used for the original measuring of the Vatican plates.

To expedite the work, an eyepiece grid (diaframma) was, in fact, used instead of a micrometer screw so degrading the accuracy of the published measures.

2. OBSERVATION

The observations of the optical counterparts of radiosources were obtained in largest part at the astrometric telescopes of Turin Astronomical Observatory whereas for some fainter and/or southern objects the 1.5m reflector at Loiano and the astrograph at La Silla were used. Table 1 lists the main instrumental parameters of the telescopes used.

Table 1: Main Instrumental Parameters of Telescopes

	Diameter (cm)	Focal length (cm)	Plate scale (arcsec/min)	Field
Astrometric reflector REOSC (Torino)	105	994.2	20.747	45' with 16x16 cm plates
Photographic refractor Morais* (Torino)	38	687.5	30.002	1.5 with 20x20 cm plates
Ritchey - Chretien reflector** (Loiano)	152	1213.3	17.000	1.2 with 16x16 cm plates
ESO-GPO Astrograph (La Silla)	38.6	399.4	51.644	2x2

* For primary reference stars and radio sources brighter than the photographic magnitude 15
** equipped with 320x512 pixel RCA CCD

The 38 cm refractor Morais was used for the observations of the primary reference stars and of the radio source with optical counterparts brighter than the photographic magnitude 15. For those fainter than this value the 105 cm reflector REOSC was used. Blue plates (Kodak 103a-O) were used on both the telescopes. The observations were carried out from 1985 to 1994. Fig. 1 shows the distribution of epochs.

The observing list consisted of 80 radiosources in the declination range from + 84^o to - 28^o. The magnitude range lies between 10 and 19. The secondary reference stars are in the magnitude range from 12 to 14. Fig. 2 displays the distribution on the sky of the observed radio stars.

3. THE TOCAMM MACHINE

As stated above the old measuring machine ASCORECORD was upgraded into an automatic and impersonal one. The major changes were as follows:

- substitution of the original glass scales (read off by the very time consuming "spiral micrometer") with two high accuracy Heidenhain optical rulers, with RS 232 computer interface

- substitution of the visual binocular head with a CCD camera, the CCD itself being now the reference "crosshair" for the measures;

- introduction of a motorized computer controlled stage for the plate carriage deplacement.

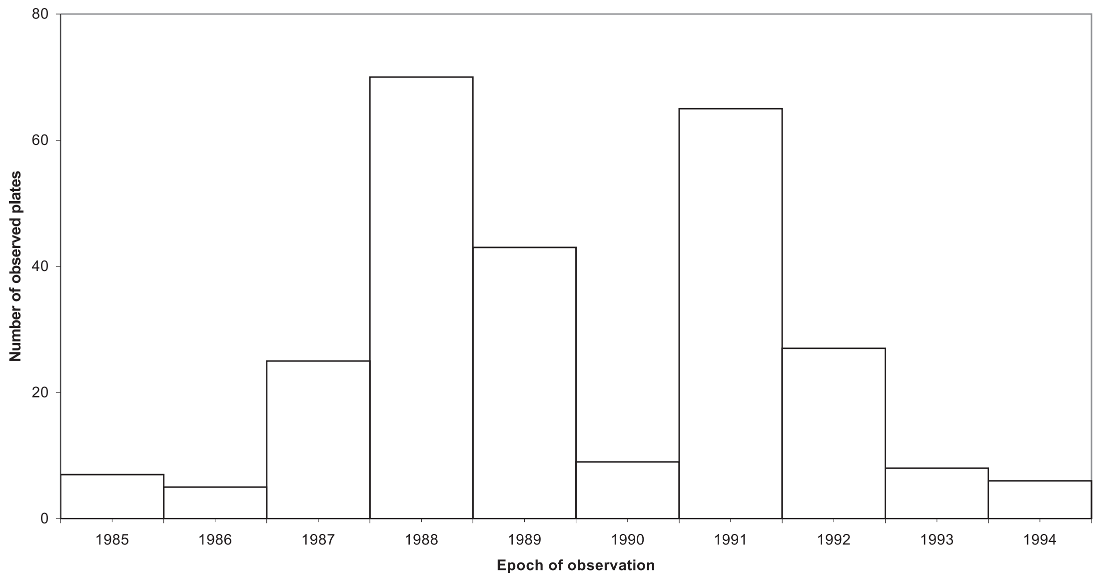

Figure 1: Epoch Distribution of Observed Plates

The CCD camera is equipped with a Kodak Kaf 260 detector of 512 by 512 pixels, covering a field of view on the plate of 2.8 by 2.8 mm. During operations the CCD temperature is controlled by a Peltier cell and kept stable within 0.5 degrees at -2^{o}C exposure time. The plate sampling is 5.5 micron/px. The Tocamm machine, after some preliminary tests carried out at the Torino Astronomical Observatory, was moved in the summer of 1998 to the Cagliari Astronomical Observatory where now is installed in a temperature-controlled room. In order to check possible temperature effects, 4 temperature sensors are mounted on the machine (one on each optical ruler, one on the CCD's mounting head and one on the machine environment). No detectable temperature drift was found for a 12 hours measuring run. Several calibration tests carried out measuring the same plate in different orientation (in one position and then after rotation of the plate of 90, 180, and 270 degrees) indicate that the positions are accurate and stable at 0.5 micron level on both co-ordinates, i.e. the intrinsic limit of the presently available optical rulers. Details are given in Del Bo et al. (2000).

4. PLATE REDUCTION

All the plates have been measured on the Tocamm machine at Cagliari Observatory. Typically each plate takes about five hours to process and the mean rate of production was about 40 plates per month. The whole work of reduction has been finished in about 14 months.

As a first step in the reduction of the data the classical model of the 6 first-order plate constants was used. The primary reference net comes from the Carlsberg Automatic Meridian Circle Catalogue.

Very preliminary estimates of the accuracy based on a sample of few Morais plates confirm that the standard deviation for repeatability is of the order of 0.5 micron (about 10 to 15 mas depending on the plate scale). The internal errors (standard deviation) of the measured positions are derived from the residuals of the reference stars and vary from 80 to 150 mas. The plate to plate consistency (by comparing positions of both radiosource and secondary reference stars)

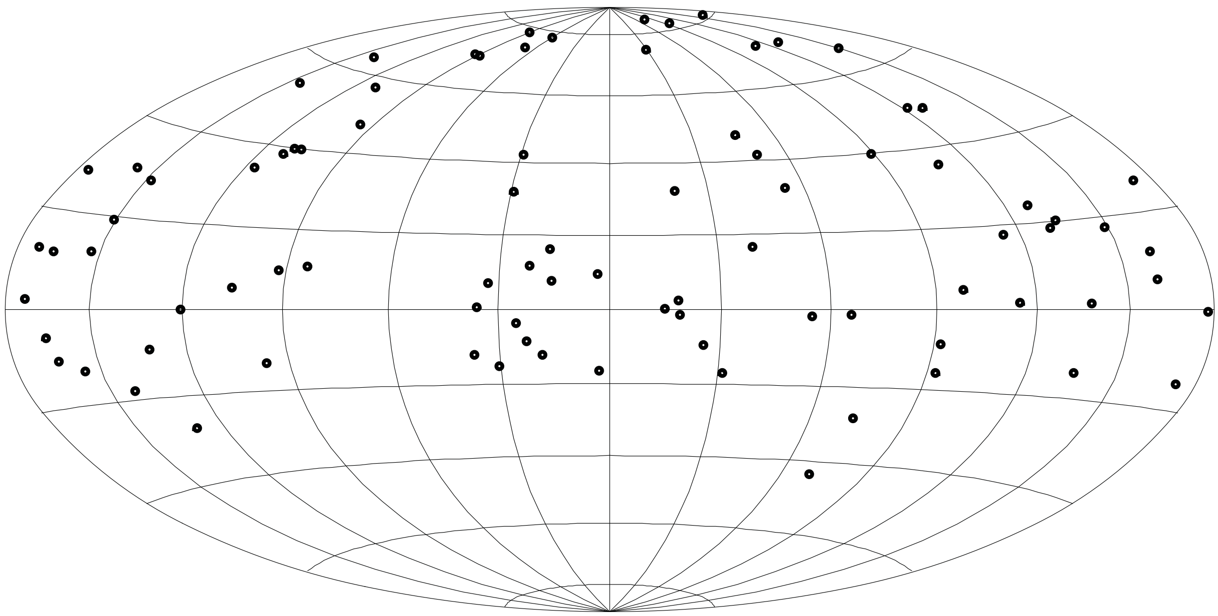

Figure 2: Distribution on the Sky of Studied Radio Sources

has been found to be less than 75 mas, on an average. These figures certainly will be improved during the successive steps based on refined reduction models. The work is in progress and the final results are expected to be completed within the first months of next year (2001).

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LINK OF DYNAMICAL AND CATALOGUE REFERENCE FRAMES: PRESENT STATE AND PROSPECTS

D. DUMA, O. BOLOTINA, L. SVACHII
The Main Astronomical Observatory of the National Academy of Sciences of Ukraine
Golosiiv, Kiev-127, 03680, Ukraine
e-mail: olga@mao.kiev.ua

ABSTRACT. A geometrical approach to linking the axes of the catalogue and dynamical reference frames is proposed. It is compared with an analitical approach. It is shown that mutual equator declination between the catalogue and dynamical systems should be taken into account when star coordinates reduced to the dynamical zero-points. The results of reducing of the Washington and Greenwich meridian observations of the Sun 1925–1982 and the Washington 1949 – 1977 and Herstmonceux 1957 – 1982 meridian observations of minor planets (1) Ceres, (2) Pallas, (3) Uno, and (4) Vesta are presented. Several recommendations concerning to the prospects of linking the catalogue and dynamical reference frames are formulated.

1. INTRODUCTION

Several approaches and mathematical simulations are presently used to link catalogue and dynamical reference frames. The analytical approach [2, 3], being more preferable, involves equations with unknown corrections to orbital elements for the differences between positions of the Sun or planets (α_0, δ_0) obtained from differential observations and (α_c, δ_c) calculated with an adopted theory of motion. The correction $\Delta\alpha_0$ to right ascensions of catalogue stars or radiosources characterizing the difference between the catalogue and dynamical equinoxes is taken into account in the equation for $(\alpha_0 - \alpha_c)$ to link the catalogue and dynamical zero-points. The correction $\Delta\delta_0$ to catalogue declinations in the equatorial zone characterizing the difference between the dynamical and catalogue equators is entered into the equation for $(\delta_0 - \delta_c)$. Introduction of the unknowns $\Delta\alpha_0$ and $\Delta\delta_0$ is testified by the fact that average estimates of $\overline{(\alpha_0 - \alpha_c)} \neq 0$ and $\overline{(\delta_0 - \delta_c)} \neq 0$ for large samples of differences $(\alpha_0 - \alpha_c)$ and $(\delta_0 - \delta_c)$ obtained with improved orbital elements of a planet. Equations for the relationship between the zero-points and for the improvment of the Earth's orbital elements from observations of the Sun are [3]

$$[\alpha_0 - \alpha_c,\ \delta_0 - \delta_c] = [\Delta\alpha_0,\ \Delta\delta_0,\ \Delta L_0,\ \Delta\mu,\ e\Delta\pi,\ \Delta e,\ \Delta\epsilon]M_1. \tag{1}$$

Different equations are applied in linking the catalogue and dynamical zero-points from observations of major and minor planets. An equation form depends on the orbit orientation and eccentricity, as well as on other factors. As an example, we can take the relations from [2] between the differences $(\alpha_0 - \alpha_c,\ \delta_0 - \delta_c)$ and the corrections to the orbital elements for best improving the catalogue zero-points and the elements of orbits with small eccentricity

$$[\alpha_0-\alpha_c,\ \delta_0-\delta_c]=[\Delta M_0+\Delta r,\ \Delta p,\ \Delta q,\ e\Delta r,\ \Delta a/a,\ \Delta e,\\ \Delta M_0'+\Delta\Psi_3',\ \Delta\Psi_1',\ \Delta\Psi_2',\ e\Delta\Psi_3',\ \Delta e',\ \Delta\delta_0]M_2. \tag{2}$$

The correction to the catalogue equinox $\Delta\alpha_0$ is present in (2) in $\Delta\Psi_2'=-\Delta\alpha_0\sin\epsilon$ and $\Delta\Psi_3'=-\Delta\alpha_0\cos\epsilon+\Delta\omega'$. Introduction of combinations of the unknowns in (2) is explained by the presence of close correlations between the corrections to the orbital elements of the Earth's and a planet.

Sometimes quite simple (formal) mathematical simulations are used to link the catalogue and dynamical reference frames. Batrakov et al. [1] considered differences $(\alpha_0-\alpha_c)$, and $(\delta_0-\delta_c)$ as spherical expansions

$$\alpha_0-\alpha_c=\sum_{i=0}^{2}\sum_{j=0}^{2}(a_{ij}\cos j\alpha\ +a_{ij+2}\sin j\alpha)\,T_i(\delta),$$

$$\delta_0\ -\ \delta_c=\sum_{i=0}^{2}\sum_{j=0}^{2}(b_{ij}\cos j\alpha\ +a_{ij+2}\sin j\alpha)\,T_i(\delta).$$

In this case the systematic differences in orientation of the catalogue and dynamical reference frames are difficult to interprete.

2. THE GEOMETRICAL APPROACH TO LINKING THE AXIS OF THE KATALOGUE AND DYNAMICAL REFERENCE FRAMES

The MAO, National Academy of Science of Ukraine [4, 5, 6] puts forward a geometrical approach to linking the axis of the catalogue and dynamical reference frames. The principle of the geometrical approach is as follows: the coordinates $(\alpha_0,\ \delta_0)$ observed in the system of a reference catalogue on the one hand, and the coordinates $(\alpha_c,\ \delta_c)$ calculated with the adopted theory of motion of a planet on the other hand, are used to build the principle circles on the sphere of a unit radius, the equator and the ecliptic, fixing the catalogue and dynamical equinoxes. Then we can build the oblique-angled and the rectangular catalogue coordinate systems based on the principle angles and points, as well as the rectangular dynamical system. The equations for the relationship between the observed and calculated positions of the Sun and planets can be obtained by transformation from the catalogue $(\alpha_0,\ \delta_0)$ to dynamical coordinates $(\alpha_c,\ \delta_c)$ or vice versa. The equations for angles of relative orientation of catalogue and dynamical reference frames are [4, 7]

$$[\alpha_0-\alpha_c,\ \delta_0-\delta_c]=[P,\ Q,\ R,\ \Delta\delta_0]M_3. \tag{3}$$

Here $P=\Delta\epsilon,\ Q=\Delta\pi\ \sin\epsilon,\ R=+\Delta A$ are the angles of relative orientation of the reference frames, $\Delta\epsilon$ is the correction to the inclination of the ecliptic to the equator, $\Delta\pi$ is the correction to the Earth's orbit perihelium, ΔA is the difference between catalogue and dynamical equinoxes.

Corrections to the orbital elements can be introduced in (3) to improve theories of the Earth's and planetary motions. Then we have for the Sun

$$[\alpha_0-\alpha_c,\ \delta_0-\delta_c]=[P,\ Q,\ R,\ \Delta\delta_0,\ \Delta L_0,\ \Delta\mu,\ \Delta e]M_4 \tag{4}$$

and for planets

$$[\alpha_0-\alpha_c,\ \delta_0-\delta_c]=[P,\ Q,\ R,\ \Delta\delta_0,\ \Delta L_0,\ \Delta\mu,\ \Delta e,\ \Delta\Omega_p,\ \Delta i_p,\ sini_p\Delta\omega_p,\ \Delta M_0,\ \Delta a/a,\ \Delta e_p]M_5. \tag{5}$$

Entering the corrections to orbital elements into (4) and (5) results in closer correlations between the unknowns and that is why the estimates of P, Q and R are obtained with a low precision.

3. COMPARISON OF THE ANALYTICAL AND GEOMETRICAL APPROACHES AND RESULTS

Comparison of the analytical and geometrical approaches to the linkage of the catalogue and dynamical reference frames gives rise to the following inferences.

1. The geometrical approach implies obtaining of the three angles P, Q, R, which corresponds to a well-known principle of linking the two reference frames. The analytical approach reduces to deriving the corrections $\Delta\alpha_0$ and $\Delta\delta_0$ only, the mutual inclination of the catalogue and dynamical equators are not taken into account. Therefore, the analytical approach is a special case of the method for linking the catalogue and dynamical reference frames we put forward. Different realizations of the catalogue reference frames can be reduced to ones of the dynamical reference frames by the relations

$$\begin{aligned} \Delta\alpha &= -P\, tan\delta\, \cos\alpha + Q\, \tan\delta\, \sin\alpha - R, \\ \Delta\,\delta &= P\, \sin\alpha + Q\, \cos\alpha - \Delta\delta_0. \end{aligned} \tag{6}$$

2. Constant components of the corrections to right ascensions $\Delta\alpha_0 = R + \Delta\pi\, \cos\epsilon$ and $\Delta A = R$ differ geometrically, so their estimates are different as well.

3. The quality of the linkage of the catalogue and dynamical reference frames depends on the Earth's orbit orientation in space. Consequently, the estimates of the orientation angles of the catalogue and dynamical reference frames are different when different theories of the Earth's motion are used.

The geometrical approach was used to link the axis orientation of the reference frames realized by FK5 and the theories of motion of the Sun DE200 and DE403 [9]. The Washington and Greenwich 1925 – 1982 meridian observations of the Sun were reduced on the basis of this approach. It is found that the corrections

$$\begin{aligned} \Delta\alpha &= (0.10'' \pm 0.01'')\, tan\delta\, cos\alpha - (0.04'' \pm 0.01'')\, tan\delta\, sin\alpha + (0 \pm 0.01''), \\ \Delta\,\delta &= -(0.10'' \pm 0.01'')\, sin\alpha - (0.04'' \pm 0.01'')\, cos\alpha + (0.13'' \pm 0.01''). \end{aligned} \tag{7}$$

should to be used to reduce the catalogue FK5 coordinates to the dynamical DE200 zero-points, while the corrections

$$\begin{aligned} \Delta\alpha &= (0.11'' \pm 0.01'')\, tan\delta\, cos\alpha - (0.03'' \pm 0.01'')\, tan\delta\, sin\alpha + (0.02'' \pm 0.02''), \\ \Delta\,\delta &= -(0.11'' \pm 0.01'')\, sin\alpha - (0.03'' \pm 0.01'')\, cos\alpha + (0.13'' \pm 0.01''). \end{aligned} \tag{8}$$

must be added to FK5 star positions to reduce them to the DE403 zero-points. The estimates of the angles P, Q, R in (7) and (8) testifies that the mutual inclinations of the catalogue and dynamical equators in the cases of FK5 – DE200 differ significantly as well as FK5 – DE403 ones.

The geometrical approach to solving the problem on the linkage of the catalogue and dynamical reference frames was used also to reduce the Washington 1949 – 1977 and Herstmonceux 1957 – 1982 meridian observations of minor planets (1) Ceres, (2) Pallas, (3) Uno, and (4) Vesta. We found close correlations between the orientation angles P, Q, R and the corrections $\Delta\Omega$, Δi, $\Delta\omega$ to the elements of the orientation of the planetary orbits in space [8, 10] (see the Table). An analysis of the correlations for different minor planets shows the correlations between similar corrections to be different and depend on the orientation of the planetary orbits in space. The letter factor gives rise to different bias of the estimates for the corrections to the orbital elements and, consequently, to the catalogue zero-points. We infer that the zero-points of the dynamical reference frames specified by theories of different minor planets are in rather poor agreement with one another. In other words, different dynamical reference frames are realized by theories of different minor planets.

Table. Correlations (C) between the Unknowns from Observations of the Minor Planets (1) Ceres, (2) Pallas, (3) Uno, (4) Vesta.

Unknowns	C for (1)	C for (2)	C for (3)	C for (4)
$R, \Delta\Omega_p$	-0.92	-0.94	-0.96	-0.91
$P, \Delta\omega_p$	0.84	0.88	0.86	0.80
$Q, \Delta i_p$	0.71	-0.63	0.82	0.79
$Q, \Delta\omega_p$	0.51	0.15	0.42	0.15

4. RECOMMENDATIONS AS TO THE PROSPECTS OF INVESTIGATIONS OF LINKING THE CATALOGUE AND DYNAMICAL REFERENCE FRAMES

Several recommendations as to the prospects of linking the catalogue and dynamical reference frames can be formulated on the basis of our investigations.

1. High-accurate position observations of the Sun, major and minor planets might be further proceeded. Although observations of the Sun are difficult to carry out and less accurate as compared with observations of minor planets, they nevertheless are most suitable mathematically for the link between the catalogue and dynamical reference frames. Observations of Venus and Mars [3] are most preferable as for as observations of major planets are concerned. In order to reduce the correlations between the unknowns in the equations and to upgrad the accuracy of the determination of the orientation angles it is advisable to observe minor planets far from opposition. At present such observations can be obtained with the CCD technique and large telescopes.

2. The list of the minor planets selected for observations can be made in dependence of the correlations between the unknowns on the orientation of planetary orbits in space.

3. The more rigorous geometrical approach providing estimates of the three orientation angles is worthy of the linkage of the catalogue and dynamical reference frames.

4. In our opinion, orientating reference frames should be carried out separatedly from improving orbital elements for refining theories of motions of Solar system bodies.

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THE NON-PRECESSIONAL MOTION OF THE EQUINOX : A PHANTOM OR A PHENOMENON ?

V.V. VITYAZEV[1], E.I. YAGUDINA[2]
[1] Sobolev Astronomical Institute
St.-Petersburg University, 198904, St.-Petersburg,
Petrodvorets, Bibliotechnaya pl.2, Russia
vityazev@venvi.usr.pu.ru
[2] Institute of Applied Astronomy, 10 Kutuzov quay, St.Petersburg, 191187, Russia
eiya@quasar.ipa.nw.ru

ABSTRACT. The paper presents a review of the equinox corrections derived from observations of the Sun and planets in XIX and XX centures. It is found that corrections to the Newcomb's equinox are well correlated with the curve $\Delta T = ET - UT$ on the time interval near 170 years. This correlation prompts an explanation that the so called fictitious motion of the equinox may be caused by neglect of transition from UT to ET in treating the observed (not computed) declinations of the Sun. In this way it may be stated that the fictitious motion of the equinox, discovered about 100 years ago, was the first evidence of the irregular rotation of the Earth.

1. INTRODUCTION

Contrary to the ICRS, where directions of the principal axes are fixed once and forever, the pre-ICRS concept, realized in the ground-based optical catalogues, implied that these directions should be tied to the Vernal Equinox and the equatorial plane at some chosen epoch. In the long series of catalogues, from the first commonly recognized fundamental catalogue – the Tabulae Regiomontanae (TR) by Bessel (1930) up to the FK5 – the direction of the x-axis (the equinox of a catalogue) was changed several times. The first change was made in 1882 by Newcomb, who introduced in his catalogues the equinox $N_1 = B + 0.^s016$, where B - the equinox of the TR. In 1931 two important papers (Kahrstedt, 1931; Morgan, 1931) were published. Both authors found that for the beginning of the XX century the equinox N_1 required significant negative corrections $K = N_1 - 0.^s050$ for $T = 1913$ and $M = N_1 - 0.^s038 - 0.^s007(T - 1900)/100$ correspondingly. Due to these results the R.A. zero point in the FK3 and FK4 was taken to be $E = N_1 - 0.^s050$ while the GC had got the equinox $E = N_1 - 0.^s040$.

Later on, Fricke (1982) found that at the epoch 1950 the x-axis of the FK5 was to be returned almost to the direction of the Newcomb's equinox. Moreover, Fricke derived the rate of the secular motion of the equinox with respect to the zero point of the FK4. These results are summarised in his equation

$$E_{FK5} = K + 0.^s035 + 0.^s085(T - 1950)/100,$$

from which the direction of the x-axis in the ICRS was derived at the epoch $J2000.0$.

In his survey of the equinox determinations over 250 years Blackwell (1977) described the effect of equinox motion by a parabola with its vertex at an epoch short after 1900. This data complemented by the observations of minor planets in 1960 - 1997 (Vityazev and Yagudina, 1997) are presented in Fig.1. This plot gives evidence that at least from the stand point of statistics the secular motion of the equinox does exist.

The first hints on the motion of the equinox may be traced down to the end of the XIX century, and they gave rise to speculations about its origin. Two approaches are visible in the attempts to explain the motion of the equinox. In the first one the authors are trying to explain the effect by pure astrometric reasons. In this connection it is instructive to cite Morgan (1950): *The uncertainties in the equinox correction arise from large personal equations in observing the transits of stars and the limbs of the Sun and planets; personal equations varying with magnitude and the difference between day and night seeing; personal equations changing with the methods of observing, eye and ear, key and chronograph, hand or motor driven travelling threads.* and Duma (1995): *The reasons of the non-precessional equinox motion are a consequence of imperfection of the Earth's motion theory and observational reductions.*

In the second approach the physical reasons are proposed. Thus, according to Balakirev (1980): *there exists a real rotation of the plane of the ecliptic about an axis lying in this plane. The axis of rotation is nearly identical with the nodal line of the galactic plane on the ecliptic, and the period of rotation is about* 3×10^8 *years".*

The main goal of this paper is to propose an explanation in some sence intermidiate between the two extremes. It is based on the fact that the irregularity of the Earth rotation was not properly taken into account in treating the observations of the Sun.

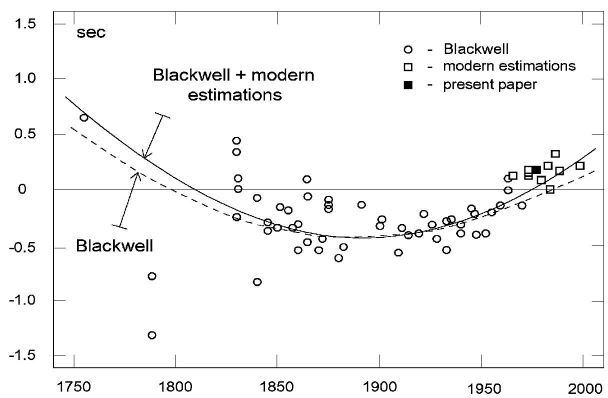

Figure 1: Approximation of the equinox motion by parabola (1755–1997).

2. TREATING THE OBSERVED POSITIONS OF THE SUN The basic equation to derive the equinox correction E is

$$\tan \delta_{obs} = \sin(\alpha_{obs} + E) \tan \epsilon, \tag{1}$$

where δ_{obs} – the DECL. of the Sun reckoned from the equator of a catalogue, α_{obs} – the R.A. of the Sun reckoned from an arbitrary zero point, ϵ – the obliquity of the ecliptic.

Since meridian observations of stars are able to fix the equator of the fundamental system, there is no problem with the zero point of declinations. It means that to get the fundamental δ_{obs} it is sufficient to carry out the differential obsevations of the Sun (in what follows we adopt that no correction to the origin of declination system is needed). On the contrary, the transits of stars provide the R.A. system of the catalogue referred to unknown origin. Thus the differential observations of the Sun give us its R.A. only with respect to this unknown zero point. An introduction of equinox correction E into Eq.(1) balances this deficiency of the zero points.

In practice, Eq.(1) is used in the form

$$\Delta\delta sec^2\delta - \Delta\alpha\tan\epsilon\cos\alpha = E\tan\epsilon\cos\alpha + \Delta\epsilon\sin\alpha sec^2\delta. \tag{2}$$

Here the $O-C$ values are $\Delta\alpha = \alpha_{obs} - \alpha_c$, $\Delta\delta = \delta_{obs} - \delta_c$, $\Delta\epsilon = \epsilon_{obs} - \epsilon_c$, and the values α_c, δ_c, ϵ_c, are computed from the ephemeris.

Consider now the effect of differencies between Universal Time (UT) and Ephemeris Time (ET) on the analysis of observations of the Sun. The observed values $\alpha_{obs}, \delta_{obs}$ are referred to the UT, and if the values α_c, δ_c are computed for the UT timings they should be corrected for the $\Delta T = ET - UT$. It was shown (Fricke, 1980) that **equinox correction E is not affected by the neglect of ΔT in computed values**. Nevertheless, it is not true if the **observed** values are considered. The crucial point is that the transition to the ET must be done only for δ_{obs} since the α_{obs} does not depend on the rate of the Earth rotation (the α_{obs} is nothing else but an angle between the Sun and a reference star measured by a clock the rate of which is synchronised with the current rate of the Earth rotation). That is why, reducing δ_{obs} to ET we get

$$\delta_{obs}(ET) = \delta_{obs}(UT) + \dot{\delta}_{\odot}\Delta T = \delta_{obs} + \dot{\alpha}_{\odot}\tan\epsilon\cos\alpha\cos^2\delta, \tag{3}$$

since from Eq.(1) it follows that

$$\dot{\delta}_{\odot} = \dot{\alpha}_{\odot}\tan\epsilon\cos\alpha\cos^2\delta, \tag{4}$$

where $\dot{\alpha}_{\odot} = 1/365.2422 = 0^s.002737909$ per sec.

Now, from Eq.(2) we find

$$\begin{aligned}&\Delta\delta sec^2\delta + \dot{\alpha}_{\odot}\Delta T\tan\epsilon\cos\alpha - \Delta\alpha\tan\epsilon\cos\alpha = \\ &E\tan\epsilon\cos\alpha + \Delta\epsilon\sin\alpha sec^2\delta.\end{aligned} \tag{5}$$

Solution of this equation may be written in the form

$$E(ET) = E_0 + \dot{\alpha}_{\odot}\Delta T(ET), \tag{6}$$

where E_0 stands for solution of Eq(2) when $\Delta T = 0$.

This result was obtained on the basis of the Bessel's method which was in general use through-out the XIX century. The XX century determinations of the equinox correction were based on the Newcomb's method in which the averaging of $\Delta\alpha = \alpha_{obs} - \alpha_c$ over at least one year period of observations yields

$$E = <\Delta\lambda> - <\Delta\alpha>, \tag{7}$$

where the mean correction to the longitude of the Sun $<\Delta\lambda>$ is derived from the observed declinations. In this case our approach yields

$$E(ET) = E_0 + \dot{\alpha}_{\odot}\Delta T(ET) < sec^2\delta_{\odot} > . \tag{8}$$

Since $< sec^2\delta_{\odot} >= 1.04$ we may state that the equations (6) and (8) coincide withing 4 per cent (a specific accuracy of the Newcomb's method).

Thus we come to our main result: **Neglect of ΔT in treating the observations leads to dependence of the origin of R.A. in a catalogue on time. This phenomenon is mostly known as "fictitious motion of the equinox".**

In a discussion of the equinox determinations Blackwell (1977) replaced the observed corrections $< \Delta\lambda >$ with the values $\dot{\alpha}_{\odot} \Delta T$. Though this diminished the scatter of the data, this procedure is wrong since it actually prevents correcting of the data for the errors other than neglect of the ΔT.

3. CONSISTENCY WITH OBSERVATIONS

To see whether Eq.(6) is able to explain the motion of the equinox (the motion of the catalogue's R.A. origin in time) we chose the observations of the Sun from 1830 up to 1970 which Blackwell (1977) reduced to the FK4 system and presented as corrections ΔN to the Newcomb's equinox. In doing this he corrected initial data by the values $\dot{\alpha}_{\odot} \Delta T - < \Delta\lambda >$ and "night minus day" and "moving wire minus hand tapping" corrections. This data have been complemented by the observations of minor planets in 1960 - 1997 (Yagudina, 1997). According to Duma (1995) transition to the IAU 1976 value of the parallax of the Sun yields the rate of the equinox motion as much as $0^s.17$ per century, i.e. two times more than the rate found by Fricke. For this reason to all data in the XX century the correction $\Delta N_D = -0^s.015 + 0^s.085(T - 1950)/100$ was added to make the rate of the equinox motion consistent with the IAU 1976 value of the parallax of the Sun.

For fitting Eq.(6) to the data one has to evaluate the value E_0. From our analysis it follows that this value is the expected correction to the Newcomb's equinox at the epoch when both time scales ET and UT coinside. Since the scatter of observed data is large it would be dangerous to use any one of the observed values. For this reason it is more reliable to evaluate the E_0 using all available data. This yields $E_0 = -0^s.072$ and with this value the curve of the equinox motion together with the individual observed corrections are shown in the left box of Fig.2. Here we see that the observations and the values ΔT are well correlated. Still, the curve runs below the observed values in XIX century and above them in XX century reproducing very well the general trends of the equinox motion. This prompts the separate analysis for both centures. Really, with the value $E_0 = -0^s.040$ the curve runs within the set of the XIX century points, and is shifted by almost constant value from the XX century points. In its turn, the value $E_0 = -0^s.119$ makes the curve to be well consistent with the set of the XX century observations and to pass below the XIX century points at near equal distances.

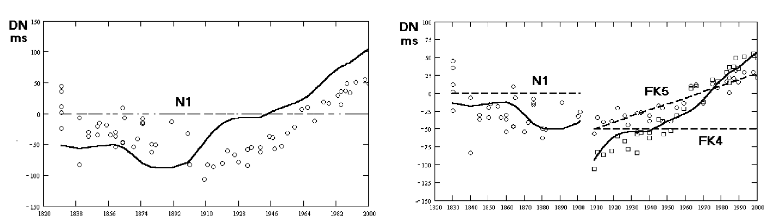

Figure 2: Left: observed equinox corrections (circles), the values ΔT against the time (bold line). Right: Continuous curves corresponding to Eq.(6) – bold line. observed equinox corrections (circles), observations corrected for the IAU 1976 parallax of the Sun (boxes).

With these values the curves of the equinox motion for 1830-1997 are shown in the right box of Fig.2. Now we can see that the curve became discontinuous at the epoch 1900, but except this both branches follow the motion of the equinox pretty well at the time span of almost 170 years. The discontinuity of the curve at 1900 may be explained by a lot of reasons among which the main are: a) the change of observational techniques and b) introduction of the Newcomb's standards for the general use. In this connection a reference is made to Kolesnik (2001) who in his intensive study of the solar and planet observations during 18-20 centuries omitted the E-term in the equation for R.A. thus forcing the correction to the longitude of the Earth to absorb the effect of the equinox motion. Thus obtained resulting curve for the correction to the longitude of the Earth exhibits well pronounced leap about $1''$ near 1900, and the measure of the leap is in good agreement with the $1''.2$ offset of our curves at the same epoch.

For further analysis we reduced the observations and the curves corresponding to Eq.(6) to the zero point of the ICRF. This was done in two steps: the first step implied transition from the FK4 to the FK5 equinox (Fricke, 1982):

$$\Delta A'(T) = \Delta N(T) - 0^s.015 + 0^s.085(T - 1950)/100,$$

while the second step was used to make the reduction to the HIPPARCOS (ICRF) system:

$$\Delta A(T) = \Delta A'(T) - \epsilon_3 - \omega_3(T - 1991.25)/100,$$

where $\epsilon_3 = 16.8\ mas$, $\omega_3 = 0.88\ mas\ y^{-1}$ – rotation and spin of the FK5 frame with respect to the HIPPARCOS (ICRF) (Mignard and Froeschle, 1997).

The results are shown in the left box of Fig.3, where one can see that our curves are again in good agreement with the observations. Moreover, the plot of residuals "Obs. - E(ET)" against date (the right box of Fig.4) gives evidence that after elimination of the values $E(ET)$ from the observations they no longer show any systematic motion of the equinox neither in the XIX nor in the XX century.

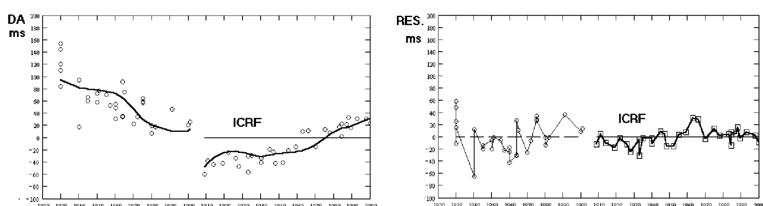

Figure 3: Left: Equinox motion with respect to ICRF. Right: Residuals Obs. - E(ET)

4. DISCUSSION

As it was mentioned earlier, from 1830 to 1989 the R.A. zero point of the fundamental catalogues was changed three times in order to keep the catalogue equinox to Vernal Equinox as close as possible. The proposed explanation reduces the problem to such very well known practics as, for example, an introduction of a leap second in UTC to mantain the UTC in such a way that the difference between the two time scales would never be more than a desired tolerance. In this way it may be emphasised that the so-called fictitious motion of the equinox, discovered about 100 years ago, was the first evidence of the irregular rotation of the Earth.

Now, several remarks should be made about the corrections applied to the individual observations. Our experience shows that no matter whether or not the corrections of the type "day minus night", etc., were applied, the Eq.(6) yields reasonable approximation of the trend in the observed values. Still, of all the corrections the reduction to the IAU 1976 value of the parallax of the Sun, proposed by Duma (1995) is of a paramount importance. In Fig.2 (circles and the dashed line) one can see that practically all observations of the Sun before 1970 and of the planets after 1970 are in excellent agreement with the linear motion of the equinox, found by Fricke (1982). It is not a surprise, for this law of the motion was derived from the observations that were treated with the Newcomb's "Tables of the Sun", based on the standards of 1896.

When corrected for new value of the parallax the observations show the speed of the equinox motion almost two times more (about $2''.5$ per century). The corrected values no longer follow the Fricke's line, now they are traced very well by our curve (boxes and the bold line in Fig.2). From this it follows that with respect to the direction of the x-axis in the ICRF (which is determined by position of the FK5 equinox at 2000.0) the motion of the equinox of new catalogues (if they will be observed) is not eliminated and the Vernal Equinox will be in motion the speed and direction of which is governed by the irregularity of the Earth rotation.

Thus we see that our explanation based on neglect of transition from UT to ET in treating the observed (not computed) declinations of the Sun makes the phenomenon of the equinox motion quite understandable on the time interval near 170 years where the results of observations are realistic.

Inspite all, our approach faces at least two difficulties. The first one is a discontinuity of the curve at 1900, and this was discussed by us earlier. The second trouble comes from the kinematical analysis of the proper motions. For the observations of the XX century the speed of the equinox motion ($0^s.082$ per century) derived from the proper motions of 512 FK4/FK4 Sup. distant stars is in excellent agreement with the secular term ($0^s.085$ per century) derived from the observation of the Sun and planets (Fricke, 1982). The retrograde motion of the equinox in the XIX century implies that the rate of the equinox motion in the proper motions of the last century must be negative. Nevertheless, kinematics of the proper motions in old catalogues gives at least zero value for the empirical term $\Delta E = \Delta\lambda + \Delta e$ (Fricke, 1977, Bakulin, 1966) and never negative. This problem requires further study, and until it is not done our approach should be considered as a guiding hypothesis.

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CONTRIBUTIONS OF THE GROUND-BASED ASTROMETRY TO THE REFERENCE SYSTEMS

M. STAVINSCHI
Astronomical Institute of the Romanian Academy
Str. Cuţitul de Argint 5, RO-75212 Bucharest, Romania
e-mail: magda@roastro.astro.ro

ABSTRACT. At the last General Assembly of the IAU, the IAU Commission 8 established a new Working Group, "The Future Development of Ground-Based Astrometry". Its main objective is to identify scientifically important programs for it. To maintain and establish reference frames and systems, IAU recommends that the scientific community continue ground-based observations as well as the space ones for the maintenance of the optical HIPPARCOS frames and for links of the frames to the ICRF. This paper intends to make a first analysis of the possibilities of the small and medium size instruments in this field of interest for the ground-based astrometry.

1. INTRODUCTION

The stars are not fixed. They change their positions in time. They move. Knowing this, we could know more about their dimensions and shapes. This is the role of the astrometry: to know these and, implicitly, to determine the geometrical and dynamical properties of the bodies from our Universe. How to do this? To have a fixed reference system on the sky or, more exactly, to have its materialization, it means a fixed reference frame. Farther the celestial objects are, better the frame will be. Quasars and distant galaxies are ideal fiducial points for such a celestial reference frame.

How to realize a reference frame? Simply: by continuous and more and more precise astrometric observations, by positions of fiducial points in the sky which may be distant stars, galaxies, or quasars. Of course, it is impossible to have a good celestial reference frame without a reliable and accessible celestial reference system, which is the theoretical concept. The results of the modern astrometry are more stars observed and measured than expected and, consequently, the catalogs which acquired an accuracy better than predicted after the launch of the first space astrometric mission, **HIPPARCOS** (**Hi**gh **P**recision **Par**allax **Co**llecting **S**atellite). The expectations of the scientists were surpassed and a window to a new kind of astrometry was opened. But the success of HIPPARCOS is important for many other areas of astronomy: identification of radio sources, stellar physics, dynamics of our galaxy.

For us the most important remains its exceptional contribution to the astrometry. It arises the natural question: before, the reference systems were the result of the ground-based observations. The star positions were determined for years within the framework of a large international campaign, known as "*Carte du ciel*". But now, when HIPPARCOS fulfilled its mission and other space missions are ready to be launched, which would be the contribution of the ground-based astrometry to the reference systems?

2. HIPPARCOS – A NEW CATALOG

The International Astronomical Union decided at its 23rd General Assembly (1997) that HIPPARCOS catalog will be the primary realization of the International Celestial Reference System, **ICRS**. The epoch of ICRS is 1998, January 1, and it is consistent with **FK5** system at J 2000.0. It is materialized by equatorial coordinates of extragalactic radio sources observed by Very Long Base Interferometry - VLBI. It is based on a *kinematical* definition (considering that the visible Universe does not rotate, that is, the most distant sources do not exhibit global motion).

But now we have HIPPARCOS catalog, resulting from the first astrometric space mission:

The mission had not a long life (between 1989 and August 1993), but its life was exceptionally fruitful. HIPPARCOS catalog is the optical counterpart of the International Celestial Reference Frame - ICRF and provided the primary realization of the ICRS in optical wavelengths. The HIPPARCOS stellar reference frame was astrometrically aligned to ICRF within ±0.6 mas at the epoch J 1991.25 and to within ±0.25 mas/y in rotation.

The HIPPARCOS system coincides with the principal axes of the ICRS. The catalog contains 118 218 stars, this means a density of ≈ 3 stars per square degree. It contains for each star all astrometric parameters, their standard errors, and the correlation coefficients. It includes a variety of accurate and homogeneous photometric information, as the Johnson V magnitude, B−V and V−I color indices and accurate multi-epoch broadband photometric data. The astrometric precision in position, proper motion and parallax are of order 1 mas or mas/yr, or better for the brighter stars.

The mean magnitude of the HIPPARCOS catalog is Hp = 8.7 and half of the catalog lies in the interval $8 < Hp < 9.5$.

It is useless to remember that the result surpassed significantly the goal of the mission. We have only to mention that, having measured the parallaxes to within 1 mas instead of 2 mas, this makes the accessible universe eight times larger than expected from the nominal mission.

Among the stars of the HIPPARCOS catalog, one can find double and multiple systems. A new surprise: more of a third of them were considered before as being single stars.

From HIPPARCOS catalog was deduced the TYCHO catalog, which contains one million stars. This one was obtained from photon counts obtained by scanning with the HIPPARCOS star mapper, carried out with the HIPPARCOS observations in the adjacent main field of view of the telescope. The mean value of internal standard errors of TYCHO catalog is 25 mas for the position components.

Such a large set of data was exploited in several steps. The most recent result is TYCHO 2 catalog, which contains about... 3 million stars. 99.9% of the stars are brighter than V = 10.0 mag. Each one is accompanied by all astrometric parameters: position (right ascension α and declination δ), angular parallax and proper motion components, in angular units per time unit.

Have a look on the main catalogs used until now (including IRS - International Reference System) and GSC (General Stellar Catalog):

Number of entries

HIPPARCOS	FK5	IRS	Carte du Ciel	GSC	TYCHO
118,218	1,500-3,000	40,000	12-15 million	25 million	1,058,332

Precision of positions (RA/Dec)(mas)

HIPPARCOS	FK5	IRS	Carte du Ciel	GSC	TYCHO
0.77/0.64	50-80	20	10,000	1,500-2,000	7-25

Precision of proper motions (mas/year)

HIPPARCOS	FK5	IRS	Carte du Ciel	GSC	TYCHO
0.88/0.74	1-2	5	-	-	-

As good it is, the proper motions of the HIPPARCOS catalog have to be linked to an extragalactic reference system. Absolute proper motions were derived from measurements of photographic plates taken with the best telescopes.

3. WHAT WILL BRING THE NEXT ASTROMETRIC SPACE MISSIONS ?

As the stars are not fixed, we have to observe them continuously. HIPPARCOS ended its activity in 1993, so their results are already old and its quality degraded year by year. The ground-based astrometry could assure the continuity of the star observation and a certain determination of the proper motions but the quality is not the same. So, other astrometric space missions are ready to be launched to accomplish this goal.

FAME (**F**ull-Sky **A**strometric **M**apping **E**xplorer) will be launched in 2004 as a result of a collaborative effort of U.S. Naval Observatory and several other institutions. It is an astrometric satellite designed to determine with unprecedented accuracy the positions, distances, and motions of 40 million stars within our galactic neighborhood. It will measure stellar positions to less than 50 μas at 9th visual magnitude and 500 μas at 15th visual magnitude.

The **S**pace **I**nterferometry **M**ission **SIM** will be launched in 2004. It will determine the positions of pointlike sources to an accuracy of 4 μas globally. We expect a deeper understanding of stellar evolution, a new definition of the Milky Way. SIM will perform 4 mas measurements on objects as dim as 20th magnitude using optical interferometric techniques with a 10 m baseline. The problem is to find suitable astrometric grid objects to support microarcsecond astrometry.

Another mission is projected to be launched between 2007 and 2010. It is **LIGHT**, conceived for stellar and galactic astronomy. It would observe about one hundred million stars up to V = 18 mag with an accuracy better than 0.1 mas in parallaxes and better than 0.1 mas/yr in proper motions, and will establish the precise photometric characteristics of the observed stars. It will observe almost all of the giant and supergiant stars belonging to the disk and halo components of our Galaxy to within 10 kpc from the Sun. Perhaps LIGHT will be the precursor of a more sophisticated future astrometric interferometer satellite, like GAIA, projected to be launched no later than 2012.

The ESA candidate cornerstone mission $-$ **GAIA** will analyze the composition, formation and evolution of our Galaxy by mapping the stars with unprecedented precision. It is estimated to provide the positions, proper motions and parallaxes of at least 4$-$50 million objects, down to the magnitude V = 15 mag, with an accuracy better than 10 mas, along with multi-color multi-epoch photometry of each object.

Another project is **DIVA** (**D**eutsches **I**nterferometer für **V**ielkanalphotometrie und **A**strometrie). This is a small satellite designed to perform astrometric and photometric observations of at least one million stars. It will perform an all-sky survey to V = 10.5 mag at least. The limiting magnitude will be 15.0 mag. After two years, DIVA will provide parallaxes accurate to 0.3 mas, proper motions accurate to 0.5 mas/yr, broad-band photometry with a typical precision of 0.003 mag, it means all better than what HIPPARCOS provided.

Pulkovo Observatory has another astrometric project, **STRUVE**, expected to be launched before 2010 for at least three years. It will include about 20 million stars down to V = 19.5 mag. The proper motions of HIPPARCOS stars are to be determined with a mean accuracy of at least 0.6 mas.

Taking into account the results obtained by the space missions until now and especially the projected missions, we have to put the natural question:

4. THERE STILL IS A PLACE FOR THE GROUND-BASED ASTROMETRY ?

There are three kinds of opinion: the optimists don't care of the spectacular results of space astrometry. They like to continue to observe the stars every clear night in order to determine their positions with their classical instruments, to compile catalogs and, finally to obtain a reference frame. They don't care of the well-known limits of the ground-based astrometry.

There are the pessimists: they want to stop all the observations on the ground, in spite of a certain homogeneity, which these ones could assure, and of certain programs, which could be done only from the ground. They don't ask: which instruments are to be closed, which ones have to continue and what will happen if all of them will be closed suddenly and very soon?

But there are the realist ones, too: they understand that the competition with the space missions is very hard. They understand that no matter how sophisticated the instruments would be, they are however limited by the atmospheric turbulence, the atmospheric refraction, the instrumental errors. At the same time, the fact that they are moving with the Earth is both an advantage (we need to have a good terrestrial reference frame) and a shortcoming (the Earth rotation is far from being regular). But this is not a reason to stop all observations, even if some instruments became obsolete or good for teaching of astronomy, or just very beautiful museum objects. In this case one should not hesitate to stop their operations. But the last two decades, about 80% of the existing types of instruments have been replaced by new ones or been modernized to the extent that they became new instruments, and this has led to a major increase in accuracy. These ones could continue to compete the space astrometry, but consequently to this renewal. The instruments are now much more powerful and sophisticated, hence much more expensive to build and to run. This limits their number and increases the need for a careful programming of observations.

Not only the instrumentation has changed, but also the reduction techniques have considerably evolved.

It seems that the competition with space astrometry pushed the ground-based astrometry in various directions, gaining in accuracy and usefulness. Our role is to identify the true direction of the classical ground-based astrometry and its capacity to compete the space missions. If these objectives are attained, the astronomy will gain in precision and all its field of interest will be satisfied.

So, we have to answer this question: who is right? The pessimist, the optimist, or the realist? Who could answer this dilemma? Maybe the new WG on "The Future Development of Ground-Based Astrometry" (chair: M. Stavinschi, co-chair: J. Kovalevsky), which has been set up during the 24th General Assembly of the International Astronomical Union (August, 2000).

The main objective of this WG is to identify scientifically important programs that can be achieved using ground-based astrometric or related observations, and to study what kind of modifications, upgrades or additions to the existing instruments should be performed in order to provide useful astronomical information with required accuracy, keeping in mind what the future astrometric satellites will contribute.

A final objective that we see for this working group is to identify programs that could be made on instruments that are either insufficiently used or working on projects that have no longer significant value for the present day astrometry. A major reason for this is that these instruments can be used as they are or with not too expensive modifications to train students in astronomy how to use telescopes and, at the same time, to contribute in a significant way to astronomy.

One of the topics in which ground-based astrometry could still contribute is the maintaining of the reference systems.

Even the Resolutions of the IAU specify this. So, the IAU Resolution B1.1 has as subject the "Maintenance and Establishment of Reference Frames and Systems. It says:

"The International Astronomical Union recommends *that the scientific community continue with high priority ground- and space- based observations (a) for the maintenance of the optical Hipparcos frames and frames at other wavelengths and (b) for links of the frames to the ICRF.*"

Let us try to identify some contributions of the ground-based astrometry to the reference frames.

Yet, there are some objects, which could be still observed from the ground:

The Dynamical Reference Frame (defined by the planetary and lunar ephemeris) is no longer used for the determination of nonrotating frame. It is made to be coincident with ICRF. It has been used until now as fundamental system (FK5 system) and will continue to be maintained for comparison with the extragalactic reference system − a major theoretical objective. Only very precise observations are useful for this goal, which is also true for the preparation of space missions and their operational fulfillment.

Some very well observed asteroids contribute, along with the major planets, to the definition of the dynamical reference system. The major part of the uncertainties of the inner planets ephemerides (of order 1 km) is due directly to the uncertainties in the masses of the asteroids, which perturb the inner planets. Consequently, the ephemerides must be updated continuously with current, accurate observational data.

If we are talking about the ideal reference frames (celestial, intermediate, or terrestrial) we need to better know the orbital motion of the Earth around the barycenter, the precession and nutation of the axis of rotation, the variations in the rate of the Earth rotation and the wobble of the axis of figure around the axis of rotation.

The contribution of the VLBI for determining the precise equatorial coordinates of extragalactic radio-sources and, implicitly, of the ICRS, is obvious. There are the satellite techniques (SLR, GPS), too.

To link HIPPARCOS to ICRF it is necessary to observe the optical counterparts of compact radio-sources, using plates from Schmidt telescopes or the prime focus of a large telescope.

The Southern Hemisphere has to be observed more and more.

The catalogs have to be linked to the HIPPARCOS catalog.

The Terrestrial Reference Frame has to be densified. A densification of the Celestial Reference Frame is necessary, too. We need better links between it and the stellar and dynamical reference frames, between ICRF and reference frames at other wavelengths.

The "instantaneous" HIPPARCOS proper motions differ sometimes from "long-period" ground-based ones because many stars (the so-called astrometric binaries) move nonlinearly due to some hidden massive satellites. In these cases one observes the nonlinearly moving photocenters of unresolved stellar systems. Thus, the HIPPARCOS catalog is not a perfect realization of the

ICRS at optical wavelengths. It is clear that a direct combination of the HIPPARCOS catalog with ground-based astrometric results can improve its proper motions.

Useful will be the radio-astrometrical determination of radio-stars, independently recorded by HIPPARCOS, with respect to angularly nearby radio-sources, included in the ICRF list (US VLBI Network, NASA Deep Space Network, NRAO VLBA, European VLBI Network).

We need to observe the optical counterparts of compact radio-sources, using plates from Schmidt telescopes, prime focus of a large telescope, HST (fine Guide Sensors): separations between H stars and extragalactic objects.

We have to extend the ground-based programs of observations at optical wavelengths, at various zones of the sky and magnitude thresholds. Here is the role of automated transit circles: CAMC, CMASF, scanning telescopes: AMC, Fast, Meridian 2000, NAOJ, SDSS, photographic astrographs: FON, NPM, SMP, and CCD astrographs: UCA, UCAC - System.

We don't neglect to continue the reduction of Schmidt survey plates.

A good role of the ground-based astrometry is that of observation of radio-stars to maintain the HIPPARCOS link, e.g. CONFOR (**CON**nnection of **F**rames in **O**ptical and **R**adio Regions), program connecting radio and optical Reference System.

A method to set up a link between FK5- System and the radioastronomical coordinate system consists in photographic and meridian observations of extragalactic radio/optical sources and intermediate reference stars, of course very faint stars ($mag > 19$). Optical positions of compact extragalactic radiosources could be still determined by using 1.56 m, 1 m and 60/90 cm telescopes with CCDs (e.g. China and Axial Meridian Circle of Nikolaev).

5. CONCLUSION

Continuous ground-based astrometric observations giving accurate positions are a fundamental objective of astrometry, which has indirect effects on all other measurements of motions of celestial bodies: any rotation of the reference system is wrongly interpreted as a motion of the bodies under study.

We have to add that the astrometrists have to do something, not only practical, but theoretical, too. We have to extend and clarify the application of relativistic concepts to astrometry and geodesy. We need to ensure that the theoretical models and the procedures for the reduction and analysis of the observations are consistent with an adopted relativistic theory. Even the theoretical bases of the links between the reference frames need some improvements.

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THE INFLUENCE OF ADOPTED ITRF ON ACCURACY OF DAILY EOP ESTIMATES

O. TITOV
Saint-Petersburg University, Astronomical Institute of SPbU
Bibliotechnaya sq., 2, Petrodvorets, Saint-Petersburg, Russia
e-mail: olegtitov@mail.ru

ABSTRACT. VLBI observations made by parallel NEOS-A and CORE-A networks provide independent EOP time series for the same epoch. Disagreement between the EOP time series has been detected. Significant biases are calculated from 71 daily sessions between January, 1997 and April, 2000. It is shown the biases can be canceled after empirical corrections for permanent tide.

1. INTRODUCTION

It was shown by many authors, for example, D.MacMillan, C.Ma, [2000] and M.Sokolskaya, E.Skurihina, [2000] that the daily EOP estimates from parallel VLBI sessions (NEOS-A and CORE-A) are not in a good agreement. The possible sources of the observed biases between NEOS and CORE results are listed D.MacMillan, C.Ma, [2000] :

1. Non-linear station motion;
2. Troposphere delay and troposphere gradient delay errors;
3. Errors in the tidal ocean loading model;
4. Errors in the local pressure loading model;
5. Errors in the underlying ITRF.

The last reason - errors in the underlying ITRF due to unadequate correction for permanent tide is discussed this paper.

2. DATA ANALYSIS AND DISCUSSION

71 parallel sessions (networks NEOS-A and CORE-A) operated from 28, January, 1997 till 18, April, 2000 have been processed using OCCAM software. The adjustment procedure corresponds to operational IVS approach, therefore ITRF and ICRF are fixed and only five EOP are estimated. Kalman filter technique considers clock offset as well as wet troposphere delay as a stochastic parameters like random walk. Clock rate is estimated as a constant parameter for 24-hour session. Clock acceleration and troposphere gradient are not estimated. Adopted correction for permanent tide does not correspond to IERS Conventions 1996 [D. McCarthy, 1996]. So, restitution for subdiurnal Love numbers recommended by IERS Conventions has not been applied. All resulted daily EOP values have been included into consideration inspite of the fact that a set of VLBI stations slightly changes from session to session. It is appeared that the EOP time series from parallel sessions (networks NEOS-A and CORE-A) demonstrate a bias for the same epoch. The Table 1 showes the statistics of the differences for pole components

and UT1-UTC.

Table 1. NEOS-A - CORE-A Bias Differences (71 sessions) (h=0, l=0)

Component	Bias(muarcsec)	Sigma
PM-X	-291	42
PM-Y	- 8	27
UT1	237	24

The bias for Y pole component is a small but for X pole component and UT1 the bias values exceed 3-sigma level essentially. It means that underlying ITRF is not self-consistent, therefore the single EOP systems from independent VLBI networks can not be combined into unified system.

One of the component of the ITRF construction is a convention about the permanent tide modelling. The appoach for solid tide model calculations supposes that restitution for permanent tide correction with Love numbers for semidiurnal component to be done. Unfortunately, the restitution for semidiurnal Love numbers with opposite sign (h=-0.6078, l=-0.0847) recommended by IERS Conventions 1996 does not provide us a better agreement between the EOP systems. For the reason all Analysis Centers do not apply for the correction routinely. Nonetheless, implementation of Love numbers for permanent tide correction with other values will shift the resulting EOP systems and will change the estimates in Table 1. Therefore, we have tried to determine so estimates of the Love numbers that could reduce the offsets in Table 1. New values (h=0.78, l=-0.02) have been found empirically. It is appeared that adding of the empirical values instead of ones recommended by IERS Conventions (h=-0.6078, l=-0.0847) improves agreement between the EOP systems. The corresponding statistics is given in Table 2.

Table 2. NEOS-A - CORE-A Bias Differences (71 sessions) (h=0.78, l=-0.02)

Component	Bias(muarcsec)	Sigma
PM-X	- 37	47
PM-Y	- 43	26
UT1	20	24

It is obvious that the biases became better. Offsets for X component and UT are less than 1-sigma level. Offset for Y component has increased but does not exceed 2-sigma level. Nonetheless, internal accuracy of daily EOP values became worse. It means that more detailed analysis have to be done in order to solve the problem.

3. ACKNOWLEDGMENTS

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SENSITIVITY OF THE ARCLENGTH METHOD OF REDUCTION OF VLBI ASTROMETRIC DATA TO SOME ASTROMETRIC MODELS

DE BIASI, M.S.
Facultad de Ciencias Astronómicas y Geofísicas (UNLP)
Paseo del Bosque s/n, 1900 La Plata, Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina [1]
e-mail: debiasi@fcaglp.fcaglp.unlp.edu.ar

ARIAS, E.F.
Bureau International de Poids et Mesures, Sèvres, France
Astronome Associé au DANOF, Observatoire de Paris, France
e-mail: farias@bipm.org

ABSTRACT

We analyse the effect of the Earth rotation when the arclength method of reduction of VLBI data is applied to the case of quasi- simultaneous observations of the two sources composing a pair. We investigate the method and we develop the equations in two reference frames defined differently: one is attached to a single baseline, and the other to two.

1. ANALYSIS

The arclength method proposes a new strategy of analysis of VLBI observations with the aim of reducing the correlations between the astrometric and geodetic parameters, which are entirely present in the classical reduction software. It introduces as observable the arclength between two radiosources observed simultaneously. The new observable is derived from the delay and the delay rate (Dravskikh et al, 1975a, 1975b).

The fact that most VLBI stations are not equipped for the ideal strategy of simultaneos observations makes us to study the case of quasi-simultaneous observations, where a time interval is elapsed between the observations of the sources of a pair.

We start by adopting an orthogonal frame attached to the directions of the celestial baseline vector and to the angular velocity of rotation of the Earth and fixed to the instant of observation of one of the sources of a pair. It is obvious that such a system will evolve with the Earth in the time interval elapsed between the observation of a pair. The director cosines of the geocentric directions of each source in the frame adopted are written as functions of the geometric component of the delay and delay rate. We assume a priori values of the modules of the baseline and of the angular velocity and of the arc as known and we derive the equation of observation from the general expression of an arc developed up to the first order in the small dimensionless corrections to the a priori values adopted. The equation of observation is independent of the

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directions of the baseline and of the angular velocity of rotation of the Earth, and it should be linear in the unknowns. Its coefficients depend on the adopted priori values, on the observables and on the terms of non simultaneity representing the Earth rotation.

The main point of the quasi simultaneous observations is to obtain the equation of condition linear in the unknowns. In this analysis we use real data from seventeen IRIS VLBI sessions (september to december 1998), and select two baselines: Richmond-Mojave, with a large East-West component ($\delta_b = 14^circ$) and Westford-Richmond with 50^circ declination. 13 radiosources at declinations between 10° and 80° were selected to construct 22 pairs, the arclengths ranging from 8° to 179°. A maximum delay of 1 hour was fixed between the observations of the two sources (De Biasi & Arias, 1998). The a priori values for the station coordinates are those in ITRF97, for the source coordinates those in ICRF-Ext.1. The non simultaneity terms were calculated using the classical transformation between the terrestrial and the celestial frames; EOP CO4 series were used with the addition of diurnal and subdiurnal terms in x, y and UT1-UTC. The clocks and the were not estimated, the respective parameters came from a GLORIA software solution performed for this analysis (Gontier, 2000) and later removed from the observed delay and delay rates.

We analysed the linealisation for both baselines. 40% of the observations done with the large East-West component baseline does not accept the linear form of the equation when the second source of the pair is close to the equator; this percentage decreases as we go to higher declinations. 26% of 109 observations does not accept the linear form in the equation of observation. In the case of Westford-Richmond, most cases accept the linear approach. Only 3% of 106 observations failed when the second source of the pair is at low declination (De Biasi & Arias, 1998, De Biasi,1999).

We then investigated if by adopting a reference system defined by the directions of two baselines it was possible to reduce the restrictions. To limit the degrees of freedom of the system the two baselines are almost perpendicular. The system is fixed to the instant of the observation of the first source of the pair. The a priori values adopted are the angle separating both baselines, their respective modules and the length of the arc between pairs of sources, whose dimensionless corrections are unknowns of the problem.

From the same VLBI data set we formed 62 pairs of radiosources with arclengths between 8° to 179°. Again, the interval between observations is set up to 1 hour. A total number of 2845 observations was used; only 3.4% does not accept the linear form of the equation of observation for pairs where the second source is near the equator (De Biasi, 1999).

2. CONCLUSION

When the arclength method is developed in a reference system attached to one baseline, the rotation of the Earth imposes a strong constraint to the reduction of quasi-simultaneous observations, mainly for pairs observed at baselines with East-West extended where the second component observed is close to the equator. This limitation is smaller when the reference system is attached to two baselines, where we loose the information contained in the delay rates since they do not appear in the equation of observation.

The next step is to develop the arclength method for a network of several baselines, defining a reference system attached to a "fiducial" baseline -the nean vector of the components of the network- and the angular velocity of the Earth, and use all the information contained in both delay and delay rate.

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USING THE ARC LENGTH APPROACH TO CHECK POSITIONAL STABILITY OF RADIO SOURCES

O. MOLOTAJ[1], L. PETROV[2], D. VOROBJEV[1]

1- Astronomical Observatory of University of Kyiv, Observatorna str. 3, Kyiv, 04053 Ukraine
2- Code 926, NVI, Inc./NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA
e-mail: molotaj@observ.univ.kiev.ua pet@leo.gsfc.nasa.gov

ABSTRACT. The arc length method for investigation of the stability of radio source positions was tested using 50 NEOS-A VLBI sessions from 1995. Twenty stable sources were selected.

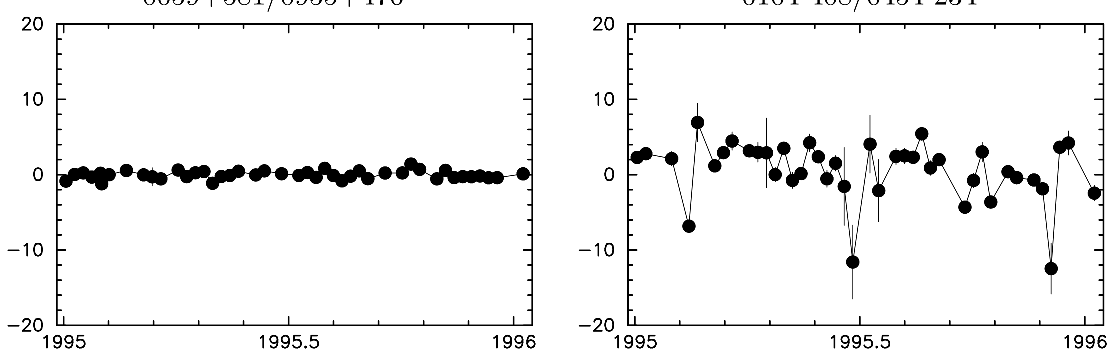
In the early 80s when regular VLBI observations of extragalactic sources began, we believed that quasars would not exhibit apparent proper motion since they are located at gigaparsec distances. Later, it was found that all sources have structure at the level of 1-10 nanoradians which in many cases is variable on time scales of years. It was found that some quasars do show non-linear apparent motion at a level exceeding 10 formal errors of the estimates of their coordinates. To our shame, up to now source structure is not taken into account in routine VLBI data analysis. In order to alleviate this problem, we separate sources onto two groups: sources with stable apparent position and sources with unstable position.

We used the arc-length approach in order to classify the source positions stability. First, positions of all sources were obtained from each 24 hour VLBI session independently keeping nutation angle and UT1 fixed to the a priori values. Then we computed arc-lengths using the estimates of source positions and their covariances. Of course, errors in a priori nutation angles and UT1 will add arbitrary rotations to the daily catalogues of source positions but they will not affect arc-lengths which are invariant to rotation.

In order to test this approach, a short series of VLBI observations consisting of 50 diurnal sessions of the 1995 NEOS-A observing campaign, with about 50000 observations of 145 radio sources, was analyzed. In each session 26 to 53 different sources were observed, and 12 sources were observed in every experiment. Arc-lengths and their formal uncertainties were obtained for each pair of sources observed in the same session using the estimates of their positions and the time series of arc-lengths were generated. Two examples are shown in fig. 1.

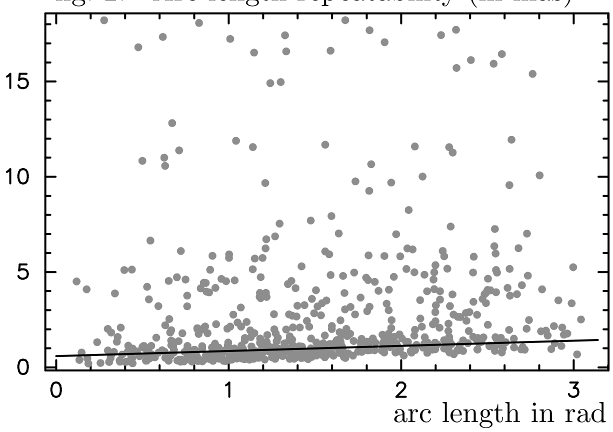
fig. 1: Arc length evolution (in mas)

0059+581/0955+476   0104-408/0454-234



We computed the average lengths of all arcs over all observations, subtracted these averaged lengths and formed small residuals. The averaged rms of residuals over the entire dataset was about 0.02 mas. Then we computed the averaged rms of residual arc-lengths over all arcs of each session. They ranged between 0.01 and 0.06 mas. It was noticed that these statistics increased for the two sessions in February and for the experiments in September–November.

fig. 2: Arc length repeatability (in mas)



We investigated arc-length repeatability for each arc over the annual data set (fig. 2) which we defined as an unweighted rms of residual arc-lengths. We found that the repeatability depends on the number of observations of the sources which form the arc, the smaller the number of observations, the larger the repeatability. It appeared that the distribution of the arc length repeatability is far from normal: 90% of the points are grouped around the regression line $4.9 \cdot 10^{-9} + 1.3 \cdot 10^{-9}\, L$ where L is arc-length in radians while there is an excess of arcs with larger repeatability.

In order to select the most stable sources from the full set of 145 sources with about 3000 arcs we used the following procedure. First, all arcs with less than 30 observations were discarded. Then we selected 200 arcs with the lowest repeatability values: the arc which repeatability deviates by no more than 0.5 mas from regression line mentioned above. Then we investigated how frequently each source appeared in the arc which belong to the set of 200 stable arcs. The sources which formed the stable arcs more frequently were put into the group of stable sources.

We found 20 stable sources which satisfy this criteria and list them in Table 1. These source are mainly included in 200 stable arcs. It is notable that these stable sources are distributed over all three ICRF radio source categories: defining, candidate, other.

Table 1: **List of stable radio sources.** ICRF status: D[efining], C[andidate], O[ther]. Sources observed in all 50 sessions are marked by asterisk

IAU name	ICRF status	*	IAU name	ICRF status	*	IAU name	ICRF status	*	IAU name	ICRF status	*
0014+813	D		0727-115	O	*	1228+126	C		1611+343	C	
0059+581	O		0804+499	D		1308+326	D	*	1638+398	O	
0202+149	C		0923+392	O		1334-127	O	*	1745+624	D	
0552+398	C	*	1128+385	D		1357+769	C		1749+096	C	*
0642+449	D		1219+044	D	*	1606+106	D	*	2145+067	D	*

AN IR ASTROMETRY PROBLEMS AFTER THE 2MASS AND DENIS

L.K. PAKULIAK[1], N. EPCHTEIN[2], A.S. KHARIN[1], N.A. LISITSA [1]

[1] Main Astronomical Observatory National Academy of Sciences of Ukraine
Golosiiv, 03680, Kiev-127, Ukraine
e-mail: pakuliak@mao.kiev.ua

[2] Observatoire de la Cote d'Azur
Department Fresnel, BP 4229, F06304 Nice cedex 04
e-mail: epchtein@obs-nice.fr

1. INTRODUCTION

The completion of two near-infrared surveys DENIS and 2MASS has been planned for the year 2000 (Epchtein 1997; Skrutskie 1997). As a result a vast catalogue with positions and four bands photometry for about 10^9 near-infrared objects would prepared.

The positional accuracy of this catalogue has to be much better then that of the IRAS PSC and some IR astrometry problems could be resolved on the base of it. On our opinion besides the IRAS PSC position melioration, next two problems can be solved with the help of the 2MASS and DENIS data

- extension of IR reference catalogue (Kharin & Molotaj, 1999)
- verification of preliminary IR/OPTIC and IR/RADIO identifications

The last one, namely the verification of the preliminary identified ISRF sources with their counterparts from two infrared catalogues (Kharin, this volume), is under investigation in Golosiiv observatory.

The technique of such verification with the implication of the DENIS data has been developed and is testing now on the example of the DENIS/ACT identification. Some results of this investigation are presented here.

2. THE DENIS DATABASE AT THE CDS

DENIS is a project to survey the all-southern sky in three wavelength bands (Gunn-i 0.82μm; J, 1.25μm; and K_s, 2.15μm) with limiting magnitudes 18.5, 16.5 and 14.0, respectively. The observations are performed with the 1m-ESO telescope at La Silla (Chile).

The survey is carried out by observing strips (spherical trapezium or triangles) of 30 grad in declination and 12 arcminutes in Right Ascension with an overlap of 2 arcminutes between consecutive strips. More than 2000 strips have to be obtained during all-southern survey.

The data are reduced and implemented to the final databases at The Centre de Donnees Astronomiques de Strasbourg (CDS). 102 strips are contained now in the DENIS database at the CDS.

Files, named denis.sam, of DENIS database contain up to 61 parameters in every records that consist up to 40 fields which has to be transported from CDS.

3. THREE STEPS OF VERIFICATIONS METHOD PROCESSING

The process includes three steps of DENIS data processing and comparison with an astrometric catalogue for the verification of identification. On the first stage the transportation of DENIS data from CDS by internet pipeline and transformation them into the form, convenient to the following treatment have been made.

On the second one the preliminary identification of sources has been carried out by comparison of positions in source catalogues with the criterion of differences established by estimation of the files position accuracy.

The third step of data processing implies the verification of above preliminary identification, which is carrying out on the base of two files photometric data comparison.

4. SOME RESULTS OF THE EXAMPLE OF THE DENIS/ACT IDENTIFICATION AND VERIFICATION

The above said operations were used to process two strips of DENIS data, No 3971 and 3972 , which contains 179120 and 524587 records accordingly.

The ACT contains 97 and 172 objects in the borders of these two trapezia accordingly and after preliminary identification with ρ=3" limiting circle we initially have got 50 and 78 common ACT and DENIS objects for these strips. After verification by using criterion 2σ 46 and 69 objects remain in the lists accordingly (Fig 1).

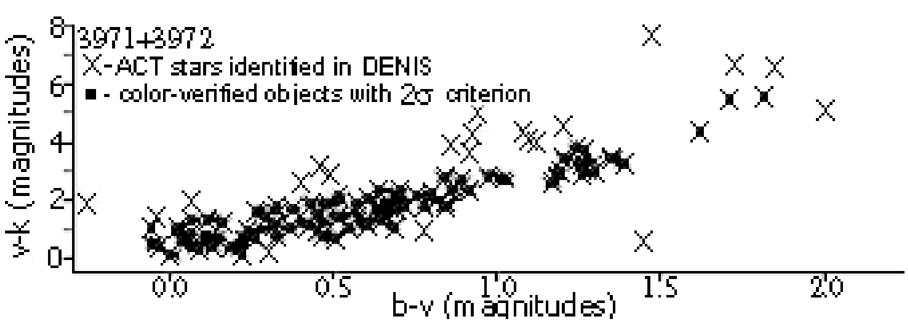


Figure 1: Common ACT-DENIS stars from two strips 3971+3972.

5. CONCLUSION

The CDS DENIS database contains photometric data for the three bands and colors obtained with these data. It permits to develop the method of verification of DENIS objects preliminary identification with their counterparts from some other catalogues which have photometric data in the other bands. The effectivity of the method was examined by verification of preliminary identification of the ACT and DENIS objects contained in two spherical trapeziae of 3971 & 3972 strips.

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ACTIVITIES OF THE IVS ANALYSIS CENTER AT BKG, GERMANY

V. THORANDT, G. ENGELHARDT, D.ULLRICH
Bundesamt fuer Kartographie und Geodaesie (BKG)
Karl-Rothe-Strasse 10-14, 04105 Leipzig, Germany
e-mail: vt@leipzig.ifag.de and engelhardt@leipzig.ifag.de

ABSTRACT. The VLBI group at BKG was established in 1994. In close cooperation with the VLBI group at the University of Bonn, Germany, global solutions as a contribution to the IERS annual solutions were produced. The BKG catalogue system contains about 6000 X-band databases and so-called superfiles for analysis purposes. It covers the time span from 1976 to 2000. Since March 1, 1999, the inauguration date for the IVS, the VLBI group at BKG has been operating as an IVS Analysis Center together with the VLBI group at the University of Bonn. At present so-called session series Earth orientation parameters and IVS-IERS annual solutions for EOP, TRF and CRF are produced with the Mark III/IV VLBI analysis software CALC/SOLVE.

1. THE PROCESSING OF SINGLE VLBI EXPERIMENTS

The Mark III/IV VLBI analysis software CALC/SOLVE (GSFC, NASA, 2000) is used for processing of VLBI data at BKG. The first processing step of single experiments (IRIS-S, EUROPE, COHIG) is the transfer of the version 1 X-band and S-band databases and the related log files from the Bonn correlator at MPIfR (Max Planck Institut fuer Radioastronomie) to the BKG Leipzig. In the second step the program APRIORI provides the necessary ephemeris, UT1 and polar motion information. After that CALC calculates theoretical values, partial derivatives, contributions and corrections according to the geophysical models. As of now CALC 8.2 Y2K is in use, but it is planned to upgrade to CALC 9.11. The fourth analysis step is to run F-SOLVE to fit the databases with the partial derivatives, theoretical values, contributions and corrections calculated by CALC and to add an ionosphere correction by using the S-band database. It also includes the group delay ambiguity resolution for X- and S-band databases. The result is the version 4 of X-band database. Currently F-SOLVE version 2000.03.31 is in use. In step 5 the program XLOG prepares the log files for the calibration of the databases with meteorological and cable data. The output of this step are the calibration files for DBCAL, the next processing step. It enters the weather and cable data into the database by interpolating the measured data for the observation epoch. Output is the version 5 of X-band database. F-SOLVE provides the final parameterization according to the purpose of the experiment and the least squares solution. It includes the inspection of the residuals and procedures for outliers elimination. The output files are the listing of results and after update the X-band database version 6. The last step is to upload the databases and related files into the incoming area of the BKG IVS Data Center and into the local data area. After mirroring the files are available at all primary data centers.

2. THE SESSION SERIES EOP PRODUCT

After the preprocessing of each new VLBI experiment a new solution on the basis of delays with the latest version of the databases is produced to derive session series EOP for the IVS. The features of the solution, called bkgi0001.eops, are described in the technical description and are available in the IVS data centers in the directory ivsdocuments (e.g. ftp://ftp.leipzig.ifag.de/vlbi/-ivsdocuments/bkgi0001.eops.txt). The solution is put into the so-called IVS incoming area. After checking the syntax the solution will be moved to the regular data center area (e.g. ftp://ftp.leipzig.ifag.de/vlbi/ivsproducts/eops/bkgi0001.eops). The solution includes the estimated EOP (xpol, ypol, ut1-utc) and the nutation offsets (dPsi, dEps) including the corresponding uncertainties and correlation. Together with the solution a solution statistics summary file (bkgi0001.stats.eops) is given. First comparisons with results of the IVS analysis center GSFC based on differences to the IERS C04 series (IERS, 1999) give an impression of the EOP variations between both solutions. Apart from some outliers both solutions fit quite well. But two other facts are remarkable. Firstly, after test solutions with different a-priori TRF, NAVY 1998-10 (USNO, 1997) and ITRF97 (Boucher C. et al., 1999), an offset of about 20 microseconds in UT1-UTC can be seen. Secondly, the differences in the nutation component dPsi are approximately three times bigger than the ones in the nutation component dEps. That corresponds quite well with the difference in the estimated formal program errors of dPsi and dEps.

3. THE IVS-IERS ANNUAL SOLUTION

In addition to the session solution a combined global solution is computed to estimate the EOP, station coordinates and velocities as well as the radio source positions. This annual solution is a contribution to the IERS. The solution type for the TRF is called bkgtra99 and corresponds to a quasi free network solution. The technical description of this solution is available in e.g. ftp://ftp.leipzig.ifag.de/vlbi/ivs-iers/bkgtra99.des. The file format is SINEX (Solution Independent Exchange Format) and can be found in the IVS data structure under e.g. ftp://ftp.leipzig.ifag.de/vlbi/ivs-iers/bkgtra99.snx.gz. The solution type for the EOP and CRF is called bkgira99 and the corresponding technical description can be found in e.g. ftp://ftp.leipzig.ifag.de/vlbi/ivs-iers/bkgira99.des. The format for the two files is IERS standard EOP format and IERS standard source position format, respectively. These two files are also available in the IVS data structure under e.g. ftp://ftp.leipzig.ifag.de/vlbi/ivs-iers/bkgira99.eop and ftp://ftp.leipzig.ifag.de/vlbi/ivs-iers/bkgira99.rsc.

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PRACTICAL ASPECTS OF THE NEW CONVENTIONS FOR ASTROMETRY AND CELESTIAL MECHANICS IN THE RELATIVISTIC FRAMEWORK

M. SOFFEL
Lohrmann Observatory,
Dresden Technical University,
01062 Dresden, Germany

ABSTRACT. The new conventions for astrometry and celestial mechanics in the relativistic framework concern the definition of a Barycentric Celestial Reference System (BCRS), a Geocentric Celestial Reference System (GCRS) and a definition of relativistic potential coefficients. Various practical implications of these resolutions are discussed.

1. COORDINATES, METRIC TENSOR AND THE IAU RESOLUTIONS

Here we discuss some practical aspects of the two Resolutions concerning relativity in astrometry and celestial mechanics that were adopted at the IAU General Assembly in Manchester in the year 2000. The two relevant ones can be found under Resolution 3 and 4 in the proceedings of IAU Colloquium 180 (Johnston et al., 2000). Resolution three concerns the definition of Barycentric Celestial Reference System (BCRS) and Geocentric Celestial Reference System (GCRS). The BCRS is defined with coordinates $(ct, x^i) = x^\mu$ where $t =$ TCB and the spatial coordinates might be called XCB. In relativity coordinates only have some meaning if the corresponding metric tensor $g_{\mu\nu}$ is specified. The GCRS is defined with coordinates $(cT, X^a) = X^\alpha$ where $T =$ TCG and the geocentric spatial coordinates migth be called XCG. The geocentric metric is denoted by $G_{\alpha\beta}$. Note that the transformation from the BCRS to the GCRS

$$x^\mu \longleftrightarrow X^\alpha$$

is a four dimensional space-time coordinate transformation. Resolution 4 provides post-Newtonian definitions for M_E, J_2^E etc., i.e., for post-Newtonian potential coefficients of the Earth in the GCRS. They can be used to write $G_{\alpha\beta}$ in terms of M_E, J_2^E etc. in the region outside the Earth.

In relativity the fundamental objects are coordinates and the metric tensor. Usually one writes $x^\mu = (ct, x^i) = (ct, x, y, z)$ so $\mu = 0, 1, 2, 3$ and the metric tensor $g_{\mu\nu}(x^\mu)$ is a 4×4 matrix with

$$ds^2 = \sum_{\mu,\nu} g_{\mu\nu} dx^\mu dx^\nu .$$

Here ds^2 is the infinitesimal distance between two neighbouring point in space-time, x^μ and $x^\mu + dx^\mu$. The metric tensor determines

- clock rates

- the propagation of light rays (ASTROMETRY)
- the motion of solar system bodies (CELESTIAL MECHANICS).

2. ASTROMETRY

In this section we would like to discuss some implications of our Resolutions with respect to the description of astrometric measurements.

2.1 The celestial sphere

Usually one considers the solar system to be isolated, i.e., one neglects other stars, galaxies etc. In other words one usually forgets about the cosmological problem. In that case the BCRS is a *global* coordinate system that contains all the 'far away regions'. Consider some light-ray in the BCRS and follow it back in time, i.e., for $t \to -\infty$. That region where all light-rays originate is called *past null infinity*; it might also be called the celestial sphere. Since the BCRS is a gobal coordinate system it contains the celestial sphere.

The situation, however, is very different for the GCRS. The origin of the GCRS is *accelerated* with

$$g \sim \frac{GM_S}{(\mathrm{AU})^2}$$

and one can show that any reasonably defined accelerated coordinates become meanigless for $d \gg c^2/g$ (e.g., Misner et al., 1973). Accelerated coordinates are only *local* ones and the remote celestial sphere cannot be described in the GCRS.

2.2 Observables - coordinate quantities

In relativity only one basic astrometric observable exists: the angle of two incident light-rays as seen by some observer. This observable can be derived in any coordinate system as a *coordinate independent object*. How this works can be found in the literature (e.g., Soffel 1989). Usually, however, in the field of astrometry one talks about a large number of quantities, such as

- parallax, proper motion and radial velocity

that are pure coordinate objects. For that reason it is essential to fix the coordinates (and the corresponding metric tensors) by convention.

Other coordinate quantities are extremely useful for the description of global geodynamics in the GCRS. The GCRS coordinates X^a clearly can be used for the introduction of diverse quantities, like polar coordinates r, θ, ϕ and a corresponding directional sphere, coordinate vectors $\mathbf{X} = (X, Y, Z)$, the equator of $\mathbf{X}$ etc.

2.3 Orders of magnitude

Let us now come to orders of magnitude of various effects in astrometry. In Special Relativity (no gravitational fields) e.g., the aberration effect on the apex angle ($\beta = v/c$) is given by

$$\Delta\theta = \beta \sin\theta - \frac{1}{4}\beta^2 \sin 2\theta + [\beta^3 - \mathrm{terms}] \,.$$

One important implication is the following: to determine first-order aberration with μas accuracy *one needs* v to 1.4 mm/s!

As far as post-Newtonian effects are concerned some *maximal* deflection angles amout to: $1.75''$ due to the mass of the Sun M_S, 240 μas caused by the oblateness of Jupiter $J_2^{\rm Jup}$, 10 μas due to $J_4^{\rm Jup}$. The square of the solar mass leads to a 2PN effect that *maximally* is of order 11 μas. In other words: for future astrometric missions at the μas level all of these effects have to be taken into account. Moreover, the time has come where one has to think seriously about cosmological effects in astrometry. Further references to this subject can be found e.g., in Johnston et al., 2000.

3. CELESTIAL MECHANICS

Let us finally come to the problem of celestial mechanics. Also here one single basic observable exists: the travel time of some electromagnetic signal between various points in space as measured by some observer. This fundamental observable can be derived directly from the metric tensor and the equations for photons and the observer. Obviously in celestial mechanics one talks about diverse quantities such as orbits of satellites, planets, spacecrafts etc., that are mere coordinate quantities. The same applies for concepts like the ecliptic or some celestial equator. An ecliptic might be defined as $t =$ const. coordinate plane by means of $\mathbf{x}_E(t_0)$ and $\mathbf{v}_E(t_0)$ in the BCRS. I.e., one is forced to consider the ecliptic only as a BCRS coordinate object related with the Earth's ephemeris. A transformation of this object into the GRCS yields nothing reasonable. On the other hand some celestial equator might be defined in the GCRS only.

That is the reason why the *equinox* should be considered as an obsolete quantity and should not be used in the future.

As far as equations of motion of solar system bodies are concerned relativistic potential coefficients are suggested in Resolution 4. Note that it is at least a definition of a physical post-Newtonian mass of some solar system body that one needs for relativistic celestial mechanics. It is important to note that without such a definition one might easily introduce spurious, i.e., non-physical acceleration terms in the equations of motion. Post-Newtonian equations of translational motion can be found in the literature (DSX I-IV). In satellite motion it is mainly the Schwarzschild acceleration caused by the mass of the Earth and the Lense-Thirring acceleration due to the Earth's total angular momentum that dominate the relativistic effects in satellite orbits. Also expressions for post-Newtonian tidal forces can be found in the literature.

Keeping only the masses in the the BCRS equations of motion it can be shown that they reduce to the well known EIH-equations that are already used for the JPL DE-ephemerides. Work on the post-Newtonian rotational equations of motion is in progress.

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THE CELESTIAL POLE AND UT1 IN THE ICRS : THE NEW IAU RESOLUTIONS

N. CAPITAINE
DANOF/UMR8630-CNRS
Observatoire de Paris, 61 avenue de l'Observatoire, 75014, Paris, France
e-mail: capitain@danof.obspm.fr

ABSTRACT. Important resolutions have been adopted by the IAU at the 24th General Assembly (Manchester, August 2000) concerning the celestial pole of reference and the origin on the instantaneous equator for defining UT1, in consistency with the International Celestial Reference System (ICRS). Resolution B1.7 defines the "Celestial Intermediate Pole" (CIP) in the Geocentric Celestial Reference System (GCRS) and in the International Terrestrial Reference System (ITRS) in order to replace the "Celestial Ephemeris Pole" (CEP) for the new IAU 2000 precession-nutation model which is recommended in IAU Resolution B1.6; this definition specifies the way for taking into account the high frequency terms in polar motion and nutation. Resolution B1.8 concerns the use of the "non-rotating origin" (Guinot 1979) in the GCRS and ITRS for defining UT1 and recommends their use in the transformation between the GCRS and the ITRS together with that of the celestial and terrestrial coordinates of the CIP. This paper reports on these resolutions and presents their basis as well as their practical application.

1. INTRODUCTION

At the 23rd GA of the IAU (Kyoto, August 1997), a new International Celestial Reference System, ICRS, realized, at a sub-milliarsecond accuracy, by the direction of extragalactic radiosources (Ma *et al.* 1998) has been adopted starting 1 January 1998, in place of the FK5, which is based on positions and proper motions of stars and referred to the equinox. Such a change in the celestial frame has been associated with significant improvements in the theory and observations of Earth rotation. It was therefore necessary to consider the computational consequences of this new situation (Capitaine 1998), especially on precession, nutation and Earth Rotation. This has been done during the past three years and has resulted in IAU Resolutions adopted at the 24th IAU General Assembly (Manchester, August 2000). First, a new precession-nutation model, proposed by the IAU/IUGG Working Group on the "Non-rigid Earth nutation theory", has been recommended in Resolution B1.6. Second, proposals for new Earth orientation parameters (EOP) and pole, consistent with the ICRS, have been under discussion in the framework of the subgroup T5 [2] named "Computational consequences" of the IAU Working Group "ICRS" which was implemented by the 23rd GA. The work of T5 (Capitaine 2000 a and b), which has been followed and complemented by large discussions at the IAU Colloquium 180 (Washington, March 2000), has led to the proposal of two resolutions which have been adopted

[2] T5 membership and list of contributors are given at the end of the paper.

at first by the colloquium participants, then at the Joint Discussion 2 at the 24th IAU GA and finally voted by the IAU. Resolution B1.7, which concerns the definition of the celestial pole, provides an extended definition of the Celestial Ephemeris Pole (CEP) in the high frequency domain, named the "Celestial Intermediate Pole" (CIP); Resolution B1.8, which concerns the adoption of the "non-rotating origin" (Guinot 1979) for defining UT1, recommends its use in the coordinate transformation between the Geocentric Celestial Reference System (GCRS) and the International Terrestrial Reference System (ITRS) together with the use of the celestial and terrestrial coordinates of the CIP.

The following sections give the official text of these two IAU resolutions and then explain their basis as well as their practical application.

2. THE IAU 2000 RESOLUTIONS ON THE CELESTIAL POLE AND THE CELESTIAL AND TERRESTRIAL EPHEMERIS ORIGINS

The IAU resolutions recommending a new definition of the celestial pole of reference and new origins on the moving equator for defining UT1 are given below.

2.1 IAU Resolution B1.7 : Definition of Celestial Intermediate Pole

The XXIVth International Astronomical Union General Assembly,

Noting

the need for accurate definition of reference systems brought about by unprecedented observational precision, and

Recognising

1. the need to specify an axis with respect to which the Earth's angle of rotation is defined,
2. that the Celestial Ephemeris Pole (CEP) does not take account of diurnal and higher frequency variations in the Earth's orientation,

Recommends

1. that the Celestial Intermediate Pole (CIP) be the pole, the motion of which is specified in the Geocentric Celestial Reference System (GCRS, see Resolution B1.3) by motion of the Tisserand mean axis of the Earth with periods greater than two days,
2. that the direction of the CIP at J2000.0 be offset from the direction of the pole of the GCRS in a manner consistent with the IAU 2000A (see Resolution B1.6) precession-nutation model,
3. that the motion of the CIP in the GCRS be realised by the IAU 2000 A model for precession and forced nutation for periods greater than two days plus additional time-dependent corrections provided by the International Earth Rotation Service (IERS) through appropriate astro-geodetic observations,
4. that the motion of the CIP in the International Terrestrial Reference System (ITRS) be provided by the IERS through appropriate astro-geodetic observations and models including high-frequency variations,
5. that for highest precision, corrections to the models for the motion of the CIP in the ITRS may be estimated using procedures specified by the IERS, and
6. that implementation of the CIP be on 1 January 2003.

Notes

The forced nutations with periods less than two days are included in the model for the motion of the CIP in the ITRS.

The Tisserand mean axis of the Earth corresponds to the mean surface geographic axis, quoted B axis, in Seidelmann, 1982, Celest. Mech. 27*, 79-106.*

As a consequence of this resolution, the Celestial Ephemeris Pole is no longer necessary.

2.2 IAU resolution B1.8 : Definition and use of Celestial and Terrestrial Ephemeris Origins

The XXIVth International Astronomical Union General Assembly,

Recognising

1. the need for reference system definitions suitable for modern realisations of the conventional reference systems and consistent with observational precision,
2. the need for a rigorous definition of sidereal rotation of the Earth,
3. the desirability of describing the rotation of the Earth independently from its orbital motion, and

Noting

that the use of the "non-rotating origin" (Guinot, 1979) on the moving equator fulfills the above conditions and allows for a definition of UT1 which is insensitive to changes in models for precession and nutation at the microarcsecond level,

Recommends

1. the use of the "non-rotating origin" in the Geocentric Celestial Reference System ((GCRS) and that this point be designated as the Celestial Ephemeris Origin (CEO) on the equator of the Celestial Intermediate Pole (CIP),
2. the use of the "non-rotating origin" in the International Terrestrial Reference System (ITRS) and that this point be designated as the Terrestrial Ephemeris Origin (TEO) on the equator of the CIP,
3. that UT1 be linearly proportional to the Earth Rotation Angle defined as the angle measured along the equator of the CIP between the unit vectors directed toward the CEO and the TEO,
4. that the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS, and the Earth Rotation Angle,
5. that the International Earth Rotation Service (IERS) take steps to implement this by 1 January 2003, and
6. that the IERS will continue to provide users with data and algorithms for the conventional transformations.

Note

The position of the CEO can be computed from the IAU 2000A model for precession and nutation of the CIP and from the current values of the offset of the CIP from the pole of the ICRF at J2000.0 using the development provided by Capitaine et al. (2000).

The position of the TEO is only slightly dependent on polar motion and can be extrapolated as done by Capitaine et al. (2000) using the IERS data.

The linear relationship between the Earth's rotation angle θ and UT1 should ensure the continuity in phase and rate of UT1 with the value obtained by the conventional relationship between Greenwich Mean Sidereal Time (GMST) and UT1. This is accomplished by the following relationship:

$$\theta(UT1) = 2\pi\,(0.779\,057\,273\,2640 + 1.002\,737\,811\,911\,354\,48 \times (Julian\,UT1\,date - 2451\,545.0))\,.$$

3. THE CELESTIAL INTERMEDIATE POLE

3.1 The need for an extended definition of the CEP

The IAU-1980 nutation theory is referred to the Celestial Ephemeris Pole (CEP) which was actually defined by its realization through the nutation series. Following the IERS1996 Conventions (McCarthy 1996), the CEP is realized by the precession-nutation model (including terms of periods greater that 4 days), plus the "celestial pole offsets" which are estimated by observations and refer to the GCRS; these corrections, which contain the deficiencies in the precession-nutation model, provide the whole retrograde diurnal motion with respect to the ITRS, or, equivalently, the whole long period motion of the CEP with respect to the GCRS. Similarly, the "pole coordinates" provide, when estimated from observations with a time resolution of one or a few days, the whole long period motion of the CEP with respect to the ITRS.

The CEP is thus currently defined through its long period motion both in the GCRS and the ITRS for a time resolution of the observations of one or a few days. It can be considered to be an intermediate pole separating, by convention, the motion of the pole of the ITRS into a "celestial part" including all the variations of periods greater than 2 days in the GCRS (i.e. with frequency $-1/2 < \sigma_{GCRS} < +1/2$, in cpd) and a "terrestrial part" including variations in polar motion of period greater than 2 days in the ITRS (i.e. with frequency $-1/2 < \sigma_{ITRS} < +1/2$, in cpd). Such a definition does not give any convention for the high frequency motions (periods shorter than two days) in the GCRS or the ITRS and the convention is not valid for sampling intervals shorter than 1 day (Brzeziński & Capitaine 1993).

Since the adoption of the CEP, significant improvement in precision and time resolution of the observations as well as in the development of theory have been achieved. Intensive VLBI observations provide EOP with a submilliarsecond accuracy over a few hours, whereas diurnal and semi-diurnal terms are considered in the theory at a μas accuracy both of nutation (Bretagnon *et al.* 1997) and pole motion (Herring & Dong 1994).

A new definition of the pole of reference is thus necessary in order to be in agreement with the current precision and resolution of the modern observations and with the accuracy of the recent models including high frequency variations both in the GCRS and ITRS.

3.2 Proposal for a modern definition of the pole of reference

A modern definition of the pole of reference must take into account the predictable prograde diurnal and prograde semi-diurnal nutations as well as the sub-diurnal tidal variations in polar motion. It is important to note that the prograde diurnal nutations can also be considered as long periodic prograde and retrograde variations in polar motion and the prograde semi-diurnal nutations as prograde diurnal variations in polar motion (Bizouard *et al.* 2000). Similarly, the retrograde diurnal tidal variations in polar motion are actually included in the recent nutation models. A clear convention is therefore necessary for the high frequency domain and especially for the overlapping between the motions in the GCRS and the ITRS in this frequency domain.

Several approaches have been considered within the work of T5 in order to give a definition of the pole of reference by extending the definition of the CEP in the high frequency domain through additional conventions. Mathews (1999) proposed a definition of the CEP which keeps the symmetry in the frequency band between the terrestrial and the celestial motions and a procedure for extracting the high frequency signal in processing the observations together with the current celestial pole offsets and the current polar motion. The advantage of such a choice is the symmetry in the frequency domain, but the disadvantage is that the estimated terms do not correspond to the predictable high frequency motions, which are actually prograde within the GCRS and both prograde and retrograde in the ITRS. The proposal has then been transformed to a similar procedure but with the whole motion of the pole expressed in the GCRS (Mathews & Herring 2000); such a convention corresponds to the adoption of the pole of the ITRS as the pole of reference without consideration of any intermediate pole.

However, the proposal which has been preferred within the work of T5 is rather to consider the pole of reference to be the pole of the intermediate equator (between those of the GCRS and the ITRS), of which motion, with respect to the GCRS, is mainly produced by the external torque on the Earth, with a limitation to the long periodic part. The definition is thus extended such that only the nutations with period greater than two days are considered in the GCRS, whereas the diurnal and semi-diurnal nutations are taken into account as a part of the polar motion, together with the long period polar motion and the high frequency tidal variations in polar motion. Such a definition represents an extension of that of the CEP by considering the whole high frequency motion as a motion in the ITRS, except the diurnal retrograde motion which is still totally considered in the GCRS. The name "Celestial Intermediate Pole" (CIP) has been proposed to emphasize the role of intermediary of this pole between GCRS and ITRS.

The definition of the CIP is thus unchanged from that of the CEP for the celestial motion and is sharpened by considering the whole high frequency motions both in the GCRS and the ITRS, except the retrograde diurnal terms, as being conventionally a part of the polar motion. The advantages are that : 1) the extension in the frequency domain is consistent with a deterministic approach, 2) the definition is not dependent on further improvements in the model, but can be given as accurately as necessary by a model, 3) the definition is not dependent on the techniques of observation, 4) the change from the current CEP will have minimal impact on users.

3.3 Definition and realization of the Celestial Intermediate Pole

Following the proposal above, the new pole which has been recommended in IAU Resolution B1.7, the "Celestial Intermediate Pole" (CIP) is such that its name emphasizes the fact that it is defined neither by a physical property, nor by a model, but that it appears as an intermediate pole in the coordinate transformation between the GCRS and the ITRS.

The definition of the CIP separates, by convention, the motion of the pole of the ITRS (i.e. the pole of the Earth's mean surface geographic axis) into two parts : 1) the celestial part including the whole long period terms in the GCRS, 2) the terrestrial part including the whole long periodic polar motion as well as all the daily and sub-daily variations in polar motion; such variations in the ITRS originate both from tidal variations in polar motion and from prograde diurnal and semi-diurnal nutations.

The motion of the CIP in the GCRS is provided by the IAU 2000 A model for precession and forced nutation for periods greater than two days plus the celestial pole offsets estimated by observations. As the GCRS is not linked to the pole of J2000, the CIP has to be offset from the direction of the pole of the GCRS in a manner consistent with the IAU 2000 precession-nutation model; this means that this offset is estimated from observations together with the parameters of the precession-nutation model. The motion of the CIP in the ITRS is provided by EOP observations and takes into account a predictable part specified by a model including the high frequency variations (except the retrograde diurnal terms in the ITRS). The corrections to this model can be estimated by extracting the high frequency signal using a procedure similar to the one described by Mathews & Herring (2000).

4. THE CELESTIAL AND TERRESTRIAL EPHEMERIS ORIGINS

4.1 Change from the FK5 to the ICRS and computational consequences

Until the adoption of the ICRS, the conventional celestial system, the FK5, which is based on positions and proper motions of bright stars, was oriented so that at the "epoch", the positions are referred to the best estimate of the location of the mean pole and mean equinox. The proper motions of stars were evaluated so that, for the adopted model of precession, they provide the best access to the mean pole and mean equinox of epoch, at any other date. The mean pole and mean equinox of date, which had a major role in the realization of the FK5, were therefore naturally considered as fundamental points of reference for the EOP.

The ICRS, adopted by the IAU as the International Celestial Reference System since the 1st January 1998, is defined such that the barycentric directions of distant extragalactic objects show no global rotation with respect to these objects. The ICRS was aligned with the FK5 at J2000.0; however, no attempt has been made to refer the positions of the sources to the mean pole and mean equinox at J2000.0 and it was decided that further improvements of the ICRS will be accomplished without introducing any global rotation. This corresponds to a conceptual change, which is the abandonment of the link of the conventional celestial reference system with the motion of the Earth. Such a change requires to adopt a definition of the equatorial system without any relation to the orbital motion of the Earth and consequently to abandon the equinox as the origin on the equator. Moreover, as the mean pole is no more a fundamental point of the celestial frame of reference, precession has no reason to be considered separately from nutation, these two motions being observationally not separable.

New EOP have to be defined to be in consistency with these conceptually fundamental changes in the conventional celestial system.

4.2 Proposal for new EOP referred to the GCRS and new origin on the moving equator

The current precession angles (Lieske *et al.* 1977) as well as Greenwich mean sidereal time (Aoki *et al.* 1982) are defined in the FK5 System and the current nutation angles (Seidelmann 1982) as well as the equation of equinoxes necessary to provide Greenwich sidereal time and therefore UT1, are referred to the ecliptic of date. Formulations of these parameters combine the motions of the equator and of the ecliptic with respect to the ICRS and combine the precession and nutation of the equator with the Earth rotation.

In order to take advantage of the fundamental properties of the GCRS, it is necessary to abandon the current parameters in the FK5 System and to select a new departure point on the moving equator, replacing the equinox. It is also necessary to consider more basic quantities for precession-nutation and Earth rotation, reducing the number of parameters by combining precession and nutation and referring them to a fixed plane and no more to the ecliptic of date. Several possibilities have been considered, within the work of T5, and compared (Capitaine 2000 b), among which Euler angles between the GCRS and the ITRS and the celestial and terrestrial coordinates of the pole (Capitaine 1990).

A comparison of the parameters shows that Euler's angles, or equivalently the celestial coordinates of the z-axis of the ITRS (Mathews 2000), which reduce to three the number of EOP, do not use any intermediate pole and consequently include both high frequency and low frequency components of the motion of the Earth's axis of figure with respect to the GCRS. Their estimation from observations would therefore need a large number of parameters. On the contrary, the use of the celestial and terrestrial pole coordinates of the CIP, separate the celestial components from the terrestrial components according to the frequency criteria defining the CIP. Such a procedure, in which the coordinates in the GCRS appear in a symmetric form as the coordinates in the ITRS, facilitates the estimation of the parameters from observations.

The celestial pole coordinates, $X = sind \cos E$, $Y = sind \sin E$ (E and d being the polar distance and azimuth of the CIP in the GCRS) provide the position of the CIP in the GCRS; they include precession, nutation, the coupling effects between precession and nutation and the offsets ξ_o, η_o, $d\alpha_0$ of the precession-nutation models at J2000 with respect to the pole and equatorial origin of the GCRS. They fullfill all the requirements for EOP in the GCRS and can be written as : $X = \bar{X} + \xi_0 - d\alpha_0 \bar{Y}$, $Y = \bar{Y} + \eta_0 + d\alpha_0 \bar{X}$, $\bar{X}$ and $\bar{Y}$ being the coordinates of the CIP derived from the precession and nutation model.

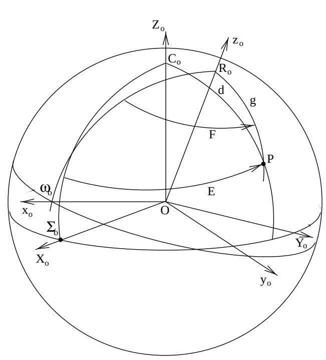

Figure 1 : The coordinates of the CIP in the GCRS and the ITRS (Capitaine 1990)

Several origins on the moving equator have also been considered, within the work of T5, for replacing the equinox, among which the "non-rotating origin" (NRO) (Guinot 1979), which is defined by a kinematical property of "no rotation" around the axis of the CIP, with respect to the reference system, when the pole is moving. The position of the NRO with respect to the GCRS, given by the quantity s (such that $\dot{s} = (\cos d - 1)\dot{E}$) is dependent on precession and nutation, whereas the position of the NRO with respect to the ITRS, given by the quantity s', is only slightly dependent on polar motion (Capitaine *et al.* 1986).

A comparison of the use of the possible origins for defining the Earth's angle of rotation on the moving equator shows that such an angle referred to any geometrical origin includes, inevitably, the effect of the instantaneous rotation of this origin around the equator with respect to the GCRS, due to the precession-nutation of the equator. On the contrary, the angle referred to the NRO has the advantage of including, by definition, only the Earth's rotation, clearly separating it from the precession-nutation of the equator. This provides a rigorous definition of sidereal rotation of the Earth. Moreover, it must be noted that the use of the NRO in the ITRS is the only way to define the "instantaneous origin of longitude". The use of the NRO both in the GCRS and the ITRS defines the "stellar angle", θ, (Guinot 1979) and the resulting definition of UT1 through a linear relationship with this angle, remains valid even if the adopted model for precession and nutation is revised (Capitaine *et al.* 1986).

4.3 The use of Celestial and Terrestrial Ephemeris origins and coordinates of the CIP

The NRO can be designated as the "Celestial Ephemeris origin"(CEO) if defined with respect to the GCRS. Similarly, it can be designated as the "Terrestrial Ephemeris Origin" (TEO) if defined with respect to the ITRS. The Earth Rotation Angle, can be defined by the angle measured along the equator of the CIP between the CEO and the TEO.

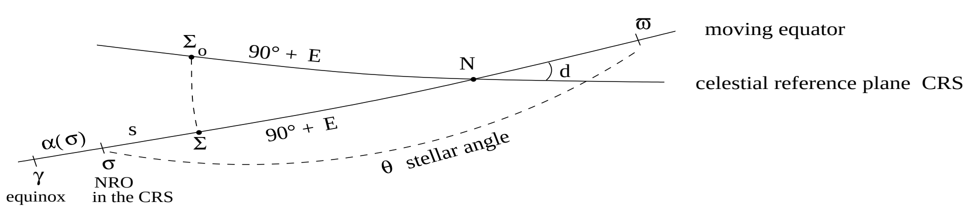

Figure 2 : The Celestial (CEO) and Terrestrial (TEO) Ephemeris Origins

The coordinate transformation between the ITRS and the GCRS, recommended by Resolution B1.7, is given by the product of rotations specified by the coordinates X, Y of the CIP in the GCRS, the coordinates x, y of the CIP in the ITRS, the quantities s and s' and the Earth Angle of rotation, θ as :

$$R_3(-E).R_2(-d).R_3(E).R_3(s).R_3(-\theta(UT1)).R_3(-s').R_1(y_p).R_2(x_p).$$

Development as function of time of X and Y at a μas accuracy after one century are provided by Capitaine *et al.* (2000) using the IERS1996 precession and nutation and the celestial offsets at J2000 (IERS Annual Report for 1997). Such developments include a constant term, a polynomial form of t, a sum of periodic terms and a sum of Poisson terms. These developments can be easily modified to be consistent with the IAU 2000 precession-nutation model as soon as it is available. The numerical development of s, providing the position of the CEO, as well as the linear relationship between θ and UT1 have been derived from the IERS Conventions 1996 by Capitaine *et al.* (2000) and the definition of UT1 has been shown to be insensitive to changes in models for precession and nutation at a microarsecond level. The position of the CEO, compatible with the IAU 2000 model for precession and nutation of the CIP, can be derived as soon as it is available; the numerical value of s' can be extrapolated using the IERS data.

5. CONCLUDING REMARKS

The adoption of the ICRS since the 1st January 1998 by the IAU, associated with significant improvements in the theory and observation of Earth rotation, have required an extended definition and realization of the CEP as well as more basic EOP parameters. Such requirements are taken into account through two IAU resolutions which have been adopted at the 24th General Assembly of the IAU (Manchester, August 2000). IAU Resolution B1.7 defines the "Celestial Intermediate Pole" (CIP), which will replace the "Celestial Ephemeris Pole" (CEP) in the new IAU 2000 precession and nutation model, as an intermediate pole in the coordinate transformation between the GCRS and the ITRS and this resolution specifies the way for taking into account the high frequency terms in polar motion and nutation. Resolution B1.8 recommends the use of the "non-rotating origin" (Guinot 1979) in the GCRS, as well as in the ITRS for defining UT1, together with the use of the celestial and terrestrial coordinates of the CIP in the coordinate transformation between the GCRS and the ITRS.

The T5 membership was N. Capitaine (Chair, France), V. Dehant (Belgium), A.-M. Gontier (France), B. Kolaczek (Poland), J. Kovalevsky (France), J. Lieske (USA), C. Ma (USA), D.D McCarthy (USA), O. Sovers (USA), J. Vondrak (Czech. Republic). Other contributors to the discussion have been C. Bizouard (France), P. Bretagnon (France), A. Brzezinski (Poland), S. Loyer (France), P. Mathews (India), M. Rothacher (Germany), H. Schuh (Germany), Y. Yatskiv (Ukraine).

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PRESENT STATUS OF ASTRONOMICAL CONSTANTS

T. FUKUSHIMA
National Astronomical Observatory
2-21-1, Ohsawa, Mitaka, Tokyo 181-8588, Japan
e-mail: Toshio.Fukushima@nao.ac.jp

ABSTRACT. Given was the additional information to the previous report on the recent progress in the determinations of astronomical constants (Fukushima 2000). First noted was the revision of L_G as $6.969\ 290\ 134 \times 10^{-10}$ based on the proposal to shift its status from a primary to a defining constant (Petit 2000). Next focused was the significant update of the correction to the current precession constant, Δp, based on the recent LLR-based determination (Chapront *et al.* 2000) as -0.3164 ± 0.0030 $''$/cy. By combining this and the equal weighted average of VLBI determinations (Mathews *et al.* 2000; Petrov 2000; Shirai and Fukushima 2000; Vondrák and Ron 2000) as -0.2968 ± 0.0043 $''$/cy, we derived the best estimate of precession constant as $p = 5\ 028.790 \pm 0.005$ $''$/cy. Also redetermined were some other quantities related to the precession formula; namely the offsets of Celestial Ephemeris Pole of the International Celestial Reference Ssystem as $\Delta\psi_0 \sin\varepsilon_0 = (-17.0\pm0.3)$ mas and $\Delta\varepsilon_0 = (-5.1\pm0.3)$ mas. As a result, the obliquity of the ecliptic at the epoch J2000.0 was estimated as $\varepsilon_0 = 23°26'21.''405\ 9 \pm 0.''000\ 3$. As a summary, presented was the (revised) IAU 2000 File of Current Best Estimates of astronomical constants, which is to replace the former 1994 version (Standish 1995).

1. INTRODUCTION

The report of IAU Working Group on Astronomical Standards (WGAS) on the issue of astronomical constants has appeared already (Fukushima 2000). Since its publishing, made were only a few revisions, whose details will be reported here. One is a minor correction of the numerical value of L_G, which was to be consistent with the other proposals submitted simultaneously (Petit 2000). Another is the revision of the correction to the precession constant based on the very recent re-analysis of LLR observations (Chapront *et al.* 2000).

2. SCALE CONSTANTS

We admit that there was a misprint in the value of L_G appeared in Fukushima (2000). The correct value (Petit 2000), which has been adopted in the IAU 2000 Resolution B1.9, is

$$L_G = 6.969\ 290\ 134 \times 10^{-10} \tag{1}$$

which has no uncertainty. Or, more rigorously speaking, it is no more one of primary constants determined from measurements but has become a defining constant, the constant defining the TCG. Thus, by combining its value with the recent determination of L_C (See Fukushima 2000) as

$$L_C = 1.480\ 826\ 867\ 4 \times 10^{-8} \pm 1.4 \times 10^{-17}, \tag{2}$$

we now have a new estimate of another scale constant, L_B, as

$$L_B = 1.550\ 519\ 767\ 7 \times 10^{-8} \pm 2.0 \times 10^{-17} \tag{3}$$

which is to be used to convert the (obsolete) TDB-based determinations to the TCB based ones. As an application, we computed the TCB-based value of GM_E from that given in Groten (2000) as

$$GM_E = 3.986\ 004\ 441\ 5 \times 10^{14} \pm 8 \times 10^5 \mathrm{m}^3\mathrm{s}^{-2} \tag{4}$$

3. PRECESSION CONSTANT

As was already stated, we have obtained a more reliable value for the correction to the precession constant, which was derived by the LLR observation recently (Chapront *et al.* 2000). Then, we expanded the table showing the history of Δp determination given in Fukushima (2000) and listed it here as Table 1. Note that, in the references of this article, we have listed only those whose information were not available before the publication of the previous work (Fukushima 2000). The latest LLR-based determination (Chapront *et al.* 2000) of Δp was close to but clearly different from the VLBI-based ones as

$$\Delta^{(\mathrm{L})} p = (-0.316\ 4 \pm 0.003\ 0)''/\mathrm{cy}. \tag{5}$$

On the other hand, the VLBI-based best estimate (Fukushima 2000) was

$$\Delta^{(\mathrm{V})} p = (-0.296\ 8 \pm 0.004\ 3)''/\mathrm{cy}, \tag{6}$$

Unfortunately, there exists still significant difference between these two determinations. We simply take their mean to derive the best estimate of Δp as

$$\Delta p = (-0.306\ 6 \pm 0.004\ 8)''/\mathrm{cy}. \tag{7}$$

By adding this to the IAU 1976 value of precession constant, we now have the best estimate of the general precession in longitude as

$$p = (5\ 028.790 \pm 0.005)''/\mathrm{cy}. \tag{8}$$

At the same time, the recent estimates of the offset of Celestial Ephemeris Pole at the epoch J2000.0, $\Delta\psi_0 \sin\varepsilon_0$ and $\Delta\varepsilon_0$, seem to converge to a single pair of values being independent on the observation type. See Table 2.
By adopting a similar procedure as we did in deriving Δp, we obtained the offsets as

$$\Delta^{(\mathrm{V})}\psi_0 \sin\varepsilon_0 = (-16.7 \pm 0.5)\mathrm{mas}, \quad \Delta^{(\mathrm{V})}\varepsilon_0 = (-4.9 \pm 0.3)\mathrm{mas}, \tag{9}$$

$$\Delta^{(\mathrm{L})}\psi_0 \sin\varepsilon_0 = (-17.3 \pm 0.4)\mathrm{mas}, \quad \Delta^{(\mathrm{L})}\varepsilon_0 = (-5.4 \pm 0.2)\mathrm{mas}, \tag{10}$$

$$\Delta\psi_0 \sin\varepsilon_0 = (-17.0 \pm 0.3)\mathrm{mas}, \quad \Delta\varepsilon_0 = (-5.2 \pm 0.3)\mathrm{mas}, \tag{11}$$

Note that thus obtained $\Delta\varepsilon_0$ is the correction *not* to the IAU 1976 value, $23°26'21.''448$, but to the angle between the ecliptic and the reference plane of International Celestial Reference System (ICRS). Quoting the result of investigation of Chapront *et al.* (1999), where the obliquity of the inertial mean ecliptic to the ICRS equator was estimated as

$$\varepsilon_0(\mathrm{ICRS}) = 23°26'21.''411\ 00 \pm 0.''000\ 05, \tag{12}$$

we now have the best estimate of the obliquity of the ecliptic at J2000.0 as

$$\varepsilon_0 = 23^\circ 26' 21.''405\,9 \pm 0.''000\,3. \tag{13}$$

This is significantly different from the value of JPL's DE series, $23^\circ 26' 21.''412$.

4. CONCLUSION

By collecting the results on the two topics described in the previous sections, we updated the former IAU File of Current Best Estimates of astronomical constants (Standish 1995). The revised list is illustrated in Table 3. Here the references for the items differ from the previous version (Standish 1995) are :
(1) Tholen and Buie (1997) for the mass ratio of Pluto+Charon to that of the Sun, $M_{\mathrm{S}}/M_{\mathrm{P}}$,
(2) DE405 for τ_{A} and $M_{\mathrm{M}}/M_{\mathrm{E}}$,
(3) IAG 1999 for the geodetic constants, a_{E}, J_2, $1/f$, and W_0,
(4) CODATA 1998 for G,
(5) Petit (2000) for L_G, and
(6) this article for GM_E, p, and ε_0.

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Table 1: Corrections to Precession Constants

Method & Reference		Δp ($''$/cy)		$\Delta\varepsilon_1$ ($''$/cy)	
		Value	σ	Value	σ
V	Fanselow *et al.* (1984)	−0.38	0.09		
V	Herring *et al.* (1986)	−0.239	0.013		
V	Zhu *et al.* (1990)	−0.38	0.05	+0.017	0.017
V	Sovers (1990)	−0.196	0.013		
S	Andrei & Elsmore (1991)	+0.01	0.15		
V	Herring *et al.* (1991)	−0.32	0.10	−0.04	0.05
L+V	Williams *et al.* (1991)	−0.27	0.04		
V	McCarthy & Luzum (1991)	−0.27	0.02	−0.005	0.007
P	Miyamoto & Soma (1993)	−0.27	0.03		
V	Walter & Ma (1994)	−0.36	0.11		
T	Williams (1994)	−0.2368		−0.0244	
L+V	Charlot *et al.* (1995)	−0.30	0.02	−0.020	0.008
V	Herring (1995)	−0.30	0.01	−0.024	0.005
V	Souchay *et al.* (1995)	−0.321	0.003	−0.026	0.001
V	Walter & Sovers (1996)	−0.31	0.01		
O	Vondrák (1999)	−0.154	0.004	−0.013 1	0.001 8
L	Chapront *et al.* (1999)	−0.344	0.004		
P	Vityazev (2000)	−0.28	0.08		
O	Vondrák and Ron (2000)	−0.216	0.005	−0.009 3	0.001 8
V	Petrov (2000)	−0.295	0.002	−0.027	0.000 9
V	Vondrák and Ron (2000)	−0.299 0	0.001 3	−0.022 0	0.000 7
V	Mathews *et al.* (2000)	−0.300 1	0.000 8	−0.024 7	0.000 3
V	Shirai & Fukushima (2000)	−0.293 0	0.000 5	−0.024 3	0.000 2
L	Chapront *et al.* (2000)	−0.316 4	0.003 0		

Note: The symbols of the methods are; V for the VLBI data, S for the short baseline radio interferometry, L for the LLR data, P for the proper motion analysis, T for the theoretical consideration, and O for the optical observation of latitude variations.

Table 2: Offsets of Celestial Ephemeris Pole at J2000.0

Method & Reference		$\Delta\psi_0 \sin\varepsilon_0$ (mas)		$\Delta\varepsilon_0$ (mas)	
		Value	σ	Value	σ
V	Herring (1995)	−17.3	0.2	−5.1	0.2
L	Chapront *et al.* (1999)	−18.3	0.4	−5.6	0.2
O	Vondrák & Ron (2000)	−12.3	0.7	−9.2	0.6
V	Vondrák & Ron (2000)	−17.10	0.05	−4.95	0.05
V	Mathews *et al.* (2000)	−16.18		−4.53	
V	Shirai & Fukushima (2000)	−16.889	0.013	−5.186	0.013
L	Chapront *et al.* (2000)	−17.3	0.4	−5.4	0.2

Table 3: IAU 2000 File of Current Best Estimates

Class & Item	Value (Uncertainty) [Unit]	Reference
DEFINING		
k	$1.720\ 209\ 895 \times 10^{-2}$	IAU 1976
c	$2.997\ 924\ 58 \times 10^{8}$ [ms^{-1}]	CODATA 1998
L_{G}	$6.969\ 290\ 134 \times 10^{-10}$	Petit (2000)
PRIMARY		
L_{C}	$1.480\ 826\ 867\ 4(14) \times 10^{-8}$	Irwin and Fukushima (1999)
p	$5.028\ 790(5) \times 10^{3}$ [$''$/cy]	This article
ε_0	$8.438\ 140\ 59(3) \times 10^{4}$ [$''$]	This article
τ_{A}	$4.990\ 047\ 863\ 9(2) \times 10^{2}$ [s]	DE405
$M_{\mathrm{M}}/M_{\mathrm{E}}$	$1.230\ 003\ 45(5) \times 10^{-2}$	DE405
$M_{\mathrm{S}}/M_{\mathrm{Me}}$	$6.023\ 6(3) \times 10^{6}$	Andersen *et al.* (1987)
$M_{\mathrm{S}}/M_{\mathrm{V}}$	$4.085\ 237\ 1(6) \times 10^{5}$	Sjogren *et al.* (1990)
$M_{\mathrm{S}}/M_{\mathrm{Ma}}$	$3.098\ 708(9) \times 10^{6}$	Null (1969)
$M_{\mathrm{S}}/M_{\mathrm{J}}$	$1.047\ 348\ 6(8) \times 10^{3}$	Campbell and Synott (1985)
$M_{\mathrm{S}}/M_{\mathrm{Sa}}$	$3.497\ 898(18) \times 10^{3}$	Campbell and Anderson (1989)
$M_{\mathrm{S}}/M_{\mathrm{U}}$	$2.290\ 298(3) \times 10^{4}$	Jacobson *et al.* (1992)
$M_{\mathrm{S}}/M_{\mathrm{N}}$	$1.941\ 224(4) \times 10^{4}$	Jacobson *et al.* (1991)
$M_{\mathrm{S}}/M_{\mathrm{P}}$	$1.352\ 1(15) \times 10^{8}$	Tholen and Buie (1997)
a_{E}	$6.378\ 136\ 6(1) \times 10^{6}$ [m]	Groten (2000)
J_2	$1.082\ 635\ 9(1) \times 10^{-3}$	Groten (2000)
GM_{E}	$3.986\ 004\ 415(8) \times 10^{14}$ [$\mathrm{m}^3\mathrm{s}^{-2}$]	This article
W_0	$6.263\ 685\ 60(5) \times 10^{7}$ [$\mathrm{m}^2\mathrm{s}^{-2}$]	Groten (2000)
ω	$7.292\ 115\ 0(1) \times 10^{-11}$ [rad s^{-1}]	Groten (2000)
G	$6.673(10) \times 10^{-11}$ [$\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$]	CODATA 1998

Note: The exponent expressions are fully introduced. The units of uncertainties are the last digit of the values shown. The value of τ_{A} shown here is that after the scale transformation was applied. The value before transformation, originally given DE405, is 499.004 783 806 1... The geophysical values are those for the tide free system (Groten 2000). Suffices of radii and masses indicate the celestial objects; E for the Earth, M for the Moon, S for the Sun, Me for Mercury, V for Venus, Ma for Mars, J for Jupiter, Sa for Saturn, U for Uranus, N for Neptune, and P for Pluto. Note that the planetary masses *except* the Earth include the contribution of their satellites.

INTERNATIONAL VLBI SERVICE FOR GEODESY AND ASTROMETRY (IVS)

W. SCHLÜTER[1], N. VANDENBERG[2]
(1) Bundesamt für Kartographie und Geodäsie,
Fundamentalstation Wettzell,
D-93444 Kötzting
e-mail: Schlueter@Wettzell.ifag.de

(2) NVI, Inc/NASA Goddard Space Flight Center,
Code 920.1, Greenbelt, MD 20771, USA

1. GENERAL REMARKS

The IVS (International VLBI Service for Geodesy and Astrometry) was established in February 1999 in order to support VLBI programs for geodetic, geophysical and astrometric research and operational activities. IVS coordinates the observations, the data flow, the correlation, the data analysis and the technology developments. Today not only science, research and development are making use of the results but also practical applications are dependent on IVS products. Thus, acting within the frame of IAG and IAU, IVS has to guarantee provision of the required results on a regular, timely basis including Earth orientation parameters, position and velocity vectors.

Beginning in the mid 80s the VLBI technique assumed the important and primary role for Monitoring Earth Orientation Parameters (EOP) and it realizes and provides the link between ICRF and ITRF with the highest reliability and accuracy. At that time the VLBI Community was organized as the VLBI Subcommission within IAG Commission VIII - CSTG (International Coordi-nation of Space Techniques for Geodesy and Geodynamics), which is also Subcommission B.2 of COSPAR. The president of CSTG, Gerhard Beutler, encouraged by the success of the IGS (International GPS Service), proposed in 1997 to organize the SLR/LLR and the VLBI sub-commissions of the CSTG into services comparable to IGS. T. A. Clark, in his function as chairman of the CSTG VLBI Subcommission, drafted the Terms of Reference (ToR) in October 1997. The final version of the ToR was worked out by a Subcommission Steering Committee composed of James Campbell (chair), Yasuhiro Koyama, Chopo Ma, Arthur Niell, Axel Nothnagel, Jim Ray and Nancy Vandenberg. The Terms of References (ToR) were presented and approved at both the CSTG Executive Committee and IERS Directing Board meetings in Nice/France in April 1998. They are available under http://ivs-cc.gsfc.nasa.gov.

2. SUMMARY OF THE IVS COMPONENTS

A Call for Participation was released jointly by CSTG and IERS on June 1, 1998. The proposals were evaluated and accepted by the Steering Committee. In summary IVS has today

- *30 Network Stations*, concentrated in USA, Europe, Japan and a deficit in the southern hemisphere,
- *3 Operation Centers* namely NASA-GSFC, NEOS, Geodetic Institute of the University of Bonn,
- *7 Correlators* operated by NEOS (Washington)/USA, NASA(Haystack)/USA, BKG-MPI/Germany, GSI/Japan, CRL/Japan, IAA/Russia, JIVE/Netherlands,
- *6 Data Centers* established at NASA-GSFC/USA, Observ. Paris/France, BKG/Germany, CNR/Italy, CRL/Japan and Agenzia Spaziale/Italy,
- *20 Analysis Centers*, five of them provide IVS core products (IAA/Ru, GSFC/US, USNO/US, BKG/D, OP/F) and 15 Associate AC perform investigations or provide related products,
- *9 Technology Development Centers* supporting the recording techniques MK III and MK IV, K4 and S2, and
- *1 Coordinating Center* operated by NEOS, a cooperation of USNO and GSFC.

All together there are 76 components representing 30 Member Organizations in 15 countries and more than 230 individual Associate Members. IVS has 31 Member organizations, and 4 Affiliated Member organizations.
IVS is recognized as a Service of the International Association of Geodesy (IAG) since July 1999 when the General Assembly was held in Birmingham/England and is also recognized as a Service of the IAU since the XXIVth General Assembly, Manchester/England, August 2000 (resolution annexed).

3. IVS OBJECTIVES AND PRODUCTS

The IVS objectives in general are to

- support geodetic, geophysical and astrometric research and operational activities,
- promote research and development activities in all aspects of the geodetic and astrometric VLBI technique and
- interact with users of VLBI products.

The products are strongly related to

- the contribution to the International Terrestrial Reference Frame (ITRF),
- the realisation of the International Celestial Reference Frame (ICRF) and
- determination of Earth Orientation Parameters (EOP).

Nowadays the IVS is a technique center for the International Earth Rotation Service (IERS) and has close interactions with IERS. The VLBI technique uniquely provides the parameters for the CRF and is the only technique to determine the celestial pole.

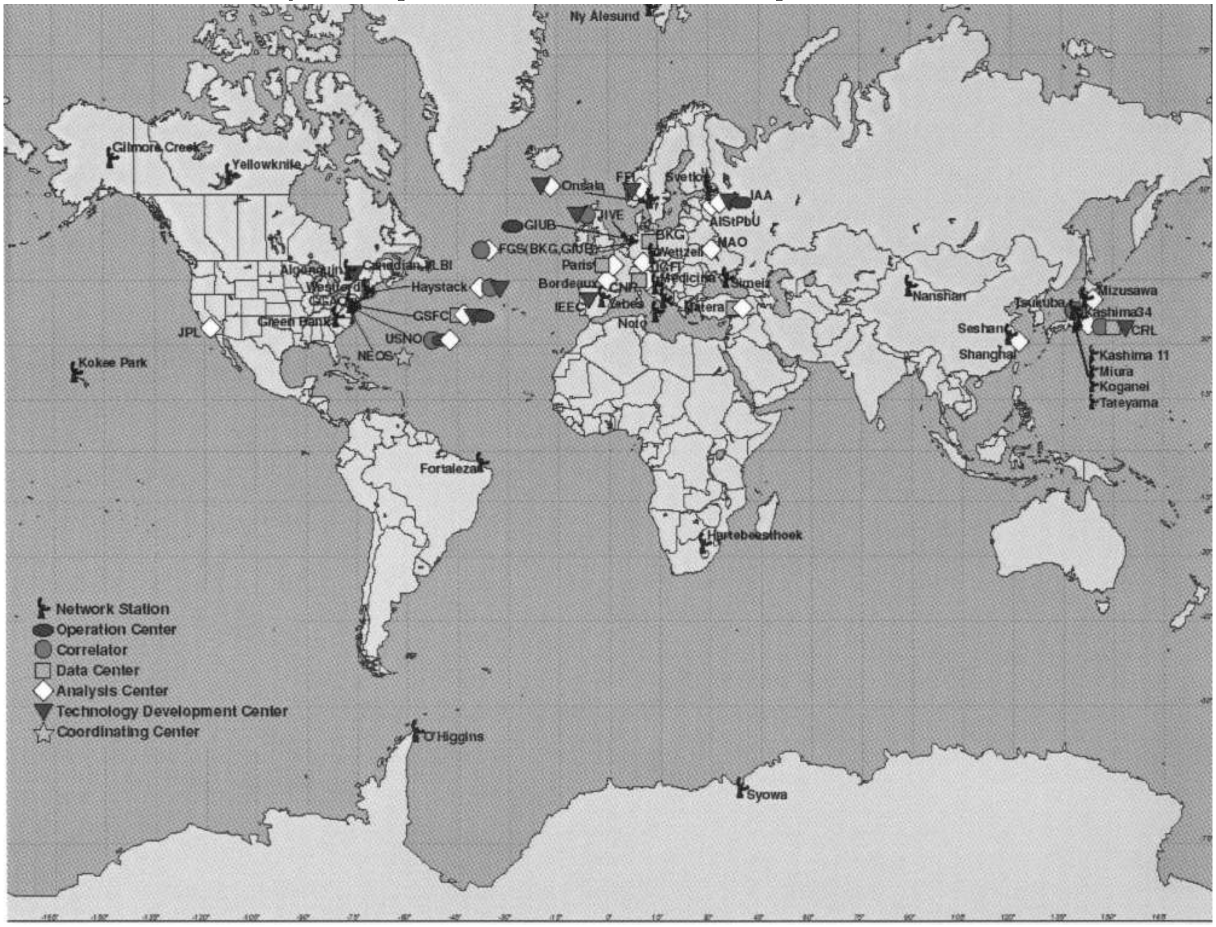

Fig. 1. Overview of the distribution of the IVS components

IVS is producing Earth Orientation Parameters (EOP) from 24h observation sessions regularly and periodically carried out such as NEOS and IRIS. The parameters in particular are the celestial pole coordinates d?, d? polar motion parameter x_p, y_p, and DUT1. In addition DUT1 is derived from 2h observation sessions carried out quasi daily by Wettzell and Green Bank, nowadays Wettzell and Kokee Park.
CRF solutions are regularly derived in order to determine quasar positions or to detect proper motions of quasars. Station positions and velocities are derived from all the observations and are a strong contribution to the ITRF.
It is planned to provide EOP with subdaily resolution and baseline length evolutions. The products are available via ftp://cddisa.gsfc.nasa.gov ftp://ftp.leipzig.ifag.de or ftp://ivsopar.obspm.fr.

4. IVS ACTIVITIES

The 1st Directing Board meeting was held in Wettzell on February 11, 1999 in order to establish the Service and to initiate activities under the flag of the IVS. As of the inauguration date of IVS, on March 1, 1999, its web site was available at http://ivs.gsfc.nasa.gov.
Soon thereafter, a solicitation for IVS data and analysis was released to obtain proposals from the Operation and Analysis Centers on the provision of products such as correlation results, EOPs, and combined analysis. Those products derived by the Analysis Centers were designed

to become "official" IVS products. In the same solicitation the call for an Analysis Coordinator was released.

The IVS Annual Report was published in August 1999 (electronically) and September 1999 (printed). The intention of the Analysis Report was to provide a document on the status of all components. A procedure was created to standardize the layout, which supported and accelerated the publication of the Annual Report 1999. The 2000 IVS General Meeting was organized and held in Kötzting, Germany, during February 21-24, 2000. It was a successful meeting with more than 120 participants registered. The goals of the meeting were determined by a program committee. The main character of the 1st General Meeting was addressed towards young researchers. Overview talks and tutorials were held before the sessions, in order to introduce the session topic to those who work in different areas. An IVS Analysis Workshop was combined with the General Meeting, held in Kötzting, Germany, February 24, 2000, and an initial meeting of a Working Group for mapping the phase centers of GPS transmission antennae was held. It has to be mentioned that the proceedings of 2000 IVS General Meeting were published in June 2000. The proceedings published nearly all the papers and tutorials and are a very valuable tool, especially for people starting to work in VLBI.

At the 3rd Directing Board Meeting, held in Wettzell before the General Meeting, slight modifications to the Terms of References were made in order to clarify the status of the Analysis Centers and to include Affiliated Members. Affiliated Members will be informed about IVS activities without having obligations to IVS.

Personnel fluctuations in the Directing Board (DB) have to be mentioned. The representative of the IAG, Gerhard Beutler, when he was elected as Vice President of IAG, withdrew from the board after the 2nd DB meeting, held in Birmingham, July 19, 1999. He was the initiator of the IVS and we have to express our thanks to him. James Campbell, one of the most experienced VLBI experts, was nominated by IAG to be the new IAG representative. Axel Nothnagel, representing the Analysis Centers on the DB, started his work as Analysis Coordinator (AC) as of October 1, 1999. Up to October, 1999 the function of the AC was jointly carried out by Marshall Eubanks, Chopo Ma and Nancy Vandenberg. Marshall Eubanks, representative of the Operations Centers and Correlators, has founded a new e-business, which demands his full attention. He withdrew from the DB and was replaced by Kerry Kingham. We have to thank Marshall Eubanks for all his important contributions in the field of VLBI and for IVS.

The current members of the DB and their functions are:

Ex Officio :
IAG representative : James Campbell
IAU representative : Nicole Capitaine
IERS representative : Chopo Ma (IERS in transition)
Coordinating Center : Nancy Vandenberg

Coordinators :
Analysis Coordinator : Axel Nothnagel (Oct.1,1999)
Network Coordinator : Ed Himwich
Technology Coordinator : Alan Whitney

Representatives :
Analysis and Data Centers : Chopo Ma (Oct. 1, 1999)
Operation Centers and Correlators : Kerry Kingham
Networks : Shigeru Matsuzaka
Networks : Wolfgang Schlüter (chair)
Technology Development Centers : Tetsuro Kondo

At Large Members : Wayne Cannon, Paolo Tomasi

5. NEXT STEPS TO INCREASE RELIABILITY

In order to increase the reliability of the IVS data and its quality some improvements will soon become effective. IVS is monitoring station performance at all steps in the data flow to help identify problems early and make sure they get fixed. As part of this effort IVS has developed a database that collects performance organized by station. This simplifies keeping track of any problems at each station, knowing when a station has any problems, and whether they have been resolved. IVS is also collecting detailed information on station configurations, including collocation with other techniques, in a central database. The configuration will provide a ready reference for each stations capabilities. The collocation information will be useful for geodetic analysis efforts where the results from more than one technique are compared. We are planning an operations workshop to provide training for operators; such workshops will be held every two years. IVS plans to develop performance standards so that the stations can evaluate their own performance locally and make any changes necessary to improve it. A pool of spare parts will help to overcome or minimize periods of failure, in case severe technical problems occur and the repair require spares which locally might not be available.

The transition from MKIII to MKIV, which currently needs resources for the implementation, will soon be completed. It is expected that the throughput, quantity and quality of the correlation will increase. A good and immediate feedback to the network stations will help to detect upcoming problems and failures.

The products of IVS are not based on the results of only one Analysis Center. Up to five Analysis Centers will provide independent solutions from the 24-observation sessions. Employing different software and slightly different strategies, even when using the same standards, can lead to slightly different results. As long as no obvious error is detected, no solution can be declared the best. In order to overcome the discrepancies between the solutions, comparisons and combination procedures have to be applied, which will provide the most reliable results as a combination of the solutions. The fact that five solutions will be available will help eliminate outliers, which finally will increase the reliability of the combined products significantly. Comparison and combination of the products also creates competition and will improve the analysis models.

In order to improve the quantity of data, extensions and changes in the observation scenario have to be considered. A tremendous improvement will result from the gradual implementation of the CORE (Continuous Observation of the Rotation of the Earth) program. Up to now CORE is based on the MKIII/MKIV technology only and in 2000 covered 2.1 days of observation per week. It is planned to increase the number of days up to 3 days in 2001, 4 days in 2002 and finally 6 days in 2003. More automation and remote control of network stations have to be considered as weekend observations will be involved and manpower on weekends will be problematic.

It also has to be strongly considered to extend the CORE program to the other recording techniques and to include stations and correlators equipped with K4 and S2 systems.

Improvements in the Network configuration have to be considered. There is a concentration of the network station in the Northern hemisphere. The network should be better balanced, which requires activation of existing stations for IVS or implementation of new stations in the Southern hemisphere. A new station will be established soon by the Bundesamt für Kartographie und Geodäsie/Germany in Concepcion/Chile. The Transportable Integrated Geodetic Observatory (TIGO) will be installed in the beginning of 2001 and will start providing observations in spring 2001. Additional stations might encouraged to be involved in the IVS activities from related groups such as the EVN (European VLBI Network) or APT (Asia Pacific Telescope).

6. TECHNOLOGY DEVELOPMENTS

The primary technology interests of IVS have focused in three areas :

1. Definition of a VLBI Standard Interface
2. Integration of new high data-rate VLBI systems into the networks
3. Exploration of electronic international transmission of VLBI data (e-VLBI)

Of these, the primary focus of the IVS Technology Coordinator over the past $1^{1/2}$ years has been the development and specification of a VLBI Standard Interface. The incompatibility of various VLBI data systems has long been recognized as posing a serious obstacle to the realization of the full potential of VLBI observations. Sporadic efforts have developed over the years to define a common interface standard which would allow observations recorded on different VLBI data systems to be processed at a common correlator, but these efforts have foundered for various reasons. The establishment of the Global VLBI Working Group (GVWG) in the early 1990s, growing primarily from the space-VLBI community but serving the broader interests of astronomy, and the International VLBI Service (IVS) in 1998, serving primarily the geodetic VLBI community, provided an organizational framework for which efforts at standardization could proceed in a more organized and sanctioned fashion. It was from these roots that the present VSI effort was initiated, led by the IVS Technology Coordinator. The main goals of the VSI can be summarized as follows :

- Develop a standardized set of hardware and software specifications that will allows inter-operability of various VLBI data systems.
- Hide as much as possible the detailed characteristics of various systems so as not to place undo restraints or restrictions on any particular system.
- Support recording on various media (tape, disc, optical, etc) as well as real and quasi real-time data links of any type and a data rates expandable to many Gbps.

The fledgling VSI concept leading to the current specification was first proposed at the time of the GEMSTONE meeting in Tokyo in January 1999 and was discussed by a small interested group at that meeting. Support was then sought and received from IVS and GVWG to create a VSI Technology Coordination Group (VSI-TCG) comprised of experts representing all of the major world institutions involved in the development of VLBI equipment. Early in the discussions it was decided to separate the hardware and software VSI, concentrating first on hardware, hence this VSI-H specification, to be followed by a companion software specification, VSI-S. The VSI-H specification was shaped by intensive e-mail discussions, plus three (multi-hour!) international telephone conferences and a two-day international VSI meeting held at Haystack Observatory in February 2000. The VSI-H specification is complete. It is intended as a starting point from which to progress, and will be extended and amended as requirements and technology demand. It is heartening that already at least three groups are known to be developing or adapting VLBI data systems to meet the VSI-H standard. As for the other IVS technology interests, efforts have been on-going :

- New VLBI data systems have been (or are being) integrated into IVS stations, led by the Mark IV data acquisition and correlation systems capable of supporting data rates up to 1 Gbps.
- Investigations into the potential for e-VLBI are on-going, primarily focusing on the current and projected economics of wordwide real-time or quasi-real-time high-speed optical links. Prices for high- speed international connections are rapidly diminishing, though last mile connection costs to many stations will remain high.

7. PROSPECTS

The primary task of the IVS is the coordination of all its components into a highly reliable service and to guarantee high quality and timely products on a long term basis. With the development of standards for the components it can be expected that the overall data quality, quantity and reliability will significantly increase. Improvements have to be considered in the network configuration. Cooperation with related communities such as the EVN or the APT, the involvement of the other data acquisition techniques, and the standardization of the data acquisition interfaces will support an increase in the number and balance of the global distribution of network stations. New developments in technology will accelerate data exchange and increase the degree of automation in order to provide timely products. In close cooperation with the other services the strengths of each technique should be exploited to improve all geodetic products. The IVS Working Group on the mapping of GPS phase center involving the IGS and the ILRS is a good example of the attempt to find better solutions in providing and maintaining reference frames for the community.

LITERATURE

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ANNEX

IAU Resolutions B1.1 Maintenance and Establishment of Reference Frames and Systems
The International Astronomical Union
Noting

1. that Resolution B2 of the XXIIIrd General Assembly (1997) specifies that "the fundamental reference frame shall be the International Celestial Reference Frame (ICRF) constructed by the IAU Working Group on Reference Frames",

2. that Resolution B2 of the XXIIIrd General Assembly (1997) specifies "That the Hipparcos Catalogue shall be the primary realisation of the International Celestial Reference System (ICRS) at optical wavelengths", and

3. the need for accurate definition of reference systems brought about by unprecedented precision, and

Recognising

1. the importance of continuing operational observations made with Very Long Baseline Interferometry (VLBI) to maintain the ICRF,

2. the importance of VLBI observations to the operational determination of the parameters needed to specify the time-variable transformation between the International Celestial and Terrestrial Reference Frames,

3. the progressive shift between the Hipparcos frame and the ICRF, and

4. the need to maintain the optical realisation as close as is possible to the ICRF

Recommends

1. that IAU Division I maintain the Working Group on Celestial Reference Systems formed from Division I members to consult with the International Earth Rotation Service (IERS) regarding the maintenance of the ICRS,
2. that the IAU recognise the International VLBI Service (IVS) for Geodesy and Astrometry as an IAU Service Organization,
3. that an official representative of the IVS be invited to participate in the IAU Working Group on Celestial Reference Systems,
4. that the IAU continue to provide an official representative to the IVS Directing Board,
5. that the astrometric and geodetic VLBI observing programs consider the requirements for maintenance of the ICRF and linking to the Hipparcos optical frame in the selection of sources to be observed (with emphasis on the Southern Hemisphere), design of observing networks, and the distribution of data, and
6. that the scientific community continue with high priority ground- and spacebased observations (a) for the maintenance of the optical Hipparcos frames and frames at other wavelengths and (b) for links of the frames to the ICRF.

THE NEW ORGANIZATION OF THE IERS

J. VONDRÁK
Astronomical Institute and Center for Earth Dynamics Research
Boční II, 141 31 Prague 4, Czech Republic
e-mail: vondrak@ig.cas.cz

ABSTRACT. As the re-organization of the International Earth Rotation Service (IERS) approaches its end, the new IERS Terms of Reference are shortly discussed. The role and recommendations of the Proposal Review Committee (PRC) are mentioned, and the components of the new structure of the IERS, as decided at the recent IERS Directing Board meetings are described. Most of the newly selected components will be operational by the end of 2000, and the new IERS will become fully operational in summer 2001.

1. INTRODUCTION

The re-organization of the International Earth Rotation Service (IERS) was initiated at the IERS Workshop, Paris 1996, that endorsed corresponding recommendations (for details see Reigber & Feissel 1997), and the discussions that followed at the next IERS Workshop at Potsdam, 1998, namely during the 'IERS Retreat'. Shortly afterwards, the new IERS Terms of Reference were formulated and endorsed in March 1999 by the IERS Directing Board; the document is available on the IERS web site (IERS 1999). The main 'driving force' of the proposed changes was the ever increasing complexity of the service since its establishment in 1988, and the efforts to make it less centralized and even more international, with tasks and responsibilities clearly defined and distributed amongst many institutions all over the world. According to the new Terms of Reference, primary objectives of the IERS are to serve astronomical, geodetic and geophysical communities by providing

- International Celestial Reference System and Frame (ICRS, ICRF);
- International Terrestrial Reference System and Frame (ITRS, ITRF);
- Earth Orientation Parameters (EOP) that define the transformation between the ICRS and ITRS;
- relevant geophysical data (i.e., information on the distribution and motion of the atmosphere, terrestrial and oceanic water, mantle, core ...);
- Conventions (i.e., standards, constants, models, algorithms, software ...).

2. RE-ORGANIZATION OF THE IERS

The re-organization practically began in November 1999, when the Call for Participation was issued, with deadlines December 10, 1999 (for non-binding Letter of Intent) and February 28, 2000 (for the Proposal). The Letters of Intent that arrived before December 10, 1999 were then reviewed by the IERS Directing Board at its meeting in December 1999 (held during the AGU Fall Meeting at San Francisco). It was decided to set up the Proposal Review Committee (PRC) with the task to evaluate all proposals and prepare the corresponding recommendations for the next IERS DB meetings. The list of candidates of PRC was composed, consisting of about 15 knowledgeable scientists, mostly independent both of the IERS DB and proposing organizations. The PRC was then established, under the chair of I. Mueller, at the beginning of 2000. Its first recommendations were delivered and taken into consideration at the IERS DB meeting in Washington (June 2000), and some of them accepted. Nevertheless, there were still several multiple proposals for the same components whose primary scientists were further contacted by the PRC and, in some cases, asked for new joint proposals. These cases were considered and accepted later, so that the IERS DB at its meeting at Frankfurt a.M. (September 2000) was able to take final decisions. Several minor changes of the Terms of Reference were adopted by the 'old' Directing Board at its last meeting in San Francisco (December 2000), mainly reflecting slightly changed structure of the ITRS PC (see below).

3. NEW STRUCTURE OF THE IERS

IERS Terms of Reference define the following components of the new IERS:

Technique Centers (TC) are generally autonomous independent services, cooperating with the IERS. There is typically only one TC per observing technique, and it provides its operational products to the IERS. At the moment, these are the following:

- International VLBI Service (IVS);
- International GPS Service (IGS);
- International Laser Ranging Service (ILRS);
- International DORIS Service (IDS) has not yet been formed, and the technique serves as a Pilot Experiment of the CSTG.

Product Centers (PC) are responsible for the products of the IERS. They are as follows:

- Earth Orientation Product Center, responsible for monitoring long-term orientation parameters, publications for time dissemination and announcements of leap seconds. It is placed at Observatoire de Paris, under the leadership of Daniel Gambis.
- Rapid Service/Prediction Product Center, responsible for providing Earth orientation parameters on a rapid basis, primarily for real-time users. It is placed at U.S. Naval Observatory, Washington D.C., and is headed by Jim Ray.
- Conventions Product Center is responsible for the maintenance of the IERS conventional models, constants and standards. Joint proposal of U.S. Naval Observatory (Washington D.C.) and Bureau International des Poids et Mesures (Sèvres) was accepted, under the guidance of Dennis McCarthy and Gérard Petit.
- International Celestial Reference System Product Center, responsible for the maintenance of ICRS and its realization, ICRF. Joint proposal of Observatoire de Paris and U.S. Naval

Observatory was accepted, both groups being represented by Jean Souchay and Ralph Gaume.

- International Terrestrial Reference System Product Center, responsible for the maintenance of ITRS and its realization, ITRF. This was probably the most difficult IERS component to assign. Two different proposals were received, from Institut Géographique National (Marne-la-Vallée) and from Deutsches Geodätisches ForschungInstitut (Munich). Because the two institutions were not able to submit a joint proposal (as requested by PRC) by the deadline of July 15, 2000, the PRC recommended to adopt a slightly different structure than originally anticipated by the IERS Terms of Reference - one ITRS Product Center and multiple Combination/Analysis Centers, with an option of rotating the PC responsibilities between the C/AC's every four years. New Call for Participation for these components was proposed to be issued. The IERS Directing Board at its meeting at Frankfurt decided to slightly modify this scheme; it assigned IGN to become the ITRS Product Center, with Claude Boucher as its representative, and both IGN and DGFI as Combination/Analysis Centers. New Call for Participation will be issued only for additional ITRF Analysis Centers. In the discussion, a possibility of rotating the PC responsibility among C/AC's was also mentioned, but no specific time schedule for the rotation was decided.

- Global Geophysical Fluids Product Center, responsible for providing relevant geophysical data sets and related results. This center, having 7 sub-centers, was established only in 1998, and consequently no new Call for Participation was issued. It is headed by Ben Chao of GSFC.

Combination Research Centers are responsible for the development of combinations from data (or products) coming from different techniques. They are expected to provide their solutions to Analysis Coordinator. There are ten of them (the names of leading scientists are given in brackets):

- AICAS & CTU, Prague (J. Vondrák);
- FGS & DGFI, Munich (D. Angermann);
- FGS & FESG, Munich (M. Rothacher);
- FGS & GIUB, Bonn (A. Nothnagel);
- GFZ, Potsdam (S.Y. Zhu);
- FFI, Kjeller (P.H. Andersen);
- GRGS, Toulouse (R. Biancale);
- IGN, Marne-la-Vallée (P. Sillard);
- JPL, Pasadena (R. Gross);
- IAA, St. Petersburg (Z. Malkin).

Analysis Coordinator is responsible for long-term and internal consistency of the IERS reference frames and other products, for ensuring the appropriate combination of the TC products into a single set of official IERS products and for archiving them. The designated Analysis Coordinator is Markus Rothacher but, because of his new position and teaching responsibilities at the Technical University Munich, he will be able to take over his new IERS office only in summer 2001. Therefore Tom Herring (MIT) was appointed as the interim Analysis Coordinator.

Central Bureau is the administrative center of the IERS; it is responsible for the general management (according to the directives given by Directing Board), for coordinating the activities, IERS publications, archiving the products and it also serves as its communication center with the users. It is placed at Bundesamt für Kartographie und Geodäsie at Frankfurt a.M., under the direction of Bernd Richter.

Directing Board exercises general control over the activities of the IERS, its chairperson (elected by the Board from its members) is the official representative of the IERS to external organizations. It consists of two representatives of each of the Technique Centers, one for each of the Product Centers, one for all Combination Research Centers together, a representative of the Central Bureau, Analysis Coordinator, and representatives of the IAU, IAG/IUGG and FAGS. The composition of the new DB is as follows:

TC:

- IVS – Chopo Ma (NASA/GSFC), Axel Nothnagel (University Bonn);
- IGS – Carine Bruyninx (Royal Observatory Brussels), Robert Weber (TU Vienna);
- ILRS – Ron Noomen (Delft University), Peter Shelus (University of Texas);
- IDS – Pascal Willis (IGN), status of non-voting observer;

PC:

- Earth Orientation – Daniel Gambis (OP);
- Rapid Service – Jim Ray (USNO);
- Conventions – Dennis McCarthy (USNO)/Gérard Petit (BIPM);
- ICRS – Jean Souchay (OP)/R. Gaume (USNO);
- ITRS – Claude Boucher (IGN);
- GGFC – Ben Chao (NASA/GSFC);

Combinations: Sheng Yuan Zhu (GFZ Potsdam);

Central Bureau: Bernd Richter (BKG Frankfurt);

Analysis Coordinator: Tom Herring (MIT Cambridge)/Markus Rothacher (TU Munich);

IAU: Jan Vondrák (Astron. Inst. Prague);

IAG/IUGG: Clark Wilson (University of Texas);

FAGS: David Pugh (Inst. of the Oceanographic Science, Wormley).

Three candidates for the election of the new IERS DB chairperson were proposed by the Nominating Committee (I. Mueller, B. Schutz, F. Arias) assigned by the DB, and J. Vondrák was elected as the new chairman in December 2000.

4. CONCLUSIONS

Most of the new IERS components were operational by the end of 2000, and the new IERS as a whole will be fully operational in summer 2001. The necessity of a data center with mirror sites appeared during the discussions, and it will also be discussed and created in near future.

5. REFERENCES

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CONTRIBUTION OF LLR TO THE REFERENCE SYSTEMS AND PRECESSION

J. CHAPRONT, M. CHAPRONT-TOUZÉ and G. FRANCOU
DANOF/UMR 8630-CNRS
Observatoire de Paris, 61, avenue de l'Observatoire 75014 Paris
e-mail: jean.chapront@obspm.fr

ABSTRACT. The present work is based on the principles and methods which are described in (Chapront et al., 1999b). Since 1998 a new analysis of Lunar Laser Ranging (LLR) observations has been performed from January 1972 till May 2000. Taking advantage of several recent improvements in the reduction of LLR observations, new values of the lunar and solar parameters have been determined, as well as the orientation of the celestial axes and a correction to the IAU 1976 value of the precession constant in longitude. The LLR determinations of the offsets of Celestial Ephemeris Pole and the correction to the precession constant converge now to the VLBI best estimates.

1. THE NEW SOLUTION S2000

An analysis of LLR observations has been performed using the lunar theory ELP 2000-96 and the completed Moons' theory of the lunar libration. The previous analysis covered the time span January 1972 till March 1998. It is described in (Chapront et al., 1999b) which is referred below as A&A99. We shall name here the corresponding solution by S1998. In the present work, we send back to this paper for the presentation of the principles of the analysis, the discussion of the methods and several tables of results for comparison. A new analysis has been performed using the LLR observations of McDonald and CERGA until May 2000. Several improvements have been introduced in the lunar ephemeredes (circulation and libration), in the program of reduction, and the statistical treatment of the datas. The corresponding solution is named S2000. The main improvements are:

Numerical precision: The lunar ephemeredes are computed with a time argument referred to J2000.0 instead of using entire julian dates that produces in the computed distances round off errors of a few millimeters.

Libration: As described in (Chapront et al., 1999a) the analytical solution for the lunar libration has been improved by numerical complements on the basis of frequency analysis of the differences between a JPL numerical integration (DE245) and the analytical solution. As mentioned in the *Journes 1998* (Chapront et al., 1998), the quality of the libration model plays an important role in the determination of the parameters and the dispersion of the residuals. In S2000 the frequency analysis is more precise than earlier. Various numerical experiments and fits with different JPL integrations (DE403, DE405) have been done, making sure that our numerical complements produce a noise which is now below the millimeter.

Distribution of the weights: In S1998, we divided the whole set of observations in 5 groups which correspond to the various stations and instruments. The nomenclature is given in Table 1 - Column 1. A weight for each group arises from the least square fit. In S2000, we have divided the original groups above in sub-groups covering 4 years each. A more appropriate weight arises that takes into account the increasing quality of the residuals O - C (Observation minus Computation) with the time. A more refined cutting out of the groups can be performed but we observed that it does not modify sensibly the determination of the unknowns and our conclusions.
Nutation: We skip now from the old model ZMOA 90 used in S1998 to the present nutation model of the IERS Conventions 1996 (McCarthy, 1996).
Enlargement of the time span: We take benefit in S2000 of 2 additional years of highly precise observations. In order to illustrate the increase of precision, note for example that the rms in distance is of 2.5 cm, during the last 6 months of observations made at CERGA.

2. THE RESIDUALS

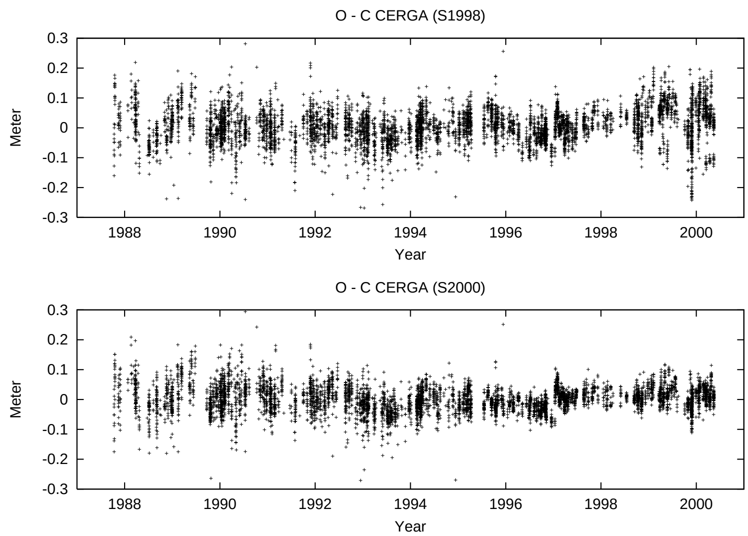
In order to appreciate the global gain of precision of our new solution S2000 compared to S1998, we show on Fig. 1 the residuals in distance O - C for the observations made at CERGA between 1987 and 2000, for the two solutions. We note for S2000 a dispersion less important on the whole interval and a better fit for the two last years (in case of S1998 the solution is extrapolated). A comparable statement can be done with the observations made at McDonald.

Table 1. Distribution of the residuals (rms) in centimeter. N is the number of normal points in each sub-group.
(1) S1998 (Table 3 in A&A99)
(2) S2000 : 1 sub-group per observatory. Improvements : numerical precision and libration
(3) S2000 : Improvements : same as (2) with sub-groups + a new nutation model + an extension of the time interval until May 2000.

OBSERVATORY *and instrument*	Sub-group	rms (1)	rms (2)	Sub-group	rms (3)	N
McDONALD	1972-1986	34.7	34.5	1972-1975	43.0	1487
2.70 m and MLRS1				1976-1979	27.3	1029
				1980-1986	29.5	992
CERGA, *Rubis*	1984-1986	18.2	18.8	1984-1986	19.6	1166
HALEAKALA	1987-1990	11.1	8.0	1987-1990	7.9	455
McDONALD	1987-1998	5.0	3.8	1987-1991	5.6	230
MLRS2				1991-1995	4.3	581
				1995-2000	3.4	1621
CERGA	1987-1998	4.8	3.8	1987-1991	5.1	1570
Yag				1991-1995	3.8	2040
				1995-2000	3.1	2754

In Table 1 we put in evidence quantitatively the increase of precision on the residuals (rms on the distance). Clearly, in each group, the recent observations have a greater weight than the oldest one. Hence the unknowns are sensibly determined with a better accuracy.

Fig 1. LLR residuals at CERGA for the two solutions S1998 and S2000.



3. THE FITS

We recall briefly the list of parameters which are fitted in the solutions (all the angles and mean motions are referred to J2000.0):

The geocentric lunar orbital parameters $W_1^{(0)}$, $W_2^{(0)}$, $W_3^{(0)}$ (values of the mean mean longitude, mean longitudes of perigee and node), $\nu = W_1^{(1)}$, Γ, E (sidereal mean motion, constants for inclination and eccentricity).

The heliocentric orbital parameters of the Earth-Moon barycenter $T^{(0)}$,$\varpi'^{(0)}$ (values of the mean mean longitude and longitude of the perihelion), n', e' (sidereal mean motion and eccentricity).

The bias parameters $\Delta W_1^{(2)}$, $\Delta W_2^{(1)}$, $\Delta W_3^{(1)}$ (observed corrections to the computed coefficient of the quadratic term of the lunar mean longitude, and the computed mean motions of perigee and node). $\Delta W_1^{(2)}$ yields an observed value of $W_1^{(2,T)}$, the tidal part of the coefficient of the quadratic term of the mean longitude (half tidal secular acceleration).

The 6 free libration parameters that we shall not consider here.

The position angles ϕ, ϵ and ψ with respect to several system of axes. Fig. 2 illustrates the relative positions of the various systems. The nomenclature is given below (see A&A99 for the basic definitions of the reference).

4. POSITION ANGLES OF THE INERTIAL MEAN ECLIPTIC J2000.

We recall here the definition of the position angles of the inertial mean ecliptic J2000.0 with respect to various 'equatorial' reference systems (R). R stands either for $ICRS$ (International Celestial Reference System), $MCEP$ (Reference linked to the Mean Celestial Ephemeris Pole) or JPL (Reference system defined by a JPL numerical integration such as DE200, DE403 or DE405). We set:

$\gamma_{2000}^I(R)$: Ascending node of inertial mean ecliptic J2000.0 on the equator of R

$\epsilon(R)$: Inclination of the inertial mean ecliptic to the equator of R

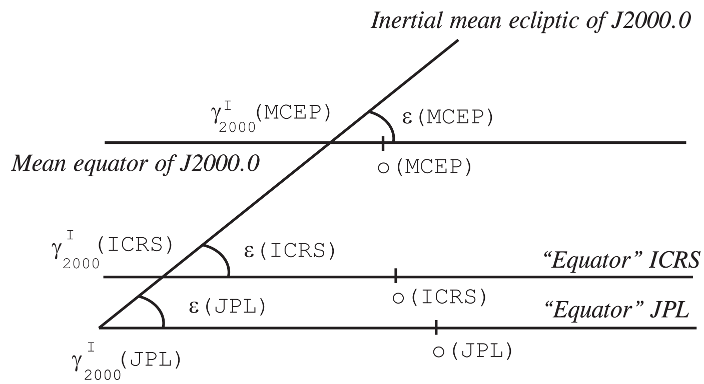
$o(R)$: Origin of right ascensions in the equator of R

$\phi(R)$: Arc $o(R)\gamma_{2000}^I(R)$

$\psi(R) = \gamma_{2000}^I(ICRS)\gamma_{2000}^I(R)$

Δp: Correction to the IAU 1976 value of the precession constant.

Fig 2. Relative positions of the mean inertial ecliptic with respect to ICRS, MCEP, and JPL



Two solutions are investigated, Sol. 1 and Sol. 2, to take respectively into consideration the reference systems $MCEP$ or $ICRS$. In the reduction of LLR observations one has to transform the terrestrial coordinates of the Moon to celestial ones. Such a transformation is done with the knowledge of the daily values of the polar motion x_P, y_P, the differences $UT1 - UTC$ and a precession nutation matrix $P \times N$ which rotates the celestial instantaneous axes to a J2000.0 fixed celestial 'equatorial' system of axes.

In Sol. 1, the $P \times N$ is provided by an analytical solution. In P enters a correction Δp to the value IAU76 of the precession constant. The reference plane is the mean equator of CEP for J2000.0. The corresponding system of axes is $MCEP$.

In Sol. 2, $P \times N$ is computed via a system of corrections to a conventional set of values for the nutations in longitude and obliquity, i.e. $d\psi$ and $d\epsilon$, which are daily values of IERS (series C04). The corresponding system is $ICRS$.

5. ORBITAL MOTION

Table 2. Corrections to values of orbital parameters fit to DE200. Units: $''$/cy for ν and n', and arcsecond for the other variables.

Variable	Sol.1	Sol.2
$W_1^{(0)}$	-0.12013±0.00016	-0.07671±0.00012
$W_2^{(0)}$	-0.06569±0.00016	-0.02224±0.00012
$W_3^{(0)}$	-0.11099±0.00061	-0.06833±0.00061
ν	-0.39430±0.00079	-0.39644±0.00063
Γ	0.00078±0.00002	0.00082±0.00002
E	0.00019±0.00000	0.00019±0.00000
$T^{(0)}$	-0.07540±0.00016	-0.03191±0.00012
$\varpi'^{(0)}$	-0.05717±0.00029	-0.01324±0.00025
n'	0.03257±0.00076	0.03075±0.00064
e'	0.00002±0.00000	0.00001±0.00000

The main results are summarized in two tables:
Table 2 shows the corrections to the orbital parameters fit to DE200 and corresponds rigorously to the same quantities listed in Table 4 of A&A99. The ascending node of the inertial mean ecliptic of J2000.0 is $\gamma^I_{2000}(MCEP)$ in Sol.1 and $\gamma^I_{2000}(ICRS)$ in Sol.2. The differences

between the angles $W_1^{(0)}$, $W_2^{(0)}$, $W_3^{(0)}$ in Sol.1 and Sol.2 are close to the value $\psi(MCEP) = \gamma^I_{2000}(ICRS)\gamma^I_{2000}(MCEP) = 0.0435''$ given below (see Table 4). For all the quantities, the differences are in general less than 3σ.

Table 3. Fitted value of the tidal part of the quadratic term of the mean longitude (in $''/\text{cy}^2$) and observed corrections to the mean motions of perigee and node (in $''/\text{cy}$).

Variable	Sol.1	Sol.2
$W_1^{(2,T)}$	-12.91852±0.00168	-12.91782±0.00164
$\Delta W_2^{(1)}$	0.03413±0.00086	0.03402±0.00084
$\Delta W_3^{(1)}$	-0.29740±0.01052	-0.29919±0.01050

Table 3 shows the bias on the variables $W_1^{(2,T)}$, $W_2^{(1)}$, $W_3^{(1)}$. It corresponds to Table 6 in A&A99. It is interesting to note the remarkable vicinity of all the three quantities determined independently in the two solutions; it was not the case in A&A99 for $W_1^{(2,T)}$ and $W_3^{(1)}$. Here the choice of the new nutation model and the distribution of weights play an important role in this improvement.

6. ORIENTATION OF THE CELESTIAL AXES AND PRECESSION CONSTANT

We gather in Table 4 our new determinations of the position angles ϕ, ϵ and ψ. It corresponds to Table 11 in A&A99. The angle ψ is obtained through the 2 different computations of the mean mean longitude of the Moon W_1 which is evaluated on one side in the $ICRS$, and on the other side in the reference system R ($MCEP$, or a JPL numerical integration). Hence, we compute the difference: $\psi(R) = W_1(ICRS) - W_1(R)$

$$\psi(R) = W_1^{(0)}(ICRS) - W_1^{(0)}(R) + [W_1^{(1)}(ICRS) - W_1^{(1)}(R)]\theta + [W_1^{(2)}(ICRS) - W_1^{(2)}(R)]\theta^2$$

θ is the time in centuries elapsed since J2000.0. The mean epoch for $MCEP$ arises directly from the least square fit; it is the weighted time tied to the distribution of weights of the sub-groups. For the JPL ephemeredes, the mean epochs are mentioned in the literature and correspond to the JPL's fits.

In $\psi(R)$ the linear term corresponds to the difference of 2 sidereal mean motions in the 2 systems: $\nu(ICRS) - \nu(R)$; the quadratic term corresponds to the difference in the tidal parts of the acceleration of the mean mean longitudes in the 2 systems: $W_1^{(2,T)}(ICRS) - W_1^{(2,T)}(R)$. For DE403 a value of ψ given in A&A99 ($0.0069''$) has been sensibly modified in S2000 ($0.0048''$). The ψ function for DE405 is:

$$\psi(405) = 0.00913'' + 0.02476''/\text{cy}\,\theta - 0.00439''/\text{cy}^2\theta^2$$

It is worth noticing that the difference of the tidal secular acceleration in DE405 and our determination is less than $0.005''/\text{cy}^2$ which gives an idea of the present uncertainty for this fundamental lunar parameter. DE405 is oriented onto $ICRS$ (Standish, 1998). The epoch is deduced from (Standish, 2000): the reference system tied to the ephemeris is based on VLBI observations (Magellan spacecraft to Venus and Phobos approach to Mars) made in between 1989 and 1994; hence, we have arbitrarily chosen 1990 Jan 1.

Using the quantities ϕ and ψ of Table 4, we make the projection on the $ICRS$ "equator" of the origin of right ascension $o(405)$ which is distant from $o(ICRS)$ by less than one mas. We find: $o(ICRS)o(405) = 0.9$mas (epoch, 1990 Jan 1). In case DE403 we obtain $o(ICRS)o(403) = 1.9$ mas (Epoch, 1985 Jan 1).

In Table 5, we bring together the best estimates for the offsets of Celestial Ephemeris Pole at J2000.0 and for the correction to the precession constant obtain by VLBI (Fukushima, 2000) and our LLR determinations. We note that our last values for Δp, $\Delta\psi sin\epsilon_0$ and $\Delta\epsilon_0$ are significantly different in S2000 (This paper) and S1998 (A&A99). Here again the nutation model and the weight distribution is a deciding factor for the improvement of the solution. Now the values for Δp obtained by LLR and VLBI converge nicely with a separation smaller than 0.2 mas/y.

Table 4. Position angles of the inertial mean ecliptic of J2000.0 with respect to 'equatorial' celestial systems (R). Units: arcsecond.

R	$\epsilon - 23°26'21''$	ϕ	ψ	Mean epoch
$ICRS$	0.41100±0.00005	-0.05545±0.00011		
$MCEP$	0.40564±0.00006	-0.01549±0.00017	0.0435±0.0004	1994 Jun 8
$DE403$	0.40928±0.00000	-0.05294±0.00001	0.0048±0.0004	1985 Jan 1
$DE405$	0.40960±0.00001	-0.05028±0.00001	0.0066±0.0003	1990 Jan 1

Table 5. Correction to the precession in longitude Δp in ''/cy and offsets of Celestial Ephemeris Pole at J2000.0, $\Delta\psi sin\epsilon_0$ and $\Delta\epsilon_0$, in arcsecond.

Method	Source	Δp	$\Delta\psi sin\epsilon_0$	$\Delta\epsilon_0$
VLBI	Fukushima	-0.2968±0.0043	-0.0167±0.0005	-0.0049±0.0003
LLR	A&A99	-0.3437±0.0040	-0.0183±0.0004	-0.0056±0.0002
LLR	This paper	-0.3164±0.0027	-0.0173±0.0004	-0.0054±0.0002

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VLBI DATA ANALYSIS CENTER AT MAIN ASTRONOMICAL OBSERVATORY OF NATIONAL ACADEMY OF SCIENCE OF UKRAINE.

S. BOLOTIN
Main Astronomical Observatory, National Academy of Science of Ukraine
Main Astronomical Observatory, Golosiiv, Kiev-127, 03680 Ukraine
e-mail: bolotin@mao.kiev.ua

ABSTRACT. We have described the current state and activities of the VLBI Data Analysis Center at the Main Astronomical Observatory, Kiev. The new generation of the VLBI data analyzing software "SteelBreeze" has been announced.

1. INTRODUCTION

The VLBI Data Analysis Center at Main Astronomical Observatory (AC MAO) was established in 1995. The primary interest of AC MAO is the software developing. It also takes part in data processing of the VLBI observations for IERS and IVS submissions, archiving the observations for local needs, etc.. VLBI data processing has a more than 10 years history at Main Astronomical Observatory:

- 1990: "KIEV-GEODYNAMICS-R" software was developed by Kurjanova, Medvedsky and Yatskiv, 1991. It was capable to make estimations of the Earth Orientation Parameters (EOP), coordinates of stations and sources processing series of selected VLBI experiments; the calculations was conducted on OS 360/370 type computer.

- 1992: It was initiated the developing of the *SteelBreeze ver.*1 software. The software was developed totally independent from previous one. Originally, it was created for MS DOS operating system, and then was migrated to Windows 3.1. The software was written on Pascal programming language.

- 1994: Direct estimations of the diurnal and semidiurnal polar motion was made from the CONT'94 VLBI observations (Bolotin, 1995). The polar coordinates (dX_p and dY_p) and $d(UT1 - UTC)$ have been estimated on the 14-days interval with about 5 min resolution.

- 1995: First submission of the "GAOUA R" solution to IERS. There were RSC, SSC and EOP estimations based on 177707 group delays in this solution.

- 1998: Started developing of the *SteelBreeze ver.*2 software. The new version of the software reflects major changes in technique of developing (e.g., *SteelBreeze ver.*2 written on the $C++$ language, running on Linux, Solaris, FreeBSD and other operating systems, uses Qt widget library for graphics user interface).

- 1999: AC MAO acts as an IVS Analysis Center.

- 2000: The source tree of the *SteelBreeze ver.*2 software was realized for public access under the GNU public license.

2. HARDWARE EQUIPMENT

All data analysis and software developing are executing on an Intel Pentium II 400MHz based computer with 192M RAM. It has a SCSI interface connected to 3 hard disk drives with 9, 4 and 6 Gb capacity (it is allowable up to 15 units). Main Astronomical Observatory has a 33.6Kb Internet connection. The computer of AC MAO is operated by the Linux/GNU system. Despite of so humble hardware equipment, it is possible to make data processing of almost all available VLBI observations. The main limitation in hardware is a narrow bandwidth of an Internet connection, which make impossible to act as an Operation Center.

3. STEELBREEZE VER.1 SOFTWARE

Features. *SteelBreeze* software was developed as a tool for geodetic VLBI data analyzing. It makes Least Square estimation of different geodynamical parameters with the Square Root Information Filter (SRIF) algorithm (see Biermann, 1977). The SRIF is using the Householder's transformation for matrix triangularization which makes it smart and insensitive for computer roundoff. The SRIF also makes possible to introduce the stochastic model for parameter estimation. *SteelBreeze* originally was written on the Pascal programming language with Object Oriented Design.

The software imports geodetic VLBI observations in different formats (NGS cards, MarkIII databases, VDB binaries). It stores the observations in its own inner binary format as well as for catalogues of radio sources, stations of observations, EOP, ephemerides, and some other data sets.

It analyses the VLBI data (time delay) of single and multiple set of sessions. The time delay is modeled according to the IERS Conventions 1996, plus additional models (tectonic plate motions, nutation models, wet and hydrostatic zenith delay, mapping function, etc). The software makes estimations of the following parameters: Earth orientation parameters, coordinates and velocities of selected set of stations, coordinates of selected set of radio sources, Love numbers, clock function and wet zenith delay.

SRIF algorithm allows to make estimations of unbiased parameters as well as stochastic ones. In the software each estimated parameter can be considered of the following type:

- global parameter: unbiased estimation for all set of selected sessions (typically useful for source and station coordinates estimating, etc.);

- local parameter: unbiased estimation at each session, the estimates on different sessions are considered to be independent (e.g., EOP);

- local parameter with time propagation: unbiased estimation at each session, the estimates on adjanced sessions are dependent according to a given rule;

- stochastic parameter: a behavior of estimated parameter is assumed varying from time to time with a given rule (it is implemented random walk process in the software), this type is useful for estimating of the clock parameters and the wet zenith delays;

- stochastic parameter with time propagation: the same as above, but adjanced estimations for different sessions are tied with the same rule.

The GAOUA 2000 R 01 solution. As an example of data processing using *SteelBreeze* software we present the GAOUA 2000 R 01 solution. The group delay data were acquired on different VLBI networks since the 6-th of January, 1998 till the 28-th of December 1999. In total 288583 dual frequency Mark III group delays from 176 sessions were processed.

The initial values of station coordinates were taken from ITRF94. The initial values of station velocities were modeled according to plate tectonic motion NNR-NUVEL1A model. The origin of the obtained terrestrial reference frame was linked to ITRF94 by requiring that the vector sum of the adjusted station coordinates be equal to the corresponding sum for a set of 15 stations. The solution SSC(GAOUA) 00 R 01 consists of the set of coordinates for 26 stations.

The initial values of the radio sources coordinates were taken from solution RSC(WGRF) 95 R 01. The right ascension origin of the obtained celestial reference frame was defined by setting the sum of right ascensions of adjusted sources equal to the corresponding sum of the sources from RSC(WGRF) 95 R 01 solution for a set of 60 radio sources. The solution RSC(GAOUA) 00 R 01 consists of the set of coordinates of 191 radio sources.

The IAU 1980 Nutation model and initial values of EOP(IERS) 97 C 04 were applied. The polar motion and UT1 were corrected for diurnal and semidiurnal tidal variations according to Herring and Dong, 1993.

The orientation between terrestrial and celestial reference frames was defined by EOP from EOP(IERS) 97 C 04 solution for the reference date December 28, 1999.

Tropospheric refraction in the local zenith direction caused by hydrostatic and water vapor components of the neutral atmosphere were modeled according to Saastamoinen, 1972. The zenith delays have been mapped to line of sight elevations with the MTT mapping function of Herring, 1992 for both hydrostatic and wet components.

Calibration data collected at each observing site were applied to correct for variations in the electrical length of the cables between the radio receivers and the data acquision systems.

The wet zenith delay and clock offset were estimated as stochastic processes and were modeled as random walk with following magnitudes of power spectral density of ruled white noise: for clock offset $20^2\ ps^2/hour$, and for wet component of tropospheric delay $1.0cm^2/hour$.

The weighted rms post-fit residuals of this solution are 38.4 ps.

High frequency oscillation in the EOP. The second example of data processing is focused on estimating of the polar coordinates and the Earth rotation with a high time resolution. For these purposes we processed the observations on the NASA-R&D network on the period of January 11-26, 1994. These observations was carried out under the terms of the extensive VLBI/GPS campaign, when VLBI stations made nearly continuous series of observations.

The procedure of data analyzing was same as for the previous example with the exception: coordinates of sources and stations was fixed, the wet zenith delay and the clock offset were estimated as stochastic parameters, polar motion and Earth rotation were estimated as stochastic parameters with time propagation to ensure continuity in their behavior. The magnitudes of power spectral density of ruled white noise of estimated parameters were assumed to:

– for clock model: $18^2 ps^2/hour$;

– for wet component of tropospheric delay: $1.0cm^2/hour$;

– for $d(UT1-UTC)$: $0.012^2 mts^2/hour$;

– for dX and dY: $0.16^2 mas^2/hour$.

This solution based on the 27975 group delays. 6279 estimations of the polar motion and the Earth rotation were obtained. The weighted rms post-fit residuals of this solution are 10.2 ps.

The estimated values of the Earth rotation variations are shown on the figure 1. The comparing between estimations and Herring and Dong model is present on the figure 2.

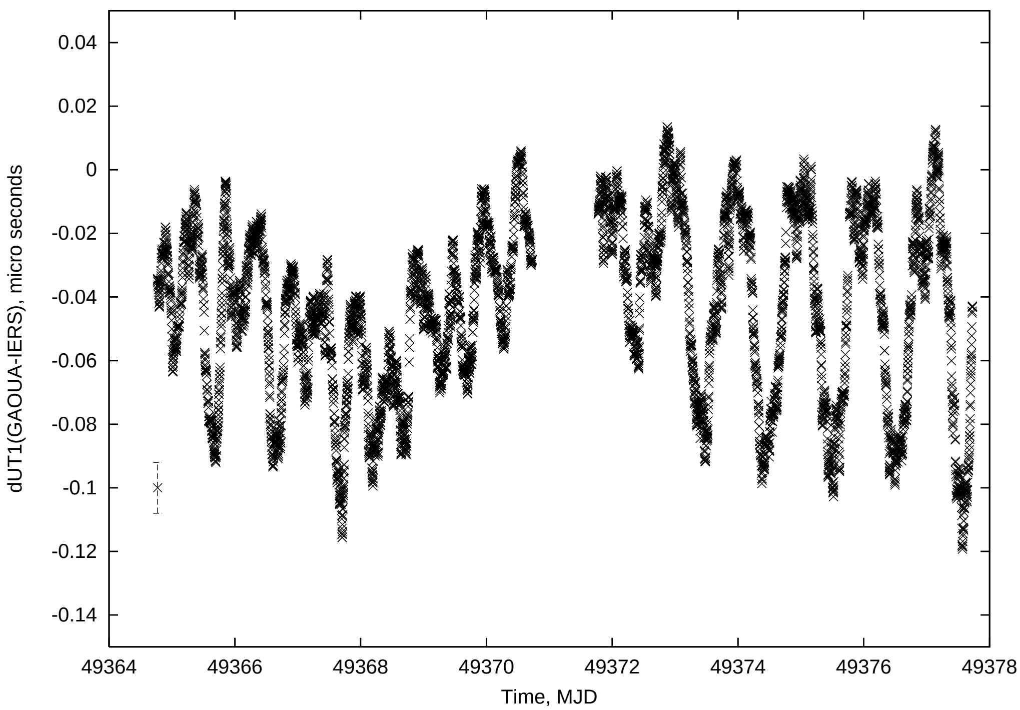

Figure 1: Estimated variations of the Earth rotation, $d(UT1-UTC)$.

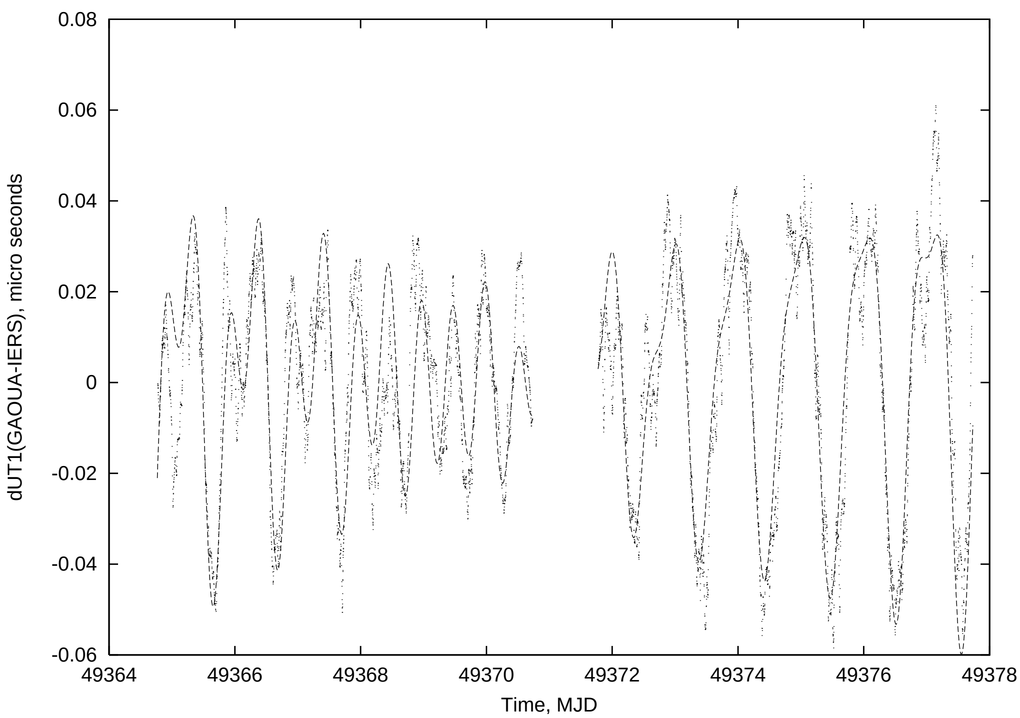

Figure 2: Subdaily variations in $d(UT1-UTC)$. The dots are the subdaily variations of Earth rotation with eliminated long periods. The dotted line is the variations in $d(UT1-UTC)$ according to Herring and Dong, 1993.

4. STEELBREEZE VER.2 SOFTWARE

Unfortunately, the *SteelBreeze ver.1* software has a set of limitations: it was written on the non-portable programming language, its design does not allow to extend it on other geodynamical techniques, there are inner limitations in capacities of databases, which may cause to fail processing sessions with large set of observations (more then 15535). Due to these reasons in 1998 the developing of the *SteelBreeze ver.1* software was stopped and the new generation of the software was created. The new *SteelBreeze ver.2* software inherits all algorithms from previous version and expands them to overcome the limitations. However, there are some changes.

The changes in the data analysis. In new version the routine of data processing is carried out for selected time interval and does not depend on intervals of sessions; it makes possible to process sessions for overlapped time interval. The algorithms of database manipulation are improved. Added new type of estimated parameters: arc; it is determined on selected time interval and does not depend on session. The graphic user interface is improved.

Features of the software developing. The difference between version 1 and 2 lies mostly in software developing technique. The *SteelBreeze ver.2* software is written on C++. It is executing on computers with different operating systems (Linux, FreeBSD, Solaris, etc.) and architecture (Intel, SPARCstation). It uses Qt user interface library. It is distributed under the GNU public license and its sources are available on the following URL:

$$http://sourceforge.net/projects/steelbreeze$$

for public access.

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LONGITUDE ORIGINS ON MOVING EQUATOR II: EFFECTS OF NUTATION

T. FUKUSHIMA
National Astronomical Observatory
2-21-1, Ohsawa, Mitaka, Tokyo 181-8588, Japan
e-mail: Toshio.Fukushima@nao.ac.jp

ABSTRACT. We obtained an explicit solution of s, the angle specifying the non-rotating origin (NRO) (Guinot 1979), for the pole uniformly rotating on a circle around an arbitrary fixed direction. Thanks to the obtained formula, we derived an approximate expression of its correction, Δs, due to the fast nutational motion of the pole by ignoring the slow precessional motion. By adopting the IAU 1980 nutation series (Seidelmann 1980) and combining the result with the previous solution for the precessional motion of the Earth's pole (Fukushima 2000, hereafter cited as Paper I), we developed a more precise expression of the global motion of the Celestial Ephemeris Origin (CEO). The current speed of global rotation of CEO amounts to $-4.149\ 688\ 1''$/yr where the contribution of the nutation is small as -38.4μas/yr but non-negligible. The negative sign shows that CEO rotates clockwise with respect to the inertial frame when viewed from the north pole. The long periodic motion of CEO is of the amplitude of the obliquity of ecliptic, around 23.5 degree, and of the period of precession, around 25800 yr. While the effect of nutation on the periodic motion of CEO looks like a series of mixed secular terms, which is simply proportional to the nutation in longitude and is of the order of some tens mas/yr.

1. INTRODUCTION

At its 24th General Assembly, the IAU passed a number of resolutions concerning the fundamental astronomy (IAU 2000). Among them, the resolution B1.8 recommends astronomers to adopt Guinot's non-rotating origin (Guinot 1979) as a new longitude origin for celestial and terrestrial reference frames. The new origins were named the Celestial Ephemeris Origin (CEO) for the celestial reference frame and the Terrestrial Ephemeris Origin (TEO) for the celestial reference frame, respectively. The aim of the replacement of the equinox by CEO is to exclude the effects of planetary precession as much as possible from the expression of right ascensions of stars and quasars. Namely, the introduction of the concept of NRO is meant to separate the effects of Earth's rotational and orbital motions, which are mixed in the current origin, the equinox, by way of the combination of the luni-solar and planetary parts of precession and nutation (Capitaine *et al.* 2000).

Of course, if for this purpose only, there are some other candidates such as (1) Σ, the point satisfying the condition ΣN = XN, (2) K, the intersection with respect to the fixed x-z plane, or (3) H, the foot of X onto the equator (Kovalevsky and McCarthy 1998). Here X and N represents the fixed x-axis and the node, the intersection of the moving and fixed equators, respectively.

See Figure 1.

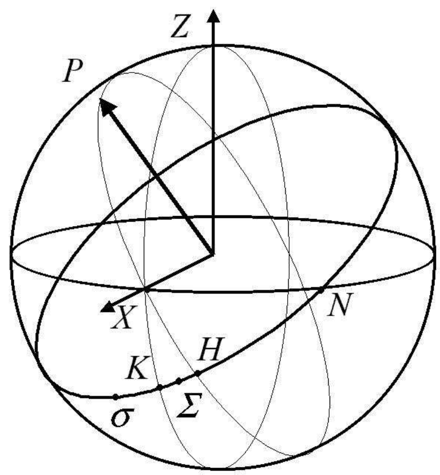

Figure 1: Longitude Origins on a Moving Equator

Note: We illustrate some longitude origins on a moving equator; N, H, Σ, K, and σ. Here N is the node defined as the intersection with the x-y plane, H is the foot of X (the x-axis) onto the moving equator, Σ is the point satisfying the condition $\Sigma\mathrm{N} = \mathrm{XN}$, K is the intersection with the x-z plane, and σ is Guinot's NRO. Note that P denotes a moving pole defining the equator.

The most important property of NRO which discriminates it from the other longitude origins is its local non-rotation with respect to the instantaneous pole. Namely, NRO is defined as the point on the moving equator so as to have no component of instantaneous rotation around the pole whatever motion the pole takes (Guinot 1979). In other words, NRO at $t + dt$ is defined such that its foot on the equator at t coincides with NRO at t. See Figure 2. However, this property never assures that NRO does not rotate globally with respect to the inertial frame. In fact, we have shown that CEO, which is a realization of NRO, does have a secular component of rotation with respect to the inertial frame (Fukushima 2000, hereafter cited as Paper I). See also Figure 4 later.

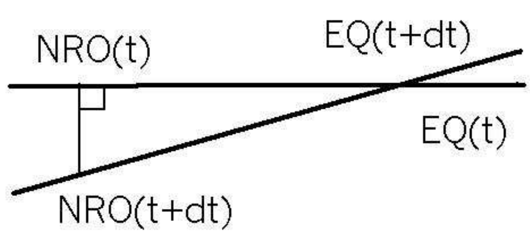

Figure 2: Geometric Definition of NRO

Note: A geometric definition of NRO is that the foot of NRO at $t + dt$ on the equator at t coincides with NRO at t (See the figure). Another definition is that NRO at $t + dt$ is the foot of NRO at t onto the equator at $t + dt$. These two definitions are equivalent with each other since the difference is of higher order and negligible in the limit $dt \to 0$. In Paper I, we examined the global motion of CEO by considering the precession only. Here in this short article, we will investigate the effects of nutation by adopting the IAU 1980 nutation theory (Seidelmann 1980).

2. INTEGRATION OF ANGLE s

The key quantity characterizing NRO is the angle $s \equiv \sigma\Sigma$ (Guinot 1979). In fact, the vector expression of NRO is given in Paper I as

$$\vec{\sigma} = \vec{\Sigma} \cos s - \vec{T} \sin s. \tag{1}$$

Here two orthonormal vectors are expressed by the pole coordinates $\vec{P} = (X, Y, Z)^T$ as

$$\vec{\Sigma} = \begin{pmatrix} 1 - X^2/(1+Z) \\ -XY/(1+Z) \\ -X \end{pmatrix}, \qquad \vec{T} \equiv \vec{P} \times \vec{\Sigma} = \begin{pmatrix} -XY/(1+Z) \\ 1 - Y^2/(1+Z) \\ -Y \end{pmatrix}. \tag{2}$$

Note that the coordinate triad $(\vec{\Sigma}, \vec{T}, \vec{P})$ replaces the classic precession/nutation matrix when Σ is used in place of the equinox (Capitaine *et al.* 2000, Eq.(8)). On the other hand, s is expressed in an integral form of the pole coordinates as

$$s = s_0 - \int_{t_0}^{t} \frac{1}{1+Z} \left(X \frac{\mathrm{d}Y}{\mathrm{d}t} - Y \frac{\mathrm{d}X}{\mathrm{d}t} \right) \mathrm{d}t \tag{3}$$

where s_0 is a certain initial value (Capitaine *et al.* 1986).

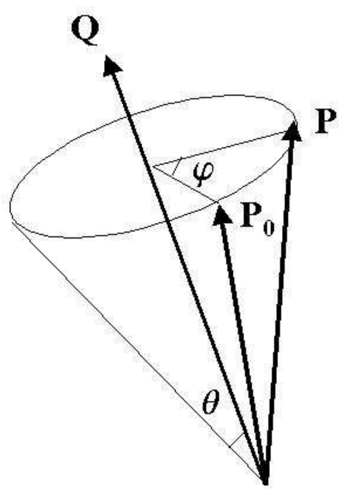

Figure 3: Uniform and Circular Screw Motion

In order to investigate the global motion of NRO, let us obtain the integral explicitly by assuming a simple model of the motion of the pole. As such a model, we consider a uniform and circular screw motion around an arbitrary fixed direction,

$$\vec{Q} = (\sin g \cos F, \sin g \sin F, \cos g)^T. \tag{4}$$

Denote the constant angle of inclination between $\vec{P}$ and $\vec{Q}$ by θ and the rotation angle of $\vec{P}$ around $\vec{Q}$ measured clockwise by φ, respectively. See Figure 3. Then the integral is explicitly obtained as

$$s = -2 \left(\sin^2 \frac{\theta}{2} \right) \varphi + 2 \tan^{-1} \left(\frac{\sin(g/2) \sin(\theta/2) \sin \varphi}{\cos(g/2) \cos(\theta/2) + \sin(g/2) \sin(\theta/2) \cos \varphi} \right) \tag{5}$$

where we chose s_0 such that $s = 0$ when $\varphi = 0$. The first term represents the secular rotation of NRO with respect to the inertial frame. This is because the reference point Σ itself has no secular rotation with respect to the inertial frame (Paper I). It is interesting to see that the secular rate contains no contribution of the direction of axis of screw motion, $\vec{Q}$, but is dependent on the inclination, θ. Note that the NRO is only locally non-rotating with respect to the instantaneous pole. Since the reference axis of NRO, the pole itself, does rotate globally, it is quite natural that the NRO also rotates globally.

This expression reduces to the solution in the case of simplified precessional motion of the Earth's pole already given in Paper I as

$$s_P = 2e\psi - 2\tan^{-1}\left(\frac{e\sin\psi}{1-e+e\cos\psi}\right), \qquad e \equiv \sin^2\left(\frac{\epsilon}{2}\right) \tag{6}$$

if g, θ, φ, and F are chosen such that $g = \theta = \epsilon$, $\varphi = -\psi$, and $F = -\pi/2$. Here ψ denotes the accumulated precession in longitude, which we assumed to be a linear function of time, while ϵ denotes the obliquity of ecliptic, which we assumed to be a constant. Using the latest IAU best estimate of astronomical constants (Fukushima 2001) giving $p = (5\ 028.790 \pm 0.005)\ ''/\mathrm{cy}$ and $\epsilon = (84\ 381.405\ 9 \pm 0.000\ 3)\ ''$, we evaluate the rate of the secular term at J2000.0 as

$$2ep = 4.149\ 649\ 7\ ''/\mathrm{yr} \pm 0.4\ \mu\mathrm{as/yr}. \tag{7}$$

Note that, however, for a short time span, say within 1000 years centered at the present, the above expression of s_P is expanded as a cubic and higher power series of ψ as

$$\begin{aligned} s_P &= \frac{e(1-3e+2e^2)}{3}\psi^3 - \frac{e\left(1-15e+50e^2-60e^3+24e^4\right)}{60}\psi^5 + \cdots \\ &\approx 36.160\ 62\ \mathrm{mas}\ T^3 - 0.56\ \mu\mathrm{as}\ T^5 \end{aligned} \tag{8}$$

where T is the time from J2000.0 measured in Julian century. This is because the linear term, $2e\psi$, cancels with the linear trend of the periodic term when ψ is small.

Now let us consider the effect of nutation on the expression of s. Capitaine (1990) made a similar study. However, her computation was not for s directly but for $\delta\theta \equiv XY/2|_{t_0} - s$.

To simplify the situation, we will ignore the precessional motion in evaluating the nutational effects. Also we neglect the second and higher-order effects of nutation, i.e. the coupling with precession and among nutation terms. More specifically speaking, we assume the mean pole as a fixed direction when computing the effects of nutations. Namely, (1) we regard $\vec{Q}$ as the mean pole, and (2) assume the motion of the true pole $\vec{P}$ around the mean pole $\vec{Q}$ to be a linear combination of uniform and circular screw motion of small amplitudes. The latter motion is no other than the nutation, which is expressed in complex Fourier series as

$$\Delta\epsilon + i\sin\epsilon_0\Delta\psi = \sum_{J=1}^{N}\Delta_J\epsilon\cos\varphi_J + i\sin\epsilon_0\sum_{J=1}^{N}\Delta\psi_J\sin\varphi_J = \sum_{J=1}^{N}\left(\theta_{+J}e^{+i\varphi_J} + \theta_{-J}e^{-i\varphi_J}\right) \tag{9}$$

where N is the number of (real) Fourier terms, which is 106 for the IAU 1980 nutation series (Seidelmann 1980). Here the complex amplitude θ_J are expressed as

$$\theta_{\pm J} = \frac{1}{2}\left(\Delta\epsilon_J \pm \sin\epsilon_0\Delta\psi_J\right). \tag{10}$$

While the nutation angle φ_J is usually expressed as a linear combination of Delauney angles as $\varphi_J = n_J^{(\ell)}\ell + n_J^{(\ell')}\ell' + n_J^{(F)}F + n_J^{(D)}D + n_J^{(\Omega)}\Omega$ where the integer coefficients $n_J^{(\ell)}, n_J^{(\ell')}, n_J^{(F)}, n_J^{(D)}$, and $n_J^{(\Omega)}$ are given in the nutation table.

Noting the smallness of the amplitudes θ_J, we ignore their third and higher powers in the above expression and obtain the approximate correction as

$$\Delta s = \Delta s_0 + \frac{-1}{2}\sum_{J=1}^{N}\left(\theta_{+J}^2 - \theta_{-J}^2\right)\varphi_J + \left(\tan\frac{g}{2}\right)\sum_{J=1}^{N}\left(\theta_{+J} - \theta_{-J}\right)\sin\varphi_J \tag{11}$$

where Δs_0 is a certain integration constant. Here the first term gives again a correction to the secular rotation. The rest terms in the correction are periodic terms in the very long run. However, for a short time span, say within 1000 years centered at the present, these look like mixed secular terms since the common factor is approximated to be a linear function of time as $\tan(g/2) \approx (\psi \sin \epsilon_0)/2$. Thus the above expression is rewritten as

$$\Delta s \approx \Delta s_0 + \Delta s_1 T + \Delta s_M T \Delta\psi \tag{12}$$

where T is again the time from J2000.0 measured in century. Here the coefficients are estimated as

$$\Delta s_1 = \left(\frac{-1}{2}\right) \sin \epsilon_0 \sum_{J=1}^{N} \Delta\epsilon_J \Delta\psi_J \left.\frac{\mathrm{d}\varphi_J}{\mathrm{d}t}\right|_{\mathrm{J2000}} \approx +3.843\ 7\mathrm{mas} \tag{13}$$

$$\Delta s_M = \frac{1}{2} p \sin^2 \epsilon_0 \approx 1.928\ 8 \times 10^{-3} \tag{14}$$

for the IAU 1980 nutation series (Seidelmann 1980). Thus the total rate becomes 4.149 688 1 $''$/yr. Note that the mixed secular terms are simply proportional to the nutation in longitude. This expression is much simpler than that provided in Capitaine (1990).

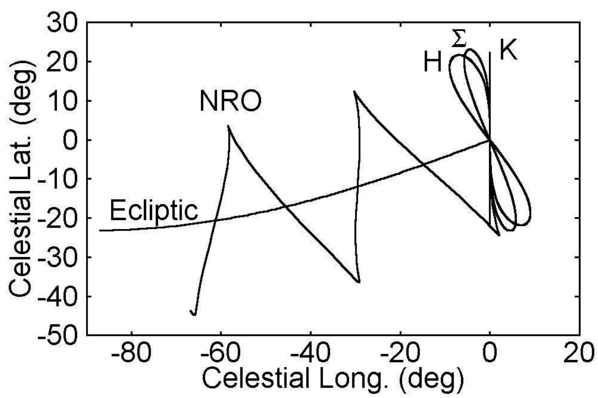

Figure 4: Orbits of Longitude Origins

Note: Shown are the orbits of the origins, K, Σ, H, and NRO, in the equatorial reference frame for the precessional motion of the Earth's pole. Note that K oscillates on the meridian of 0 longitude, Σ and H wander on 8-figure like curves centered at the x-axis, while NRO makes a zigzag motion shifting secularly along the ecliptic.

3. CONCLUSION

We obtained the effect of nutation on the angle s explicitly. Based on the obtained solution, we have redrawn the global orbits of NRO and other longitudes in Figure 4. The nutational corrections are so small that no significant difference from the figure given in Paper I is seen. The long periodic motion of CEO is of the amplitude of the obliquity of ecliptic, around 23.5 degree, and of the period of precession, around 25800 yr. While the effect of nutation on the periodic motion of CEO looks like a series of mixed secular terms, which is simply proportional to the nutation in longitude and is of the order of some tens mas/yr.

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UPDATED VALUES FOR THE EARTH C_{21} AND S_{21} GRAVITY COEFFICIENTS IN THE IERS TERRESTRIAL REFERENCE FRAME

S.M. KUDRYAVTSEV
Sternberg Astronomical Institute of Moscow State University
13, Universitetsky Pr., Moscow, 119899, Russia
e-mail: ksm@sai.msu.ru

EXTENDED ABSTRACT.

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The most recent and complete models of the geopotential including JGM-3 (Tapley et al. 1996) - the model recommended by the current IERS Conventions (McCarthy 1996), are now developed in the IERS Terrestrial Reference Frame (ITRF).

The values for the C_{21} and S_{21} gravity coefficients in the ITRF are rather small, and the attempts to find them by processing the Earth's satellites tracking data have not been quite successful yet. So, another approach has been used to derive these coefficients when developing the most recent and precise Earth's gravity models, such as the JGM series (Nerem et al. 1994, Tapley et al. 1996) and the EGM96 model (Lemoine et al. 1998).

The approach is based on the conclusion that the Earth's mean figure pole apparently closely coincides with the mean rotation pole, when both are averaged over the same multi-year span (Wahr 1987, 1990). Mean values of C_{21} and S_{21} coefficients in the ITRF are calculated by using mean values of the IERS polar motion parameters. For that, the following formulae and constants are presently employed (Nerem et al. 1994):

$$\begin{aligned} \bar{C}_{21}(IERS) &= \sqrt{3}\bar{x}\bar{C}_{20} \,, \\ \bar{S}_{21}(IERS) &= -\sqrt{3}\bar{y}\bar{C}_{20} \,. \end{aligned} \tag{1}$$

where $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are normalized gravity coefficients of degree l and order m; the coordinates $\bar{x}$ and $\bar{y}$ of the Earth's mean rotation axis are those of January 1, 1986 and equal to 0.046 arcsec and 0.294 arcsec, respectively.

The values of the $\bar{C}_{21}(IERS)$ and $\bar{S}_{21}(IERS)$ coefficients, calculated according to (1), are:

$$\begin{aligned} \bar{C}_{21}(IERS) &= -0.187 \times 10^{-9} \,, \\ \bar{S}_{21}(IERS) &= 1.195 \times 10^{-9} \,. \end{aligned} \tag{2}$$

However, the quality of the Earth's gravity field determination increases very rapidly now, and update of the expressions (1) is necessary (in the present form they yield values of $\bar{C}_{21}(IERS)$ and $\bar{S}_{21}(IERS)$ with a computation error of the order of 10^{-11}).

A more rigorous form of (1) can been obtained by calculating the Earth's inertia tensor in two close reference frames (Lambeck 1971) or by employing the general formulae for transformation of gravity coefficients of arbitrary degree and order under rotation (Kudryavtsev 1997):

$$\begin{aligned} \bar{C}_{21}(IERS) &= \sqrt{3}\bar{x}\bar{C}_{20} - \bar{x}\bar{C}_{22} + \bar{y}\bar{S}_{22} \;, \\ \bar{S}_{21}(IERS) &= -\sqrt{3}\bar{y}\bar{C}_{20} - \bar{y}\bar{C}_{22} + \bar{x}\bar{S}_{22} \;. \end{aligned} \tag{3}$$

When using (3), one gets the updated mean values of $\bar{C}_{21}(IERS)$, $\bar{S}_{21}(IERS)$ at the epoch of 1986.0:

$$\begin{aligned} \bar{C}_{21}(IERS) &= -0.190 \times 10^{-9} \;, \\ \bar{S}_{21}(IERS) &= 1.192 \times 10^{-9} \;. \end{aligned} \tag{4}$$

Thus, involving the additional terms in the expressions for calculation of the gravity coefficients does change the significant digits of their values (2) included to the latest models of the geopotential and to the current IERS Conventions (McCarthy 1996).

The proposed corection (3×10^{-12}) to the IERS' mean values for $\bar{C}_{21}$, $\bar{S}_{21}$ is much larger than the adopted cutoff (10^{-13}) for amplitudes of the time-dependent (tidal) terms of these coefficients. Morever, numerical tests prove that a similar error in the coefficients can lead to non-modelled orbital perturbations of certain Earths artificial satellites of amplitude comparable to accuracy of the current tracking measurements. For example, low-altitude satellite STARLETTE should have relevant daily perturbations along the orbit with amplitude of 2 mm; a geosynchronous satellite IUE (resonant with C_{21}, S_{21}) undergoes non-modelled along-orbit perturbations with amplitude of about 1 m and period of 525 days.

As a conclusion we can state that the formulae presently used for computing $\bar{C}_{21}(IERS)$, $\bar{S}_{21}(IERS)$ are insufficient in precision to ensure the values of the coefficients as they are reported by the current IERS Conventions (McCarthy 1996) and employed by the modern models of the geopotential. We propose to update both the formulae and values for the mean $\bar{C}_{21}(IERS)$, $\bar{S}_{21}(IERS)$ gravity coefficients in the new IERS Conventions.

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DEPENDENCE OF THE EFFECTS OF THE NON-PRECESSIONAL EQUINOX MOTION ON PLANETS ORBIT DIMENSIONS

O. BOLOTINA
Main Astronomical Observatory of the National Academy of Sciences of Ukraine
Golosiiv, Kiev-127, 03680, Ukraine
e-mail: olga@mao.kiev.ua

ABSTRACT. Factors hampering the derivation of consistent values for the effect of the non-precessional equinox motion are summarized. An assumption that the effect is not connected with the kinematic peculiarities of stars motion or Galaxy rotation is made. On the basis of results of numerous investigations dependences of the non-precessional equinox motion estimates on the mean distances to the planets or their mean diurnal motion are found.

The effect of the non-precessional equinox motion (NEM) was detected experimentally by S. Newcomb in the 19th century [1, 2]. Later the effect was repeatedly studied on the basis of the observations of the Sun, Moon, inner and minor planets, star proper motion. Summary of all determinations of the NEM effect ($\Delta\dot{A}$) known to us is presented in [6]. Note, that the effect estimations were derived by treating the experimental material in different variants and the data analysis was showed that the agreement in the estimates of the NEM effect is not satisfactorily. That is why the reality of the effect was discussed for long time.

Some facts preventing to obtain conform of the NEM estimates were indicated on the basis of the analysis of different investigations and data [6].

1. Difference of the estimates of $\Delta\dot{A}$ are due to non-eliminate systematic differences in the positional observations of the Solar system bodies.
2. Considerable differences of the estimates of ΔA obtained from the minor planets observations are detected [4].
3. Values of $\Delta\dot{A}$ depend particular on the adopted value of the constant of precession.
4. Different methods are used for studying the NEM effect.
5. Estimates of $\Delta\dot{A}$ differ when the same observations are treated using different motion theories.

On the basis of the results of [3], it is naturally to assume that the estimates of the NEM effect depend on the mean diurnal planets motion (n_i) or its mean distance (r_i). These correlations were found from the data [6]:

$$\Delta\dot{A} = \Delta\dot{A}_0 + k_n' \left(\frac{n_i - n_0}{n_0}\right) + k_n'' \left(\frac{n_i - n_0}{n_0}\right)^2, \tag{1}$$

$$\Delta\dot{A} = \Delta\dot{A}_0 + k'_r\left(\frac{r_i - r_0}{r_0}\right) + k''_r\left(\frac{r_i - r_0}{r_0}\right)^2, \tag{2}$$

where indexes "i" and "0" mark the estimates for "i"-planet and the Sun accordingly, the coefficients k'_n, k''_n, k'_r, k''_r characterize the change of the NEM effect estimates with respect to the mean diurnal planets motion $(n_i - n_0)n_0^{-1}$ and its mean distances $(r_i - r_0)r_0^{-1}$. Note, that formulae (1) and (2) are depended: it follows from the III Kepler law $n^2 r^{-3} = constant$. The estimates of $\Delta\dot{A}$ obtained by combining different objects observations and from the Moon observations were not used for equation solving due to a reason explained in [5].

We would like to point out the following from the analysis of obtained estimates and its errors.

1. Estimates of the effect of the non-precessional equinox motion are different from comparison the observed and calculated positions of the different objects. They has larger absolute magnitude for the planets that are nearer to the Sun.

2. The following equations can be used to characterize the dependence of the NEM effect on the distances

$$\Delta\dot{A} = (-0.053^s/_{cyr.} \pm 0.005^s/_{cyr.}) + (0.025^s/_{cyr.} \pm 0.006^s/_{cyr.})\left(\frac{r_i - r_0}{r_0}\right) \tag{3}$$

and on the mean diurnal motion

$$\begin{aligned}\Delta\dot{A} = (-0.0582^s/_{cyr.} \pm 0.008^s/_{cyr.}) - (0.047^s/_{cyr.} \pm 0.009^s/_{cyr.})\left(\frac{n_i - n_0}{n_0}\right) + \\ (0.015^s/_{cyr.} \pm 0.003^s/_{cyr.})\left(\frac{n_i - n_0}{n_0}\right)^2.\end{aligned} \tag{4}$$

3. It is naturally to assume on the basis of our result about the decrease of $|\Delta\dot{A}|$ to Solar system periphery that the NEM effect is not connected with kinematic peculiarities of star motion or rotation of the Galaxy. However the NEM effect may be the consequence of the disagreement of the theories of motion and planet observations.

We can confirm that the effect of the non-precessional equinox motion is real. But it connection with some physical phenomena or motion of the celestial bodies has not proved yet. It is necessary to perform a subsequent investigation of the effect.

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THE NON ROTATING ORIGIN, QUESTIONS AND ANSWERS

B. GUINOT
Observatoire de Paris
61 av. de l'Observatoire, F 75014 Paris
e-mail: guinot.bernard@wanadoo.fr

1. INTRODUCTION

The following text is an abstract of a poster whose intent was essentially didactic. The IAU Recommendation B1-8 (2000) states that one should use the Non Rotating Origin (NRO) associated to the Celestial Intermediate Pole (CIP) in the Geocentric Celestial Reference System (GCRS) and in the International Terrestrial Reference System (ITRS), under the names of Celestial Ephemeris Origin (CEO) and Terrestrial Ephemeris Origin (TEO). After recalling the kinematical definition of the NRO (see References), the poster answers some questions sometimes raised.

2. QUESTIONS AND ANSWERS

2.1. What are the advantages of the NRO over the equinox ?

- In the GCRS, the NRO depends on the motion of the CIP only.
- In the ITRS, the TEO provides a rigorous definition of the origin of longitudes on the moving equator, which was missing.
- The angle θ between the TEO and the CEO provides a rigorous definition of the sidereal rotation of the Earth and UT1 can be defined as proportional to θ.
- UT1 and θ thus defined are practically insensitive to errors of precession and nutation models (in contrast with sidereal time).
- Transformations between GCRS and ITRS are symmetrical and simple.

2.2. How the position of the NRO is obtained ?

This requires an integral over the path of the pole. The node of the moving equator in that of the reference system (origin Σ_0) being N, the NRO, denoted σ, is given by the composite arc $s = \sigma N - \Sigma_0 N$ with, using the polar coordinates E and d of the pole :

$$s = \int_{t_0}^{t} (\cos d - 1)\dot{E} dt + s_0.$$

That formula, which can be written under several forms (see references), is rigorous and valid for any value of d and E. Apart from a constant offset arising from the arbitrary choice of the integration constant, it leads to the same position of the NRO whichever be the coordinate system chosen for expressing the coordinates of the pole. Although this property is evident because the definition of the NRO is based only on the intrisic properties of the motion of the pole, a trigonometric demonstration is nevertheless provided by the poster.

2.3. One notices secular terms in the expression of quantity s giving the position of the NRO. Does it involve a secular motion of the NRO in the reference system in which it is used ?

Yes, the NRO has a secular motion. It can be easily seen in case of a circular motion of radius d of the pole P by setting the polar origin of the reference system at the center of the trajectory. After a complete revolution of P, the moving equator comes back to its initial position, but the NRO has moved by $2\pi(\cos d - 1)$. More generally, any component of the motion of P in form of a loop generates a secular motion of the NRO.

2.4. If we apply the above result to precession, we obtain an enormous drift of the NRO of about 4.2" per year, in the average. Does not it show that there is an error in the concept of the NRO ?

This drift is quite normal. Using ecliptic coordinates, and using the components of rotation vectors, it is easily shown that this drift is necessary to provide rigorously the sidereal rotation of the Earth. The trajectory of the NRO on the reference sphere has the shape of zig-zags with period 28000 years and extending over about 30 degrees in this period. The mathematical expression of that trajectory is given in the poster.

2.5. Why the usual expression of s does not show the large secular term of precession ?

This is a matter of geometry. In the usual axes of the GCRS, the velocity of the NRO is nearly perpendicular to the equator of the GCRS and its contribution to s is small.

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NOTE

Requests for copies of the poster summarized here can be sent to capitain@danof.obspm.fr. These copies are available by E-mail in rtf format or by photocopies.

A NEW APPROACH TO INTERPRETATION OF THE NON-PRECESSIONAL EQUINOX MOTION

Y.B. KOLESNIK
Institute of Astronomy of the Russian Academy of Sciences
48 Piatnitskaya str., 109017 Moscow, Russia
e-mail: kolesnik@inasan.rssi.ru

About 240000 original observations coverering the period from 1750 to 2000 have been incorporated for investigation of the equinox motion. These have been transformed onto the N70E catalogue system (Kolesnik 1997) rigidly rotated onto Hipparcos frame and compared with the DE405 ephemeris. The Jones-Clemence empirical correction (Clemence 1948) as amended by Morrison has been eliminated from ET-UT differences provided by Stephenson & Morrison (1984). Modified in this way ET-UT series were used in the comparison procedure.

Analysis was based on corrections to the mean longitude of the Earth ΔL_0 determined in short time bins separately from right ascension and declination residuals of the Sun. In the conditional equations for right ascensions the equinox correction was omitted assuming that it will be absorbed in the solution by ΔL_0. The differences between corrections inferred from the declination residuals $(\Delta L_0)_\delta$ and the right ascension residuals $(\Delta L_0)_a$ theoretically are to be considered as a shift of the origin of right ascension system of a reference catalogue with respect to the dynamical equinox ΔE=$(\Delta L_0)_\delta$ - $(\Delta L_0)_a$ at the epoch of a respective time bin. The secular variations of $(\Delta L_0)_a$ and $(\Delta L_0)_\delta$ are plotted in Fig.1. The secular variation of the equinox correction ΔE is presented in Fig.2.

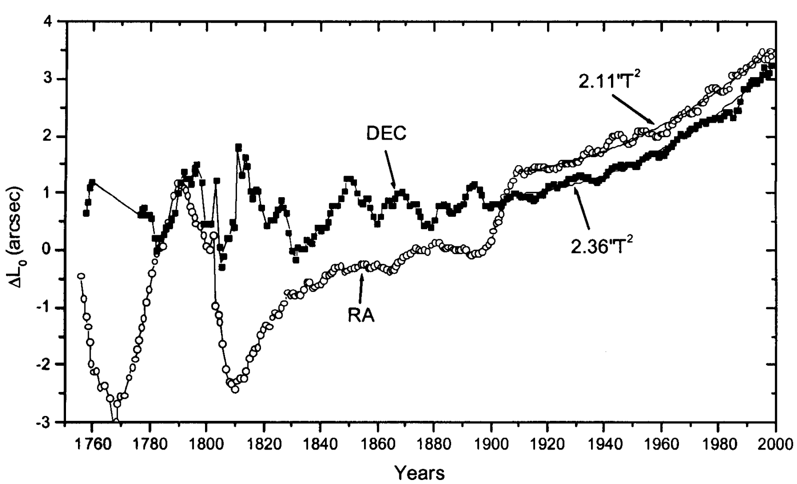

Figure 1: Secular variation of corrections to the longitude of the Sun derived from right ascensions (RA) and declinations (DEC).

As seen from the latter figure in the interval 1750-1895 the apparent equinox motion is estimated as $\Delta E = 0.83'' - 1.20''\,(t - 18.75)$ while in the interval 1910-2000 it is estimated to be zero $\Delta\dot{E} = (0.02 \pm 0.03)''/cy$. The constant term based on 18^{th}-19^{th} observations is close to Newcomb's equinox, which was corrected in the FK3. The linear term is close to Fricke's equinox motion eliminated in the construction of the FK5 system. As seen from Fig.1 the origin of the equinox motion is a systematic discrepansy between observations of the Sun in right ascension and declination that progressively changes with time in the 18^{th} and 19^{th} centuries. On the other hand, all 20^{th} century observations are to be regarded as reliable confirming that the Hipparcos-based system has no residual rotation with respect to dynamical equinox. Both intervals cannot be combined in one solution.

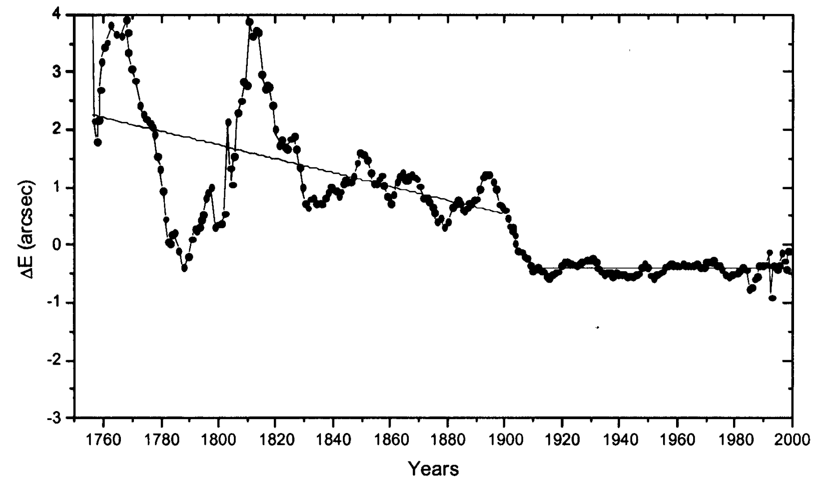

Figure 2: Secular variation of the equinox corrections in the 18^{th} and 19 th centuries as compared with the equinox motion in the 20^{th} century.

Quadratic terms of corrections to the mean longitude of the Sun in the 20^{th} century (the respective approximations are shown as solid lines in Fig.1) are estimated to be $(2.36\pm0.06)''$ as derived from declinations and $(2.11\pm0.07)''$ as derived from right ascensions. In the present context they should be interpreted as caused by the tidal acceleration of the Moon. But with an estimated acceleration of the Moon $\dot{n}$=-26$''$/cy^2 their expected value must be 0.97$''$, while with $\dot{n}$=-19$''$/cy^2, see Kolesnik (2000b) in the present volume, this must be 0.70$''$. The excess in the apparent semi-acceleration of about 1.5$''$/cy^2 should be ascribed to defects of the comparison ephemeris. Analysis of the corrections to the longitudes of Mercury and Venus in the 20^{th} century gives the analogous abnormal quadratic trends nearly proportional to the mean motions of the planets (Kolesnik 2000a). Such an observed effect is consistent with the cosmological theory by C. J. Masreliez (2000) which predicts apparent accelerations of Mercury, Venus and the Earth of nearly the same order if the Hubble time is 14 billion years.

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PRECESSION DERIVED FROM THE HIPPARCOS AND GROUND BASED CATALOGUES

V.V. VITYAZEV
Sobolev Astronomical Institute
St.-Petersburg University, 198904, St.-Petersburg,
Petrodvorets, Bibliotechnaya pl.2, Russia
vityazev@venvi.usr.pu.ru

ABSTRACT. The paper presents a redetermination of the IAU (1976) precession based on the FK5 and improved GC catalogues in combination with the HIPPARCOS catalogue. Derived corrections to the IAU (1976) luni-solar precession are $\Delta p = -0.28 \pm 0.08''/cy$ from FK5 and HIPPARCOS and $\Delta p = -0.34 \pm 0.10''/cy$ from improved GC and HIPPARCOS.

1. METHOD AND RESULTS

The IAU (1976) luni-solar precession constant was derived by Fricke from intensive study of the catalogue of 512 FK4-FK4/Sup distant stars (henceforth F12). At present, when the data from the catalogue HIPPARCOS is available, it is expedient to make a rediscussion of the Fricke's analysis. The paper presents a redetermination of precession based on the next novelties (which W.Fricke could not incorporate in his time): a) the accurate parallaxes have been taken into account; b) the galactic rotation and other kinematics have been eliminated from the proper motions of the F512 stars;

In this paper the kinematical analysis of 512 stars was made in the frames of the Oort-Linblad model and the Ogorodnikov-Milne model of a three-dimensional differential centroid velocity field (Ogorodnikov, 1930; Milne, 1932; du Mont, 1977). To solve the equations of condition we used a new method, based on orthogonal representation of proper motions – the MOTOR (Vityazev, 1999). The MOTOR, in contrast to the commonly used Least Squares Procedure, provides a test that the model is (or not) compatiple with the data. The main result is: the kinematics of 512 stars is too complicated to be described properly with the help of the both models. Fortunately, the HIPPARCOS catalogue saves the situation. Indeed, proper motions of this catalogue being tied to the ICRF are free of precessional effects. For this reason the differences $Cat. - HIPPARCOS$ depend only on precession and are free of any kinematics. Following this idea we have found that the differences $FK5 - HIPPARCOS$ yield the the correction to the luni-solar precession as much as $\Delta p = -0.28 \pm 0.08''/cy$.

The PGC, GC and N30 form a sequence of American catalogues which were created to provide researchers with absolute positions and proper motions of stars for investigating the stellar kinematics and motions of the planets. The success of such works depends on the level of the systematic errors in a catalogue. Unfortunately, the positions of the GC are overburden with large periodic errors since the authors of this catalogue, guided by wrong idea that such errors are generated by the uneven speed of the Earth's rotation, refused to correct the PGC positions by the corresponding corrections. This fault decreased the accuracy of the GC itself and spoiled the quality of the catalogue N30 for compilation of which the data from the GC had been used. An improved R.A. system of the GC (henceforth CGC, C–corrected) was created by Vityazev and Vityazeva (1985). They derived the periodical corrections ΔGC to RA of the GC on the material of 20 catalogues with epochs of observations from 1845 to 1925 and found that *the faults in the GC compiling (not observations!) prevented the GC to be more than 50 years ago as accurate (with respect to systematic errors) as the FK4 and the FK5 are now-adays.*

Now, using the correction ΔGC we calculated 512 differences

$$CGC - HIPPARCOS = (GC + \Delta GC) - HIPPARCOS$$

to obtain : $\Delta p = -0.34 \pm 0.10''/cy$.

To this we must add that our correction are consistent with results obtained by Miyamoto et al. (1993) from the kinematical investigation of 30000 K-M giants (the catalogue ACRS) : $\Delta p = -0.27 \pm 0.03''/cy$.

Our corrections are in good agreement with the corrections which were derived from the PPM and the Pulkovo proper motions, tied to galaxies (Bobylev, 1997): $\Delta p = -0.28 \pm 0.08''/cy$. and from PPM alone (Vityazev, 1996): $\Delta p = -0.35 \pm 0.05''/cy$.

Still, the more valuable is a comparison of all corrections derived from the proper motions with the results, obtained independetly with the VLBI technique (Walter, Ma, 1994) : $\Delta p = -0.36 \pm 0.11''/cy$

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THE PRECESSION OF THE EQUINOXES FROM HIPPARCHUS TO TYCHO BRAHE

D. SAVOIE
Palais de la découverte
Avenue Franklin Roosevelt, 75008 Paris
et Observatoire de Paris,
61, avenue de l'Observatoire, 75014, Paris
Savoie@danof.obspm.fr

There is hardly a work of technical astronomy written after Ptolemys *Almagest* (IInd century A. D.) that does not contain a chapter on the precession of the equinoxes[1]. The knowledge of this astronomical constant serves as the basis for the drawing up of astronomical tables and is necessary for forecasting celestial phenomena. Since its discovery by Hipparchus in the IInd century B.C. the value of this constant of the precession has been the object of numerous controversies[2], - in particular among the Arabs -, and the question was only (provisionnally) settled by Tycho Brahe at the end of the XVIth century.
The motion of precession was not known to Babylonian astronomers[3] . The merit of its discovery belongs, as has been noted, to Hipparchus. The two works which he composed on the subject - respectively entitled *On the displacement of the equinoxial and solstitial points* and *On the length of the year* - having been lost, our only source concerning the method he used is Ptolemys *Almagest* written about 150 A.D. It is not known what led Hipparchus to discover the phenomenon. Was it the result of his observations of the longitude of the star *Spica* during a lunar eclipse, or was it the result of his measurements of the length of the solar year ?
In the first chapter of Book III of the *Almagest*, Ptolemy reports that Hipparchus compared nine observations of autumn and spring equinoxes made between 162 and 128 B.C., to which he added two other observations of solstices due respectively to Meton and Euctemon in 432 B.C. and to Aristarchus in 280 B.C. In comparing all these observations, Hipparchus came to the conclusion that the solar year lasted 365,25 days, minus 1/300th of a day, or 365 days, 5 hours, 55 minutes and 12 seconds. This value of the tropical year was accepted three centuries later by Ptolemy.
One can readily proceed from the value of the tropical year to that of the sidereal year by measuring the difference in longitude. Ptolemy says nothing about the value given by Hipparchus for the sidereal year, but we know from the famous physician, Galen of Pergamos[4], Ptolemys con-

[1]PTOLEMY, Composition mathématique, éd et trad. par lAbbé Halma, 2 vol., Paris, 1813-1816, [Facsimilé, Blanchard, Paris, 1988]. PTOLEMY, Almagest, trans and notes by G.J. Toomer, London and New York, 1984

[2]See the excellent work by J. EVANS, The History and Practice of Ancient Astronomy, Oxford University Press, 1998, p.245-287.

[3]A study of nearly all the models of precession (Indian, Greek, Medieval, Copernican) will be found in R. MERCIER, "Studies in the medieval conception of precession", Archives Internationales dHistoire des Sciences, vol.26, $n°$99, 1976, Part I, p.197-220, and Part II, vol.27. $n°$100, 1976, p. 33-71.

[4]O. NEUGEBAUER, "Astronomical Fragments in Galens Treatise on Seven-Month Children", *Revista degli*

temporary, that he estimated it at 365,25 days + 1/144th, or 365 days, 6 hours and 10 minutes[5], from which is deduced a constant of precession of $36''$ per year[6], or 1° in 100 years. Although this calculation was not done either by Hipparchus, or by Ptolemy, we do owe to Hipparchus the distinction between the two kinds of year. It is furthermore known that Hipparchus held the tropical year to be variable, an idea which although outrightly contested by Ptolemy, was by contrast adopted by the Arabs in the Middle Ages, as well as by Copernicus.
The most widely known method for determining precession was that using the variation in longitude of *Spica* over time. Ptolemy writes in chap. 2 of book VII of the *Almagest* that Hipparchus, in his treatise *On the displacement of the equinocial and solsticial points*, showed from observation of eclipses of the Moon that the longitude of *Spica* had changed from 8° to 6° in relation to the vernal point of autumn, between the observation of Timocharis (active between 290 and 272 B.C.) and his own, or during an interval of time of approximately 150 years.
Hipparchus method was the following (fig.1) : the longitude of *Spica* is equal to the longitude of the Sun + the angular distance of *Spica* in relation to the Sun.
The first longitude was obtained by Hipparchus from his theory of the Sun. To measure the second term, something obviously more difficult, Hipparchus had the idea of using an eclipse of the Moon : at the instant in the middle of the eclipse, the longitude of the latter is equal to that of the Sun + 180°. Which comes out as :
the longitude of *Spica* = longitude of the Sun at the moment of the eclipse + the angular distance of the star from the Moon + 180°.

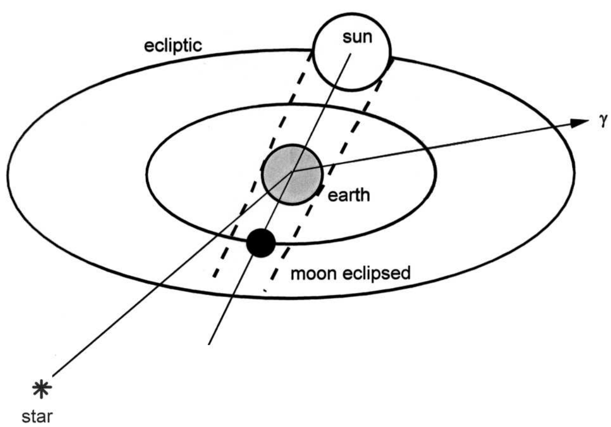

Figure 1

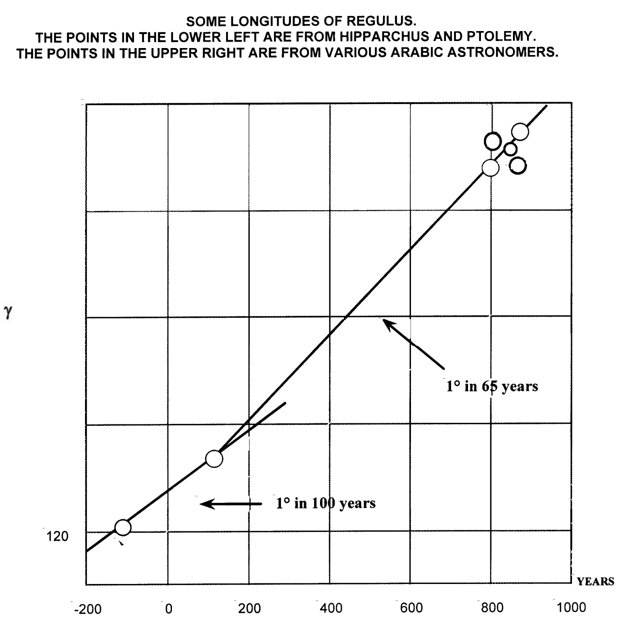

Figure 2

The conclusion drawn by Hipparchus from his measurements is resumed by Ptolemy in these terms : "We have judged from this that the stars advance towards the east at about one degree in a hundred years, as it seems Hipparchus believed from what he says in his treatise on the length of the year". (*Almagest*, VII 2). Before accepting this value himself, after a long examination to determine the constant of precession, Ptolemy criticized the reliability of Hipparchus' measurments by remarking, for example, that he twice measured the longitude of *Spica* at an

Studi Orientali, 24, 1949, p.92-94.
[5] A value used by the Babylonians.
[6] The average motion of the Sun in 24 hours is equal to $360^\circ/365$, 246666, or $0^\circ 598''$. Taking the difference between the sidereal year and the tropical year to be 14 minutes 48 seconds, the corresponding arc can be deduced, that is $36''$.

interval of eleven years with a difference of $1°15'$ between the two measurements[7]!
The method Ptolemy invented for measuring stellar longitudes is very sophisticated. He used an armillary sphere whose construction is not described here, but whose method of use is the following: shortly before the setting of the sun, Ptolemy measured the angular distance between the Sun and the Moon, then as soon as a star became visible, he measured the angular distance between the Moon and the star in question. From which he deduced :
the longitude of the star = the longitude of the Sun + the angular distance between the Sun and the Moon + the angular distance between the Moon and the star + the corrections.
The first term of the equation is obtained from the theory of the Sun. The two angular distances come entirely from observation. But as a careful astronomer, Ptolemy took account of the two corrections relating to the Moon. Between the moment when the angular distance Sun-Moon is measured and that when the star becomes visible, the Moon has moved. Hence the first correction. The second correction takes account of the diurnal parallax resulting from the motion of the Moon.
Ptolemy was mainly concerned with observing *Regulus* and *Spica*, both very close to the ecliptic, by utilising the ancient occultations of these stars as observed by his predecessors (whose results he confirmed from observations of the Pleiades and of *Bêta* Scorpion). He also concluded that if these stars had moved 2° to the east from the time of Hipparchus, this motion only affected the longitude of the stars, not their latitude. In other words the motion of precession was assimilated by Ptolemy to a slow motion of rotation the sphere of the stars from west to east round the pole of the ecliptic.
It remains that in spite of his careful observations and learned calculations, Ptolemy settled on value that was too small for the constant of precession, since its value is in reality close to $50''$ a year, or 1° in 72 years.
This difference of $14''$ quite rapidly had serious repercussions for astronomy. Indeed, as from the IXth century, the translators who translated the *Almagest* into Arabic found themselves faced with a problem of importance. In comparing their own observations with Ptolemy's measurements, the Arab astronomers ended by adopting a value of 1° in 65 years for the constant of precession (fig. 2). Now for the period from Hipparchus to Ptolemy, the value of this constant was supposed to be 1° in 100 years. If the Arab astronomers had compared their observations directly with those of Hipparchus, they would have obtained a figure very close to reality. Instead of this, by not daring to doubt Ptolemy's value, some of them were led to admit a non-uniform variation of the motion of precession, called the trepidation of the equinoxes, which can be assimilated to a false nutation of the vernal point. But furthermore, the Arab astronomers also noted that the obliquity of the ecliptic had varied since the time of Ptolemy : instead of $23°5120''$ (an overestimated value of about $10'$) for the obliquity of the ecliptic given in the *Almagest*, they found, some seven centuries after, a value of approximately $23°33'$. The Arab astronomers were thus led to conceive of a mechanism accounting for two supposed variations[8].
The theory of trepidation was attributed in the Middle Ages to the great IXth century astronomer Thabit Ibn Qurra, who observed at Bagdad. It should be recalled that from Aristotle onwards, the celestial region was conceived as an encasement of eight invisible spheres revolving round a motionless earth with the celestial bodies attached to them : seven spheres for the Moon, the Sun and the five planets and a final sphere, the eighth, in which the stars were fixed and which gave a diurnal motion to all the spheres placed below it. In the work in which he describes the motion of trepidation, whose title in Latin translation is *De motu octavae sphaerae* ("On the motion of the eighth sphere"), Thabit introduces a ninth sphere, a sphere without stars, containing the mechanism of trepidation and making it the motor of diurnal motion - medieval

[7] *Almagest*, cap. III 2.
[8] On Arabic astronomy and the reception of Ptolemys models, see R. MORELON, "Lastronomie arabe orientale (VIII-XI siècles)", *Histoire des sciences arabes*, sous la dir. de R. Rashed, vol.I, Paris, Le Seuil, 1997.

astronomers called this extra sphere the *Primum Mobile*.
Thabit explains his principle as follows : to take account of non-uniform variation of the precession and of the decrease in the obliquity of the ecliptic, he imagined, on the equator of the ninth sphere, an epicycle mechanism commanding the motion of a mobile ecliptic in relation to a fixed ecliptic inclined at $23°33'$ to the equator (fig. 3). This is a little circle with a centre at A and a radius AC to which Thabit gave a value of approximately $4°19'$. The radius AC revolves round A at an angle *bêta* which runs through the 360° of the little circle in 4057 Julian years - the same arrangement (not shown) is found diametrically opposite, for the autumn equinox. The mobile ecliptic passes through the point C and cuts the equator at *gamma*. Since the stars are fixed to the sphere which carries the mobile ecliptic, their latitude remains unchanged. The same mechanism, which makes the stars advance or recede in relation to the equinox - hence the term access and recess given to the motion - also causes the variation of the obliquity of the ecliptic.

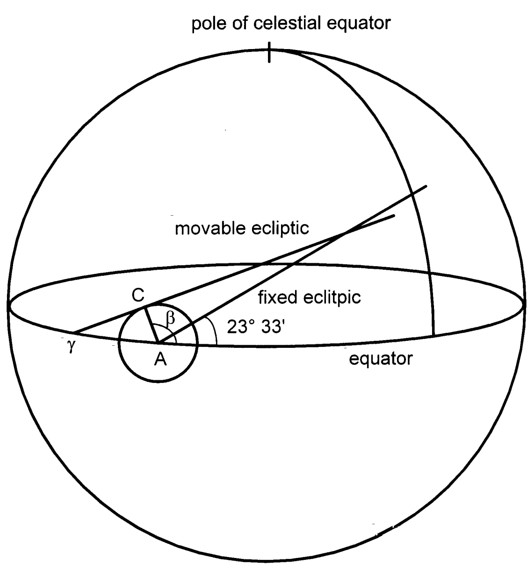

Figure 3

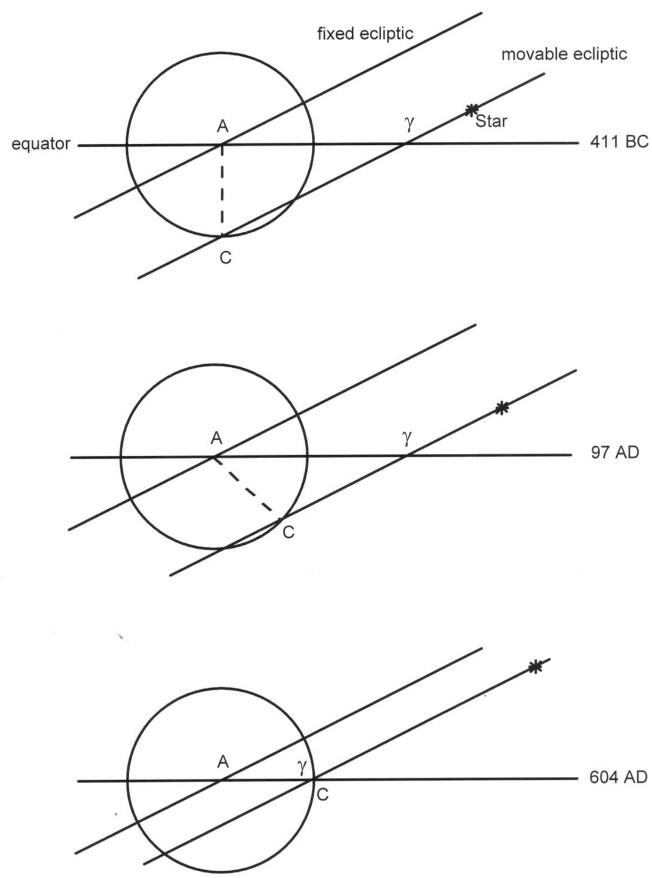

Figure 4

Figure 4 shows the effects of the mechanism at three different periods corresponding in all to a variation of the angle *bêta* through 90°. The distance between C and the star S being constant, the longitude of the star can be determined, thus the distance *gamma* S varies over time. The longitude of the star S is thus equal to *gamma* C (the equation in longitude) + the angular distance of the star at point C.
The reception of Thabits system by his contemporaries was not unanimously favorable. Some like Al Battani (end of IXth - beginning of Xth centuries) outrightly rejected it, adopting a uniform precession of 1° in 66 years. On the other hand, in Spain under Arab influence, the

theory of trepidation was enthusiastically taken up, notably via the Toledan Tables drawn up in the IXth century[9].
In Parisian astronomical circles, about 1320, more ambitious tables became current, independant of any particular calendar, and attributed to King Alphonso of Castille. Their success was such that from the XVIth century onwards, one finds practically only the *Alphonsine Tables* or their adaptations[10]. The innovation of the Alphonsine astronomers in relation to their Arab colleagues concerns the combination of two motions in the precession, the average motion and the motion of access and recession (or trepidation). By changing the values of the periods of these motions, a true nutation of the vernal point can be obtained. Thus the median point *gamma* (fig. 5) runs through the 360° of the ecliptic in approximately 49036 years, whereas its non-uniform variation over time is explicated by a mechanism of trepidation : the radius of the little circle being of 9°, it completes its revolution in approximately 7000 years. The projection of point A on the ecliptic gives the position of the true vernal point (*gamma*). The astronomers used this system during the whole medieval period, until the time of Copernicus.
Copernicus, who made few observations himself and who depended entirely on the observations of the Ancients in whom he had complete faith[11], believed, like the Alphonsines, that the rate of the precession was not uniform over time and also that the obliquity of the ecliptic varied (between two limits however). Hence the need he felt to work out a general model taking account of all the values of his predecessors.

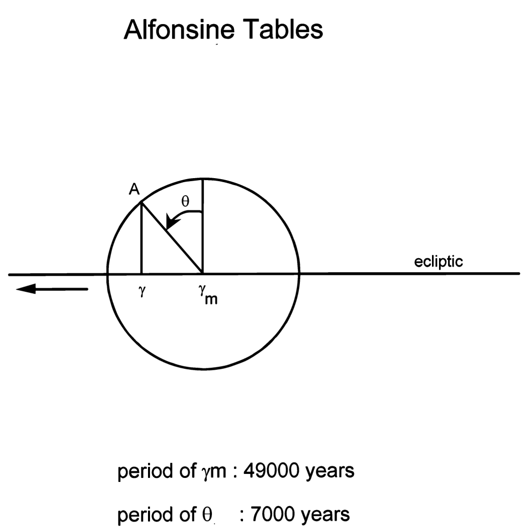

Figure 5

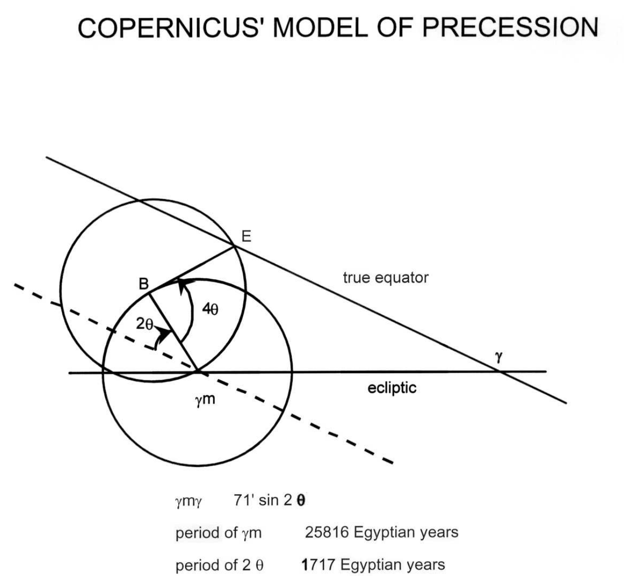

Figure 6

According to Copernicus, the "median" vernal point runs through the ecliptic in 25816 Egyptian years and the obliquity of the ecliptic varies between 23°28′ and 23°52′. He postulated that

[9]G.J. TOOMER, "A Survey of the Toledan Tables" ed. with commentary. G.J. Toomer Osiris, vol.15, 1968, p.5-174.

[10]See E. POULLE, *Les Tables Alphonsines avec les canons de Jean de Saxe*, trad. et comment. par E. Poulle, Paris, éd. CNRS, 1984, and by the same author, "The Alphonsine Tables and Alphonso X of Castille", JHA, vol.19, part 2, 1988, p.97-113.

[11]On Copernicuss theory, N.M. SWERDLOW, O. NEUGEBAUER, *Mathematical Astronomy in Copernicuss De Revolutionibus*, 2 t., Berlin-Heidelberg-New York, Springer Verlag, 1984. N. SWERDLOW, "On Copernicus Theory of Precession", The Copernican Achievement, ed. R.S. Westman, Berkeley-Los Angeles-London, 1975, p.49-98.

the period of libration of the precession was exactly double that of the precession. He distinguished an average precession differing from the true precession by an "equation of precession", a sort of false nutation of the vernal point making it oscillate from one side to the other of the median point. The equation of precession depends, according to Copernicus, on an angle called "anomally of precession" whose period is of 1717 Egyptian years[12]. On the figure 6 *gamma* m marks the vernal point, the radius *gamma* B revolves through an angle 2 *theta* measured from the median equator and the radius BE revolves in the opposite direction through an angle 4 *theta*. The combination of the two motions produces an oscillation of the true equator, hence a variation *gamma* m *gamma* and of the obliquity.

Having benefited from a long period of observations, that is over some eighteen centuries between Hipparchus and himself, Copernicus adopted a very good value for the constant of precession : approximately $50''$. But the addition of a periodic term to the uniform motion of precession had, after a while, perceptible effects, to the extent of destorting the values of the longitudes of the Sun and of the planets from before the end of the XVIth century[13]. If the *Prutenic Tables* of Erasmus Reinhold, published in 1551 and which took as their basis the Copernican parameters, quite easily replaced the Alphonsine Tables, they were in turn rapidly replaced by Keplers *Rudolphine Tables* which, together with other improvements, abandoned for good the idea of a non- uniform variation of the precession. Kepler benefited from the essential contribution of Tycho Brahe who put an end to the theory of trepidation.

In contrast to Copernicus, Tycho Brahe (1546-1601) had at his disposal a solid mass of observational data, firstly accumulated with the aid of his many perfected instruments which enabled him make the angular measurements of a few minutes arc with the naked eye, and secondly resulting from his method of discussing observations. For the first time since Ptolemy, Brahe undertook to draw up a new catalogue of stars, a task which brought him to a conclusion completely breaking with his predecessors. In his *Progymnasmata* published in 1602-1603 shortly after his death, Brahe stated that the constant of precession is $51''$ per year and the different values found by the Ancients were solely due to errors of observation. We owe to Brahe another very important discovery. Since Antiquity, the latitude of the stars was held to be invariable. Now in carefully analysing the data relative to the decrease in the obliquity of the ecliptic, Brahe showed that the latters cause was a variation in the inclination of the ecliptic to the equator. Hence it followed that the latitude of the stars must vary over time, a fact confirmed by comparing ancient observations of stars with his own[14].

To conclude, it required some eighteen centuries to pass after its discovery, for the problem of the constant of precession to be solved. And it is ironic to note that the theory of the motion of oscillation of the vernal point, which dominated medieval astronomy until Copernicus, was resuscitated by Bradley, but this time in a justified way, when the latter discovered, a century and a half after Tycho Brahe, the periodic term of the precession, that is to say the nutation.

[12]On this period, see K.P. MOESGAARD, "The 1717 Egyptian years and the Copernican theory of precession", *Centaurus*, vol.13, 1968, p.120-138.

[13]On the consequences of the Copernican model of precession, see B. MORANDO, and D. SAVOIE, "Etude de la théorie du Soleil des Tables Pruténiques", *Revue des Sciences*, 1996, p.543-567, and D. SAVOIE, *La diffusion du copernicianisme au XVIe siècle: les Tables Pruténiques dErasmus Reinhold*, thèse inédite, 1996.

[14]Tycho Brahes method is very clearly explained in J. Evans, *The History and Practice of Ancient Astronomy*, op. cit., p.281-286.

BRADLEY'S DISCOVERY OF NUTATION

J. EISENSTAEDT
Laboratoire de Gravitation et Cosmologie Relativistes
Université Pierre et Marie Curie, CNRS/ESA 7065 Tour 22-12, 4ème étage, B.C. 142,
4, place Jussieu, 75252 Paris Cedex 05, France, 75014, Paris, France
e-mail: Jean.Eisenstaedt@danof.obspm.fr

ABSTRACT. Bradley's discovery of nutation followed that of aberration which resulted form an attempt to detect the parallactic displacement of stars. Actually much of the developpment of positional astronomy at the end of the eighteenth and in the nineteenth century has its roots in Bradley's work. I will insist on the instrumental context which seems to be a most important point in Bradley's work.

1. BRADLEY'S LIFE

James Bradley was born in 1693. His father's income was limited and his education was helped by his uncle the reverent James Pound (1669-1724), rector of Wansted, Essex, who was then one of the ablest amateur astronomer in England.

Bradley received his Master of Art in 1717 and was ordained in 1719 but he was able to continue his visits at Wansted and to take part in his uncle's astronomical observations.

Pound (who was one of the earlier astronomer to use a transit in England) introduced his nephew to Edmund Halley, a friend of him, and already in 1716 Bradley had made accurate observations at Halley's request. In 1718 he was elected a fellow of the Royal Society. In 1721 he became Savilian Professor of Astronomy in Oxford. When Halley died in 1742, Bradley was appointed to succeed him as Royal Astronomer.

Bradley brilliant discoveries and work brought him preeminence among English and foreign astronomers. He was elected a member of the Académie Royale des Sciences and of the academies of Berlin, Bologna and St. Petersburg. He was giving some high awards including the Copley Medal of the Royal Society.

2. FROM HOOKE TO BRADLEY

As is well-known, the discovery of nutation followed that of aberration which resulted form an attempt to detect the parallactic displacement of stars. Since the Coppernican controversy the detection of parallax had naturally exerted a fascination on the astronomers. Many had tried to detect the motion in question, and some (including Hooke and Flamsteed) professed to have succeeded.

Actually Robert Hooke was the earliest astronomer to recognize the advantage of a zenith sector in the measurements of parallaxes.

The zenith sector was a somewhat specialized form of a transit instrument. It consisted of a telescope of long focal length supported on pivots or gimbals and left free to hang down so that it pointed to the zenith. It moved along the plane of the meridian and only a few degrees from the zenith. It had two advantages that were considered essential to accurate observations of parallaxes : like a pendulum the instrument found its own zenith automatically, by gravity. Furthermore the observations were free of atmospheric refraction.

In 1669 Hooke constructed[1] a 36 feet sector and installated it in Gresham College. He set it out in order to observe γ Draconis which passes almost directly overhead in London. As is well known he claimed to have observed a $27''$ parallax which was not accepted by his contemporaries.

As Chapman wrote it : "Almost certainly, this excessive value had been caused by the then unknown and unrecognized aberration of light..."[2]

But curiously enough it doesn't seem to be Bradley's interpretation, Bradley who, in his 1729 paper on aberration, is a bit agressive about Hooke's observations :

"I cannot well conceive that an Instrument of the Length of thirty-six feet, constructed in the Manner he describes his, could have been liable to an Error of near $30''$, (which was doubtless the Case,) if rectified with so much Care as he represents."[3]

Anyway, Hooke's method in making the observations was in some measure the same that Bradley's friend, Samuel Molyneux (1689- 1728) followed. Molyneux erected a vertical 24 feet long instrument at Kew made for him by George Graham. He made choice of the same star as Hooke and his instrument was constructed upon almost the same principles.

Bradley was associated with Molyneux from the outset and γ Draconis was observed for the first time on December 1st 1725. But at the end of 1726 Molyneux had to attend to other duties and the instrument went out of order.

3. BRADLEY'S 12 FOOT ZENITH SECTOR

In 1727 Bradley got Graham to make him an instrument of his own; this famous instrument was first erected at Wansted inside the house of Bradley's aunt, widow of the astronomer James Pound. But this house was not big enough for so long a telescope as Molyneux's. Bradley's telescope had a focal length or 12.1/2 feet, that he found "long enough to adjust the Instrument to a sufficient Degree of Exactness". He made his telescope capable of moving 6.1/4 degrees on either side of the vertical, so as to increase as much as possible the number of stars he could observe and especially he was willing to take in Capella the only Star of the first magnitude that comes so near to the Zenith.

In order to observe any annual movement of stars by observing their zenith distances at the moment that they crossed the meridian, it was of course necessary to observe them by day at certain times of the year. The observing list had therefore to be confined to those stars which passed the meridian within $6.1/4°$ on either side of the zenith, and which were bright enough to be observed by day as well as by night. The actual zenith was observed by referring to a plumb line, about the same lenght as the telescope and hanging down beside it. What Bradley was

[1] (Chapman, 1990, p.87).
[2] Ibid.
[3] Bradley 1729, postcript, p. 661

observing were changes in the angle between the apparent direction of the star and this plumb line.

Like the transit, the sector was easily cross-checked. The exact horizontality of its gimbals was capable of being adjusted, and the whole tube could be reversed by lifting it out of its gimbals and turning it through 180°. It was possible to dismount the tube altogether to test its optical collimation against land objects. Like the quadrants, its micrometer could be checked against its own scale as well as being itself "reversed".

4. BRADLEY FROM ABERRATION TO NUTATION

The explanation of aberration appears to have occurred to Bradley about September 1728 (the story of the sailing boat is well-known) and was published in the *Philosophical Transactions of the Royal Society of London* in the following year.[4]

Actually many points of Bradley's programm on nutation are still present in his work on aberration and first of all the exactness of his instrument of course.

Already in his paper on aberration Bradley is conscient that there is something bizarre in his observations, some "small alteration" as he put it :

"I have likewise met with some small Varieties in the Declination of other Stars in different Years which do not seem to proceed from the same Cause. [...]. But whether these small Alterations proceed from a regular Cause, or are occasioned by any Change in the Materials, etc., of my Instrument, I am not yet able fully to determine"[5].

And thus, from 1728 on, he continued his observations of the same stars, hoping at length to discover the real cause of such "apparent inconsistencies".

A nutation or nodding of the earth's axis had already presented itself to him as a possibility for explaining aberration. And thus it is not surprising that Bradley comes back to this explanation :

"I suspected that the moon's action upon the equatorial parts of the earth might produce these effects : for if the precession of the equinox be, according to Sir Isaac Newton's principles, caused by the actions of the sun and moon upon those parts, the plane of the moon's orbit being at one time above ten degrees more inclined to the plane of the equator than at another, it was reasonable to conclude, that the part of the whole annual precession, which arises from her action, would in different years be varied in its quantity ;"[6].

5. THE 1748 ARTICLE ON NUTATION

There is a lapse of nineteen years between the discovery of aberration and that of nutation: 1729 to 1748. There are good reasons for that. As is well known the amplitude of aberration is $20'', 49$ (between $20''$ and $20''1/2$ for Bradley) in one year. In what concern nutation the amplitude of the axes of the ellipse of nutation is $18'', 42$ and $13'', 75$ (respectively $18''$ and $16''$ for Bradley) for a period of 19 years. Thus the relative amplitude between aberration and nutation is of the order of 1/20. So that twenty years were necessary in order to get the same precision as that of aberration. The second reason for Bradley to wait for such a long time is that he was quite determined to follow the phenomenon through a whole cycle of the movement of the node of the Moon's orbit.

[4](Bradley 1729).
[5](Bradley 1729, p. 652).
[6](Bradley 1748, p. 23).

In the introduction of his article, Bradley addresses the question of the progress of astronomy "which has always been found to have so great a dependence upon accurate observations." Actually about seven pages of his paper are concerned with questions of instruments and more precisely to their "exactness". Let me just quote at one of his remarks :

"it is incumbent upon the practical astronomer to set out at first with the examination of the correctness of his instruments, and to be assured that they are sufficiently exact for the use he intends to make of them; or at least he should know within what limits their errors are confined."[7]

I have not enough time to tell you the details of the very diverse improvements realized by Bradley, (and as well I am not an expert on this matter) let me rather quote at

In his book "Dividing the circle" Allan Chapman insist on Bradley's concern with instruments :

"As one may consider Graham the founder of instrument graduation, so Bradley was to teach astronomers a new approach to the use of their instruments. Whatever the instrument – quadrant, sector, transit– Bradley tested it almost to destruction before he would rely upon it. He was fortunate enough in being able to work with the finest instruments that were available in this day, but understood that if the skill of the maker was not complemented by that of the user, nothing worthwile would be achieved."[8]

John Machin, secretary of the Royal Society, had suggested that tables might be calculated in order to model the nutation effect by supposing that the moon's attraction caused the pole of the earth to describe a small circle around its mean place. To determine the magnitude of this circle was one of the objects of Bradley's observations. The diameter was first taken as equal to $18''$ but Bradley considered that there would be a better agreement if the curve was supposed to be elliptical with its major and minor axes equal to $18''$ and $16''$.

Then, Bradley compared each observations of γ Draconis with his hypothesis (actually he reduced the observations with the help of Machin's table) and found that "only eleven that differ from the mean of these as much as $2''$, and not one that differs so much as $3''$."

As Bradley wrote it : "This surprising agreement, therefore, in so long a series of observations, taken in all the various seasons of the year, as well as in the different positions of the moon's nodes semms to be a sufficient proof of the truth both of this hypothesis and also of that which I formerly advanced, relating to the aberration of light;"[9]

But Bradley didn't limit himself to observing γ Draconis, far from it; he made a lot of observations in order to strenghten the "phenomena".

As Herrmann wrote it Bradley and Bessel are "the corner-stones of possibly the most splendid epoch in the developpment of positional astronomy."[10]

As well, in his "Revolutions in Astronomy" Freeman Dyson points at Bradley's work as the most important example of a "tool- driven revolution" opposed to Kuhn's "concept-driven revolution".[11]

[7](Bradley 1748, p.19).
[8](Chapman 1990, p. 88).
[9](Bradley 1748, p. 31).
[10](Herrmann 1976, p. 183).
[11](Dyson 1992).

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D'ALEMBERT'S THEORY OF PRECESSION-NUTATION

J. SOUCHAY
DANOF/UMR8630-CNRS
Observatoire de Paris, 61, avenue de l'Observatoire, 75014 Paris, FRANCE
e-mail : Jean.Souchay@obspm.fr

ABSTRACT. D'Alembert's treatise "Recherches sur la Précession des Equinoxes et sur la Nutation de l'axe de la terre", published in 1749, is the first one dedicated to the complete analytical determination of the combined precession-nutation motion of the axis of rotation of the Earth. Published in 1749, just two years after the discovery of the nutation by Bradley, it is impressive by its important results, its concision, and the relatively modern way of calculation adopted. After describing the causes which might have motivated D'Alembert to write this treatise, we summarize its contents.

1. INTRODUCTION

When Jean Le Rond D'Alembert (1717-1783) published his masterly work "Recherches sur la Precession des Equinoxes et sur la nutation de l' Axe de la Terre" (1749) (i.e. "Researches on the Precession of the Equinoxes and the nutation of the Earth axis"), quoted in the following for the sake of simplicity "Recherches sur la Précession", he had already achieved his most important masterpiece, "Traité de la Dynamique" (i.e. "Treatise on Dynamics" which participated significantly to the job of formalizing the new science of mechanics, thus following other pioneer works from contemporary authors like Jean and Daniel Bernouilli, Leonhard Euler, Alexis Clairaut, Joseph-Louis Lagrange. In the "Traite de Dynamique", whose the first edition was published in 1743, D'Alembert presented the basic statements of what was immortalized as the famous "D'Alembert's principle". Moreover he explained his own laws of motion, the fist one being named as the law of inertia, the second one the "parallelogram of motion", and the third one dealing with equilibrium and conservation of momentum in the case of impact situations.

Quite naturally D'Alembert wished to check his principle and the law of motions above to various situations in the fields of physics. For instance, as soon as 1744, that is three years after being named as assistant astronomer (May 1741) his "Traité de l'Equilibre et du Mouvement des Fluides" denotes his willingness to tackle problems related to fluid dynamics which was a leading and very controversial topic at this period (notice that the interest to this last topic was motivated by a recurrent believing that fluids could play a determinant place in the explanation of phenomena such as electricity, magnetism and heat). In the second part of the 1740's D'Alembert still kept on publishing a lot of major works on a great variety of subjects as "Reflexions sur la cause generale des Vents" (1747) (i.e. "Thoughts about the general cause of the Winds") which conducted him to receive a prize from the Prussian Academy, and also the well known Memoirs dealing with the vibrating strings (1747), which represents a cornerstone for differential analysis.

D'Alembert's interrogations and works in the fields of astronomy and celestial mechanics were never absent during all these years of intensive research and abundant publications. In the beginning of the 1740's the three-body problem was studied in parallel with other specialists as Euler and Clairaut. D'Alembert also produced fundamental works from 1747 to 1749 dealing with the orbital motion of the Moon (Chapront-Touze,2000), thus tackling the delicate and controversial problem of the irregularities of the motion of the apogee of the Moon first mentioned by Clairaut, among other ones.

In the natural prolongation of these studies, the treatise "Recherches sur la Précession" is remarkable for several reasons : at first we can notice its relative concision and its clearness with respect to the majority of other treatises. At second we can mention its homogeneity as well as the few number of references to anterior works, which means that all the calculations were carried out rather independantly. Moreoever we can notice the short time span required between the first calculations and the date of publication, which can be ranged, according to trustable sources, to the sole year 1749. At last the reader can be astonished by the modernity of the method used to reach what can be considered as fundamental results among which we find the theoretical explanation of the nutation phenomena, as well as the precise determination of the ratio of the mass of the Moon with respect to the mass of the Earth. We can also add that D'Alembert's introduction of the treatise "Recherches sur la Précession" has been witten in such a clear and synthetic manner, that it can be considered as a basic epistemologic starting point to whoever wishes to understand the situation in the mid 17's century concerning the knowledges related to the rotational motion of the Earth.

2. D'ALEMBERT MOTIVATIONS TO WRITE A TREATISE ON PRECESSION DES EQUINOXES

It is not so easy to identify in D'Alembert's writings what were his exact motivations to undertake its treatise on the "Précession des Equinoxes", but there is a great probability that they find their origin in several items enumerated below:

♣ Bradley's discovery of the nutation

It should be doubtful that the discovery of the nutation which was definitely demonstrated by Bradley in 1747, that is two years before the publication of the treatise, be a pure temporal coincidence with D'Alembert's calculations. This discovery is the result of a large number of observations performed by Bradley in association with Samuel Molyneux, which extended from 1725 and then during more than 20 years, and which were firstly destinated to the measurement of the parallax of stars. The attention of Bradley converged especially towards the star γ Draconis whose irregularities in the positions with respect to other stars was reckoned very early. In fact, the ironical point is that in place of the parallax, whose the amplitude is less than 1" for any star and consequently could not be detected by Bradley as given the precision of his measurements, the astronomer discovered both the effect of aberration of light in 1729 (letter to Halley), and the nutation. Indeed, concerning this last discovery, Bradley remarked that "the stars return in the same position again..." which in more details means that they accomplish a small loop which corresponds to the main oscillation of nutation with period 18.6 y. This historical discovery was presented officially to the Royal Society of Sciences by Bradley in form of a very long letter to the Earl of Macclesfield, in 1748. The fact that the period of 18.6y corresponds exacly to that of the mean motion of the longitude of the nodes of the Moon might be the crucial revelation for D'Alembert that the Moon might influence the motion of the axis of rotation of the Earth in other ways as by the sole precessional motion. This should explain the rapidity of his work, motivated by the privilege to be the first theoretician to give a complete and detailed explanation of the nutation phenomena, only just two or three years after its discovery.

♡ D'Alembert's critics of Newton's theory

A second point seems to be fundamental to explain D'Alembert's deep efforts to construct his own theory of precession. It concerns the numerous critics with respect to the historical work carried out on the subject about 60 years ago by Newton, presented in the Book III of the *Philosophae Naturalis Principia Mathematica*, which gathers also all the various studies of the famous astronomer in the fields of celestial mechanics (motion of the planets, tides, form of the Earth and of Jupiter, inequalities of the orbital motion of the Moon, theory of the comets etc...) These critics can be found in the introduction of D'Alembert's treatise on the precession, and result from his own calculations inside this treatise.

One of the most important critics is the simplification done by Newton which consists in replacing the envelope external to the globe, which is responsible of the precession phenomena, by a very thin and dense ring, consisting in fact of a "very large number of small moons". Newton predicted that the precession of the ring when isolated is as large as 45'/year, that is to say roughly more than 50 times larger than the true precession value. In the continuation, Newton pretends that the reduction by this amount is the consequence of the property that the ring is not independent, but tied to the globe. In extended paragraphs inside the introduction of his treatise on the precession, D'Alembert's explains why he is very pessimistic about the validity of Newton's calculations. According to him, Newton forgot a factor 2/5 between the gravitational action on the real envelope, and the corresponding gravitational action of the perturbing body (Moon or Sun) on the ring.

In addition, Newton did not take into account the dependence of the gravitational action on the obliquity, as well as the influence of the proper rotation of the Earth (i.e. the diurnal motion). D'Alembert also shows a very strong doubt with respect to Newton's assertion according to which the motion of the node (in fact the motion of precession) is the same, the moons constituing the ring being in contact one to each other or not. Moreover, D'Alembert lays down the fact that the motions of the ring with respect to the real envelope is not the same as the ratio of the forces which are exerted on them, as it is sustained by the author of the Principia. In addition the ratio of the force exerted by the Moon on that exerted by the Sun should be 7 to 3 as explained by D'Alembert and not 5 to 2 as it is adopted by Newton starting from imprecise data related to the variations of level of the sea according to the tides exereted by the two celestial bodies.

Following all the considerations which have just been mentioned, we understand better the global lack of confidence of D'Alembert on Newton's calculations, as well as his purpose to achieve hastily and publish his own work on precession.

♠ Other sources of motivation to write the treatise on the Precession.

In addition to the two principal factors above which undoubtedly favorized a rapid and deep investigation of D'Alembert on the question of the motion of the axis of rotation of the Earth, we can also mention some secondary ones which are related to the particular scientific context around the period 1740-1750. We must not forget that during this period d'Alembert was competing with both Alexis Clairaut (1713-1765) and Euler (1707-1783) in advancing the theory of lunar and planetary motion, and in the same time with Euler and the Bernoullis to establish the foundation of mathematical physics. Notice also that 1749 corresponds exacly to the ending phase of the controversy about the movement of the Moon's apogee which arose firstly between Clairaut and Buffon. To understand this controversy with all the details, we can refer to Chapront-Touzé (2000).

At the origin of the controversy, Clairaut, D'Alembert and Euler were working on the topic and found all together, that the motion of the apogee, according to their calculations, was half that deduced from the observations. Clairaut's interpretation of the well established discrepancy was that Newton's law of attraction should be completed by a term inversly proportional to the fourth power of the distance, whereas Buffon was a strict opponent on this thesis, being an ardent supporter of the simplicity of the laws of the nature, for metaphysical reasons. A letter from D'Alembert to Euler, dated July 20th.,1749, strongly suggests that his attempts to determine analytically the motion of the axis of rotation of the Earth, and especially the nutation, were motivated by the wish to confirm the validity of Newton's law, which was not fully accepted, as shown above. In this letter, D'Alembert explains that he wishes that "the movement of the apogee fits well with the Newtonian System, and even if it is not the case, he will not be suspicious on its validity, for it explains suitabily all the other known celestial phenomena". The interesting point there is that D'Alembert in his letter mentions, as a representative example, "the work he has just published" and which is precisely "Recherches sur la Précession".

3. THE CONTENTS AND MAIN RESULTS OF THE TREATISE "PRECESSION DES EQUINOXES ... "

The treatise "Recherches sur la Précession des Equinoxes" is consisting in 15 chapters gathered in a little more than 200 pages, when including the introduction. The whole analytical way of resolution of the problem of the determination of the combined motion of precession and nutation can be found in the sole three first chapters. We propose in the following to summerize the contents of each of the chapter, taken one by one, with their main contribution.

•The first chapter is devoted to the calculation of the torque exerted on a spheroid, to which the Earth is implicitely assimilated, by an external body when considering that each part of the spheroid is subject to a gravitational attraction whose the amplitude and the direction are given by the fundamental Newton's law of the gravitation. In this calculation D'Alembert makes full use of integral calculus which leads to a precise formula of the torque, quite conform to a modern one, and where the angle between the equatorial plane and the direction of the external body,that is to say its declination, is explicitely included.

•The second chapter gathers some basic propositions necessary to solve the problem. A first part concerns the substitution of initial forces to other ones according to geometrical principles and fundamental rules of "traditional" mechanics. In a second part, D'Alembert exposes his famous general principle of the Dynamics and proposes himself to the reader to refer to the chapter I of his "Traité de Dynamique" (1743)

• The third chapter conducts to the equations of motion resulting from the application of the author's general principle of the dynamics, to the case for which the spheroid is subject to a torque as expressed in the first chapter. These equations appear to be differential at the second order, the differenciated parameters ε and π corresponding respectively to the displacement of the axis of the Earth in longitude and in obliquity.

• The chapter IV, whose the function is to confrontate the theory elaborated in the precedent chapters with the observations, is fundamental for several reasons : at first it proposes a detailed geometrical and schematic representation of the combined motion of precession-nutation in space. At second it includes some evaluations of the relative order of amplitude of the expressions involved in the equations of motion, after taking into account the real values of fundamental parameters (obliquity, diurnal period of rotation etc...). This enables to simplify noticably these equations by neglecting some components, and finally leads to a very straightforward resolution. Thus we arrive to the most important part which can be considered as historical for it shows the total agreement between the observed loop of nutation and its analytical explanation. In other words, it is the first time that the phenomena of nutation has been explained theoretically.

•The chapter V, although being very small (two pages) can be considered itself as a fundamental and historical one for it proposes for the first time a very fine and accurate determination of the M_{Moon} to M_{Earth} ratio by two constraints which are the amplitude of the eading nutation oscillation (9") and the amplitude of the precession rate (50"/year), the first one depending only on the sole mass of the Moon, whereas the second one is depending on both the masses of the Sun and of the Moon. The modern reader can appreciate D'Alembert's estimation of the M_{Moon} to M_{Earth} ratio of 1/80, which must be compared with the real value (1/81) and the improvement with respect to Newton's estimation (1/39) computed for the tides heights.

• In chapter VI, the author gives in some details and as precisely as possible the form of the trajectory accomplished by the true pole of rotation in the space. He mentions that Bradley, in a letter, recall that according to his own observations of nutation, this phenomena might be characterized not by a circle, but by an ellipse. D'Alembert confirms this supposition, according to his calculations carried out in the precedent chapters. Moreover he shows some ambiguities in Bradley's explanations, for instance that concerning the plane of the circle or the ellipse of nutation: Bradley does not explain if this plane is parallel to the ecliptic or to the equator, and this might change the quantities of nutation in some extent.

•The chapter VII illustrates D'Alembert's preoccupation to give the geometrical formulae enabling to calculate the change of coordinates (right ascension and declination), due to both the precession of equinoxes and the nutation. These formulae, calculated at the first order for the first time in history, are exactly equivalent to the classical formulae which can be found in any modern treatise of astronomy.

•In chapter VIII, D'Alembert adds some important remarks related to his theory of the motion of rotation. An interesting part is the evaluation of the ties between the axis of rotation and the axis of figure of the Earth, tie modelized by mathematic formulae, which are quoted nowadays as Oppolzer's terms, and from which the author can argue that the difference of orientation between the two axes is very small.

•Chapter IX concerns the geodesy and shows D'Alembert's interests to the figure of the Earth and its internal structure. Starting from results extracted from Clairaut's "De la Figure de la Terre" (i.e. "On the Figure of the Earth"), D'Alembert makes calculations which brings physical constraints to the interior of our planet. These calculations are remarkable for they lead to the conclusion that the Earth cannot be solid as a whole, in other words that there should exist a fluid part inside it. Moreover, the author invokes the possibility that the Earth contains an homogoneous solid core surrounded by a fluid layer, and imagine that nutation should not act on the fluid part. In view of these calculations and these remarks the work performed here can be considered as a precursor of all the present theoretical works dealing with Earth non rigidity and its effects on the Earth rotation.

•After mentioning a difficulty susceptible to arise in the general solution of the problem of the rotation of the Earth (chapter X), D'Alembert devotes about thirty pages in chapter XI to a second method for solving this problem. This second method is rather confused and confusing, as was mentioned recently by Wilson(1987) and Nakata(2000), not only because of the old fashioned analytical expressions used, but also because of D'Alembert's concept of force, which is very difficult to understand for the modern reader. Notice that Euler(1749), in his memoir “Recherches sur la precession des equinoxes et sur la nutation de l’axe de la terre”, used a method quite similar to D’Alembert’s second method. The similitude between the works is such that D’Alembert sent a letter to Euler, dated June 12th., 1752 accusating him for having plagiarized his own ideas and calculations.

•In chapters XII and XIII, D’Alembert makes some extensions with respect to his precedent investigations, firstly by ignorating the proper rotation of the Earth in the equations of motion, and secondly by treating two fictitious cases : in one of them the purpose is to measure the precession of the Earth when its equatorial bulge is replaced by a ring placed in the equatorial plane and when the sole gravitational action of the Sun is then considered. We find here exactly the same hypothesis as Newton’s ones in the Principia.In the second fictitious case, the purpose is to calculate the precession when the Earth is reduced only to the double meniscus which surrounds the globe. These two fictitious configurations will serve to the crtitics on Newton’s work which we have already enumerated in the preceding chapter, and which are the topic of the chapter XIV.

•The final chapters XV and XVI consist respectively in conclusive remarks on the fine correspondence of the motion of the nutation as deduced both from Bradley’s observations and the author’s calculations, and in an independent study with no real link with the precession-nutation phenomena, concerning the motion of the Sun in latitude, caused by the action of the Moon on the Earth (the Earth having a small motion around the Earth-Moon barycenter)

4. CONCLUSION

We can consider D’Alembert’s treatise “Recherches sur la Précession des Equinoxes et sur la Nutation de l’axe de la terre” as the first one to explain in an analytical way both the nutation of the Earth, which had then just been discovered by Bradley, and the precession, without the various simplifications done by Newton in the Principia. Moroever it contains some very fundamental theoretical points, as the exact determination of the M_{Moon}/M_{Earth} mass ratio starting from observational data, and investigations which can be regarded as a starting point for the future and even modern reseraches,as the influence of a fluid layer inside the Earth, and the geometrical separation between the axis of figure and the axis of rotation.

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THE EVOLUTION OF THE PRECESSION AND NUTATION CONSTANTS OVER TWO CENTURIES

S. DÉBARBAT
DANOF/UMR8630-CNRS
Observatoire de Paris, 61, avenue de l'Observatoire - 75014 Paris
e-mail: debarbat@danof.obspm.fr

ABSTRACT

Considering the precession, as discovered by Hipparchus, values derived by Cassini II, Lalande and Delambre are recalled. For nutation, as discovered by Bradley, the size of the main axis of the ellipse is given for various astronomers up to the mid-nineteenth century. From the end of the nineteenth century, decisions for the numerical values of the astronomical constants are taken under the leadership of international organizations; 1896 to 1964 is the considered period.

1. INTRODUCTION

In this paper precession is used for the main term of precession in longitude and nutation is for the term discovered by Bradley. I will recall first some of the determinations made about two centuries ago. "LA PRECESSION est ce changement annuel d'environ 50"1/3 par année observé dans toutes les longitudes de toutes les étoiles fixes". This is written by Lalande (1732-1807) as a conclusion of comparisons he made between observations given for the year 1750.

2. PRECESSION ACCORDING TO CASSINI, LALANDE AND DELAMBRE

Cassini II (1677-1756) published in 1740 his "Tables", following his "Eléments d'Astronomie" in using, for the first time, according to M. Toulmonde the astronomical year zero and giving data up to 2100 in some cases.

In the Elements, Cassini recalls "Ptolémée [...] rapporte pour cet effet, le sentiment d'Hipparque, qui, par la comparaison de ses observations avec celles de Timocharis, faites 155 ans auparavant, avoit trouvé que l'Epy de la Vierge avoit conservé la même distance à l'égard de l'Ecliptique, et non pas à l'égard de l'Equinoxe,...". To derive the value of the precession to be in use in his tables, Cassini makes a detailed study covering different lengths of time.

Comparing his observations, dated 1738, with the Hipparchus values, Cassini derives from two given stars 50"32"'/year and 50"50"'/year, two values leading to the mean 50"41"'/year, he expressed under the form 1 within 71 years and a few days ; Cassini is surprised by the higher values given by Ptolemeus 52"48"' and 52"44"'(mean 52"46"') that is to say 68 years and 3 months, different from Ptolemeus's assertion of 1° per century. The values are given as they were in the paper.

Studying data from Albategnius (878 A.D.) compared with those from 1738, Cassini derives 51”6”’ (51”9”’ and 51”3”’) that is to say 1° for 70 years and 5 months. Mixing all these results, all covering a long period of time, his conclusion is 51”31”’. Coming to more recent observations to be compared with the 1738 ones, Cassini finds three Tycho Brahé’s observations 50”39”’, 49”43”’, 50”39”’ and one by Copernicus and 1738, 50”39”’. He derives the mean as 50”20”’ which he considers larger than the value, 50” only, found in using other more recent observations. Cassini assumes that perhaps there is a decreasing in the precession. Nevertheless Cassini II decides to take into account in his Tables the value 1° per 70 years, that is to say 51.4286” with four digits after the division.

About thirty years later Lalande makes a similar study in using data for the year 1750. For Hipparchus he derives 50”2/3 as he writes. When using the british catalogue of stars given for 1690 and employing 15 stars the mean value derived is 50.336”/year with an incertitude he assumes to be of the order of 1”1/2. On that occasion Lalande recalls values given by some of his predecessors Copernicus (1473-1543), 50”20”’, Boulliau (1605-1694) 51” more or less like Kepler (1571-1630), Flamsteed (1646-1719) and Halley (*ca* 1656-1743) 50”, Cassini (1625-1712) 51.43”. Having found that the value he derives by himself was exactly the same as the one given by Lacaille (1713-1762), that is to say 50”1/3, Lalande decides to take this value for the establishment of his tables.

About thirty years later, Delambre (1749-1822) made a more detailed study of the precession constant because he found previous determinations, apparently too much different - going from as low as 49.33” up to 51.00”. He considers all the stars given by Hipparchus leading him to several conclusions. First an average value of 50.0” while Le Gentil (1725-1792) had given 49”2/3 apparently too much low. Secondly Delambre discussed the date of Hipparchus observations considering the incertitude of their dates. Like other astronomers he considers that Ptolemeus has employed Hipparchus observations using only the value of the precession.

Comparing now Hipparchus data considered having been performed for the date 128 B.C. and 148 B.C., Delambre gives two values, respectively 50.343” and 50.582”, in using the positions of stars given for the year 1800. By the end Delambre will compare his observations, mostly made at his private observatory situated in the Rue de Paradis of the time, with those performed more recently and of much better quality, such as by Bradley (1693-1762), by Mayer (1723-1762) and by Lacaille. Delambre gives the value 50.1” while, in 1830, Bessel (1784-1846) adopted the value 50.21129”, a value which was used by many astronomers up to the end of the XIXth century.

3. THE NUTATION ACCORDING TO VARIOUS AUTHORS

About nutation, Lalande wrote : “LA NUTATION ou déviation est un mouvement apparent de 9” observé dans les étoiles fixes, dont la période est de 18 ans, causé par l’attraction de la lune sur le sphéroïde de la terre”. The phenomenon, mentioned first by Newton (1642-1727), was sustained by Flamsteed and Roemer (1644-1710) and detected by Bradley.

After this discovery several astronomers, both from observations and from theoretical points of view, have derived more precise values for the nutation. Such were Mayer by mid XVIIIth century with 9.66”, Maskelyne (1732-1811) who had founded the Nautical Almanac in 1767 with the value 9.55”, and Laplace (1749-1827) 9.58” leading Delambre to consider 9.6”.

In using observations performed during three revolutions of the Moon, between 1750 and 1815, Lindeman (1750-1815) gave 8.989”, while Henderson (1798-1844), one of the discoverers of the first parallaxe, determined 9.28”. As soon as in the “Connaissance des Temps” (created in 1679) for the year 1760, appeared tables to facilitate the computation of nutation for stars, to accelerate the reduction of observations. This work will led Bessel to develop a method for practical application which was, as far as I remember, in use up to the time of the computers by the mid-fifties of the present century.

4. PRECESSION AND NUTATION FROM THE MID-NINETEENTH CENTURY

The technical improvements of the astronomical instruments during the nineteenth century led to consider better values for the precession and nutation constants. This is particularly due to the return to a proposal made by Roemer by the end of the XVIIth century to have what is now called a meridian circle.

Instead of sectors or large quadrants installed on long walls in the observatories, the instruments are now placed on very stable pillars attached to the rocks. There is one pillar on each side while the refractor is mobile in the meridian plane allowing to observe from the north to the south after only a rotation around its east-west axis.

This was different from the quadrants needing to have what is called in French a "transporteur" and allowing to observe from one side to the south and from the other side to the north. It was also different from the case of the full circles placed against a wall but having the refractor on one side of the circle, not symmetrical as in the case of the meridian instrument having two circles.

It is then understandable why astronomer could derive better values for the precession having by mid-nineteenth century four decimal digits such as 50.2401" and the nutation constant being given under the form 9.2236".

As a consequence of the international enterprise launched, in 1887 at a Paris Observatory meeting and called the "Carte du Ciel", another such meeting decided, in 1896, to have homogeneous reduction methods. In order to leave for future generations the state of the sky around 1900, in an homogeneous system, it was decided to determine a set of constants to be employed for the reductions of observations for all the participating observatories. Among the set were the precession and nutation constants and, during the 1994 "Systèmes de référence spatio-temporels" Journées, I have given the historical elements of the subject.

For nutation several values were proposed : 9.2068" by Gill (1843-1914) the Astronomer Royal for The Cape, 9.214" by Newcomb (1642-1727) from the US Naval Observatory, 9.198" by Chandler (1846-1913) the discoverer of the Chandler's wobble. Trying to satisfy every one, Loewy (1833-1907), from Paris Observatory, proposed 9.21" and it was adopted by the Conference.

For precession a research was needed and Newcomb was asked to do so. A new conference is held in Paris in October 1911, upon the invitation of B. Baillaud (1848-1934) then director of the Paris Observatory. Among the pending questions, the precession constant. France had, in the preceding month of March, adopted the international meridian placed at the Royal Greenwich Observatory along the Airy (1801-1892) meridian circle, from the year 1884. Two series of 3000 stars dated 1755 and 1860 will be used and the value of the precession adopted from 1906.

First World War will consequently disturb what was supposed to be made during the years following 1911. After the creation, in 1919, of the IAU, this body will take care of the organization regarding the astronomical constants.

5. THE IAU MEETINGS FOR THE ASTRONOMICAL CONSTANTS

While researches are going on from the twenties to adopt a uniform system of standard places for stars which is made, by 1935, with the introduction of the FK3, Second World War will, a new time, interrupt the international decisions. But after the war it is seen that there are discrepancies between theory and observations ; nevertheless it is decided, in 1948 to determine "whether it is yet possible to introduce a perfectly consistent system..." and France, with Danjon

(1890-1967), propose to held in Paris in 1950 a Colloquium organized by the Centre National de la Recherche Scientifique for the fundamental astronomical constants.

Precession is in cause during the discussions between the 19 specialists with, among them, Spencer Jones and Jeffreys. Danjon gives the reason for some changes : "Une découverte fondamentale a bouleversé l'astronomie classique, celle des inégalités de la rotation de la Terre... et une constante physique très importante, la vitesse de la lumière a fait l'objet de nombreuses déterminations qui en ont précisé la valeur...". Three days of meetings and debates led to the conclusion that "...après mûre réflexion, la Conférence a décidé de surseoir à tout changement de constantes".

The subject comes more and more frequently in the various meetings to follow and, in 1955 at the Paris Observatory, Danjon delivered two talks, at the beginning of the year, on the revision of the constants and about their choice. In 1961, during the General Assembly of the IAU, Danjon proposed to held, in Paris in 1963, an IAU Symposium under the title "Le système des constantes astronomiques". At the time of the meeting, in May, Danjon is ill and J. Kovalevsky then director of the Service des Calculs et de Mécanique Céleste du Bureau des Longitudes will be the editor of the Proceedings. The astronomer Clemence from the US Naval Observatory is president of the meeting, and André Couder, as president of the Council of the Paris Observatory, welcomes the participants in the absence of Danjon.

Among the participants, Mikhailov from URSS, Jeffreys already present in 1950, Fricke from Heidelberg in charge of an introductory talk, Duncombe and Fedorov, Lévy and Guinot, Stoyko and Wilkins, Vicente and Brouwer, Shapiro and Kulikov, ... to mention a few other names. Among the constants discussed, the precession with its values according to different Earth models such as the values 9.2015", 9.2187", 9.1963", 9.1997". But besides the main term are given the coefficients corresponding to the fortnightly and semi-annual terms. About the first one of these terms Laplace had written in 1825 "J'avais négligé [...] la petite nutation dépendante du double de la longitude du nœud lunaire parce qu'elle me paraissait devoir être insensible. [...] j'en donne ici l'expression qui confirme ce que j'avais dit à son égard". To be noted the homage Laplace pays to d'Alembert mentioning that the work he made was published only one year and a half after Bradley's discovery.

On the other hand, among the best values given in 1963, are considered 9.207" by Clemence and 9.198" $\pm$ 0.002" by Fedorov, but there is also 9.2108" $\pm$ 0.0019" from Kulikov or 9.2066" $\pm$ 0.0042" by Spencer Jones. A working group is set up including Brouwer, Danjon, Fricke, Mikhailov and Wilkins as secretary. Its conclusions will have to be given at the next IAU/GA. The concluded constants will be known under the reference of that year 1964. The precession retained is 50.2564" $\pm$ 0.0222" and the nutation 9".210.

6. FURTHER CONSIDERATIONS

If we look at the values given in the past for precession all people were above. The lowest value was 50.3333" (from the 50"20"') by Lacaille and Lalande to compare with 50.2564", and with the 1896 value 50.2401". For nutation, all astronomers from the XVIIIth century gave higher value 9.6" with the exception Lindeman 8.989". Henderson with 9.28" was much better. For the values for 1896 to 1963, the closest results were given by Gill 9.2068", by Newcomb 9.214", by Clemence 9.207" and by Spencer Jones 9.2066".

Among today participants some have known and used the 1964 constants. During the following years many analysis were made in different countries and laboratories, derived from different techniques up to the new change which occurred in several steps. But it was not more during an historical period.

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FEDOROV'S ATTEMPT TO SOLVE THE NUTATION PROBLEM

YA.S. YATSKIV
Main Astronomical Observatory
National Academy of Sciences of Ukraine
03680 Golosiiv, Kyiv-127, Ukraine
e-mail: yatskiv@mao.kiev.ua

ABSTRACT. First of all brief history of the investigation of nutation carried out before the Fedorov's first studies of this problem is presented. Afterwards the main attention is paid to the Fedorov's determination of corrections to nutation coefficients from latitude observations and to the Fedorov's theory of nutation of a perfectly elastic Earth.

Fedorov showed that elastic deformations do not virtually affect the motion of the axis of angular momentum in space, i.e. the nutation of this axis. At the same time they diminish the coefficients of the expression of forced nearly diurnal motion of the pole, the so-called Oppolzer terms. Therefore Fedorov compared the expression of these terms for a perfectly elastic Earth with observations and stated: — the Earth as a whole is not a perfectly elastic body: — the theory of nutation of the Earth consisting of an elastic mantle and a fluid core has not been developed to a degree such that it is possible to test this theory by observations.

1. INTRODUCTION

1.1. ON THE DEFINITION OF NUTATION BY E. FEDOROV

Evgenii P. Fedorov gave the following definition of nutation in the Encyclopedia of Atmospheric Sciences and Astrogeology (New York, 1967): "The term nutation represents the periodic variations in the orientation of the rotation axis of the Earth in space. These variations are mainly caused by the action of the Moon and the Sun on the equatorial bulge of the Earth. For this reason the effect is called the luni-solar, or forced, nutation" (Fedorov, 1967). The term of 'periodic variations' has not been specified in this definition. However Fedorov has considered nutation as long-period variations in the orientation of the rotation axis or the angular momentum axis of the Earth in space (Fedorov, 1963).

1.2. BRIEF HISTORY OF THE INVESTIGATION OF NUTATION FROM THE 18TH CENTURY TO THE FEDOROV'S FIRST STUDIES OF NUTATION IN 50-IES OF THE 20TH CENTURY

The discovery of nutation by J. Bradley in 1747 attracted the attention of most prominent scientists of the 18th century to the problem of Earth's rotation. E. Fedorov noted that the construction of the classical theory of nutation based on the assumption of a perfectly rigid Earth had been completed by the end of the 19th century (Fedorov, 1963). In 1882 Th. Oppolzer derived the formulae of precession and nutation in the form they are used at present, namely, the angles of nutation in longitude and obliquity are represented as series with periodic terms. The greatest of these terms, which depends on the motion of the node of the Moon's orbit, is called the principal nutation term. The amplitude of the principal nutation term in obliquity is called the constant of nutation and is traditionally denoted by N. Based on the rigorous relations of the theory of rotation of a perfectly rigid Earth and knowing the constant of nutation, one can calculate all the other nutation coefficients. That is the reason why the constant of nutation was included in the system of astronomical constants until recently.

From the reduction of all available observations S. Newcomb determined in 1895 the most exact value for the constant of nutation $N = 9.210''$, which was adopted as a primary astronomical constant at the International Conference in Paris in 1896. At the same time N may be calculated theoretically based on the theory of rotation of a perfectly rigid Earth and with a knowledge of other constants (masses of the Moon and the Earth, constant of precession). Such calculations yielded values far in excess of Newcomb's constant of nutation. Various hypotheses and effects were invoked to account for this discrepancy (systematic errors of observation, relativity theory effects, elastic mantle and fluid core of the Earth, ets.). However, the discrepancy could not be eliminated, and in the 1930s W. de Sitter (De-Sitter, 1938) launched a revision of the system of astronomical constants which was completed after his death by D. Brouwer. For the constant of nutation de Sitter adopted

$$N_C = 9.2075'' \pm 0.0055'',$$

which is the average of the values found from observations. At the same time the constant of nutation calculated within de Sitter's self-consistent system of astronomical constants is equal to $9.2181''$. Thus the difference between the 'observed' and 'theoretical' values of the constant of nutation persisted.

The next attempt to critically revise the system of astronomical constants was made by G. Clemence in 1948. Based on the estimates derived by E. Przybyllok, H. Spencer Jones, H. Morgan, and others for the constant of nutation from observations, Clemence recommended to retain the 1896 value $N = 9.210''$, which differs essentially from the constant of nutation for a perfectly rigid Earth. As it often occurs in science, interest was expressed again in the studies by W. Thomson (Lord Kelvin), F. Sludskii, and H. Poincare made late in the 19th century and early in the 20th century, where it was demonstrated that the nutation of the real Earth may essentially difer from the nutation of a perfectly rigid Earth due, in particular, to the influence of a fluid core. In 1948 H. Jeffreys attempted to explain the discrepancy between the 'observed' and 'theoretical' constants of nutation, using Poincare's theory (Jeffreys H., 1948, 1949).

Fedorov's first studies treating of nutation go back to that period. In his paper "On the importance of the direct derivation of short-period nutation terms" published in Astronomicheskii Tsirkulyar (Fedorov, 1950), having emphasized the priority of F. Sludskii in the problem of the effect of Earth's fluid core on its nutation, Fedorov wrote: "Using K. Bullen's data on the oblateness of the Earth and the moment of inertia of its core, we calculated that the amplitudes of the nutation terms which include the doubled Moon's longitude in their arguments must be increased by somewhat more than 1/10 of their tabulated values. We believe that this increase may account for the presence of lunar terms in latitude variations..." Notice that the case in point was the analysis of the latitude observations at the ILS stations and the systematic variations depending on Moon's position which were detected in those observations. This first note initiated the well-known series of Fedorov's studies on the determination of nutation coefficients from the latitude observation data.

2. DETERMINATION OF CORRECTIONS TO NUTATION COEFFICIENTS FROM LATITUDE OBSERVATION DATA (OVERVIEW OF STUDIES BY E. FEDOROV AND HIS PUPILS)

2.1. LUNAR FORTNIGHT LATITUDE VARIATIONS

The first attempts to determine fortnight terms in latitude variations were made by N. Sekiguchi, H. Morgan, and N.A. Popov (see Fedorov, 1963) for reference). Fedorov reasoned that such determinations "were of great interest when regarded as a means for testing the theory" (Fedorov, 1951), and he determined for himself and in collaboration with E.I. Evtushenko the lunar fortnight terms in the variations of the latitudes of the ILS stations Carloforte, Ukiah, and Mizusawa; the terms were represented in the form:

$$\Delta\varphi = M_1 \sin(2L_M - \alpha) + N_1 \cos(2L_M - \alpha) + M_2 \sin(2L_M + \alpha) + N_2 \cos(2L_M + \alpha) \qquad (1)$$

where L_M is the mean longitude of the Moon, α is the right ascension of the star observed. The results were published in and summarized:

$$\Delta\varphi = \underbrace{0.0088}_{\pm 0.0013} \sin(2L_M - \alpha - \underbrace{14^\circ}_{\pm 8}) - \underbrace{0.0021}_{\pm 0.0007} \sin(2L_M + \alpha - \underbrace{3^\circ}_{\pm 19}) \qquad (2)$$

It was the most accurate determination of the lunar nearly diurnal wave from latitude observations, which led to conclusion that the theory of rotation of a perfectly rigid Earth was inadequate.

2.2. SEPARATE DETERMINATIONS OF THE PRINCIPAL NUTATION TERM COEFFICIENTS IN OBLIQUITY AND LONGITUDE

As noted above, within the framework of the classical theory of nutation it was sufficient to determine from observations the constant of nutation N only. Fedorov was the first to attempt to determine the principal nutation term coefficients in obliquity and in longitude separately (Fedorov. 1951). This attempt gave no grounds for any definitive conclusions, and Fedorov formulated the general problem of determining all components in the principal nutation term in view of a possible phase lag in the nutation (Fedorov, 1954, 1955). He reduced anew about 135000 observations made at Carloforte, Mizusawa, and Ukiah from 1900 through 1934 (Fedorov, 1957, 1959, 1963). This was done with original methods for determining and eliminating possible systematic errors (value of a scale division of micrometer, planetary aberration, ets.). It took Fedorov only one page to expound the results of this titanic study. He wrote(Fedorov,1963): "The constant of nutation N derived is equal to $9.198'' \pm 0.0018''$, while the generally adopted value is $9.210''$, and the theoretical value obtained from the well-known relationship between this constant, Earth's oblateness H, and the ratio μ of the Moon's mass and the Earth's mass ranges from $9.215''$ and $9.226''$. The theoretical value of n (*ratio of nutation ellipse axes* — author's comment) seems to require no correction, and a phase lag is observed in the nutation in longitude only". Fedorov cast doubt on the reality of phase lag, and he believed that it does not exceed 5 angular minutes if ever exsists (Fedorov, 1959).

The results of determination of the coefficients of the principal nutation term brought fame to Fedorov throughout the world. They formed the basis of his Doctor Dissertation and were presented in his classic book "Nutation and Forced Motion of the Earth's Pole" (Fedorov, 1963).

They were also reported at the X General Assembly of the IAU in Moscow (1958) and other scientific conferences. The world-famed scientist H. Jeffreys wrote about Fedorov's investigation: "The theory predicts a stronger effect (*of the elasticity of the mantle and the fluid core of the Earth* — author's comment) on longitude than on obliquity. E.P. Fedorov is the only astronomer who tried to estimate these effects from observations... This is a very valuable study" (Jeffreys, 1959).

2.3. FEDOROV'S PUPILS EXPLORE NUTATION FURTHER

Although Fedorov determined the coefficients of the principal nutation term from the ILS observations with a very high accuracy, he was not satisfied whith the inhomogeneity of various series of these observations which hampered the study of such long-term effects as a 18.6-year nutation. He proposed his post-graduate student S.P. Major to reduce the latitude observations (Major, 1966). The results of that investigatios were used by V.K. Taradii, also Fedorov's pupil, who repeated the work of his teacher, adding new observational material for years 1935–1941 (Taradii, 1968). Taradii obtained $N = 9.1970'' \pm 0.0010''$ and found neither a phase lag nor a deviation of ratio of the axes of the ellipse of nutation from its theoretical value. Assuming that the absence of this deviation is real, N.A. Popov and the author of this paper (who is also a pupil of Fedorov) could determine unambiguously the constant of nutation from observations of bright zenith stars at Poltava (Yatskiv, Popov, 1980). They found $N = 9.203''$.

By that time the corrections to the nutation coefficients were determined at other research centers — in Paris from the Bureau International d'Heure data, in Mizusawa from the ILS data, ets. (see, Yatskiv, 1980). Those determinations were in reasonable agreement, and it became evident that a new theory of nutation which would take proper account of some peculiarities in the Earth's structure should be adopted. It was recognized in 70-ies that the discrepancies between the nutation coefficients calculated for a more or less realistic Earth model and the coefficients derived from observations diminish by about an order of magnitude as compared to the model of a perfectly rigid Earth.

3. ON THE THEORY OF NUTATION OF A PERFECTLY ELASTIC EARTH

Foundations of any theory need a revision if some hitherto unknown discrepancies are found between the predictions of this theory and observation data. Such was the situation in which the theory of rotation of a perfectly rigid Earth found itself in the middle of the 20th century. It was found to fail to agree with observations, and it did not meet the requirements for the accuracy of astronomical reductions. Fedorov thought it more reasonable to compare observation results to the deductions of the theory of rotational motion of a perfectly elastic Earth (he recognized, of course, that the Truth lies somewhere between these two extremes) . He developed such a theory on the following assumptions:

a) the Earth as a whole is a perfectly elastic body;

b) mechanical properties of the Earth are functions of the distance from its center only (Fedorov, 1963).

Fedorov showed that elastic deformations do not virtually affect the motion of the axis of angular momentum in space, i.e., the nutation of this axis. At the same time they diminish the coefficients of the terms in the equation of the forced nearly diurnal motion of the pole, the so-called Oppolzer terms,[3] in the same ratio. It was difficult to compare the deductions

[3] In 1882 Th. Oppolzer demonstrated that the instantaneous axis of rotation of a perfectly rigid Earth and the axis of angular momentum execute, in addition to the nutation motion, a nearly diurnal motion in the Earth's body under the action of the tidal forces of the Moon and the Sun.

of this theory with observations, as the Oppolzer terms have a form similar to expression (1). Thus it is impossible to determine separately from observations the effects of inaccuracies in the fortnight nutation term and in the Oppolzer term. It was believed prior to the Fedorov's investigations that the coefficients of the Oppolzer term calculated from the theory were not subject to any corrections — they were taken into account and then corrections to nutation coefficients were determined. Fedorov assumed that it was the motion of the axis of angular momentum with respect to a coordinate system fixed in Earth's body which was determined from latitude observations (the nutation of this axis needs no refinement to allow for elastic deformations), and he compared the variations of form (2) with the expression he derived for the forced latitude variations for a perfectly elastic Earth:

$$\Delta\varphi_T = +0.0051'' \sin(2L_M - \alpha) \tag{3}$$

It is obvious from a comparison of (2) and (3) that the Earth "as a whole is not a perfectly elastic body" (Fedorov, 1963). He understood that there were some arbitrariness in this approach to the comparison between theory and observations. Since the observer is connected to the Earth's mantle, the data obtained from observations refer to the motion of this mantle only rather than to the Earth as a whole. Moreover, he tried to draw some qualitative conclusions as to the interaction between the Earth's core and mantle. Nevertheless, Fedorov had to recognize that "the theory of the rotational motion of the Earth consisting of an elastic mantle and a fluid core has not been developed to a degree when it is possible to test the basic assumptions of this theory by comparing its deductions to observation data". Such a test became possible due to the further development of the theory of nutation in the works by H. Jeffreys and R. Vicente, M.S. Molodenskii and others. Fedorov followed attentively those investigations and kept in touch with their authors.

4. FROM WOOLARD'S THEORY OF NUTATION TO WAHR'S THEORY OF NUTATION

In 1953 E. Woolard published his classical theory of rotation of a perfectly rigid Earth (Woolard, 1953). The nutation expansions and coefficients he proposed were used in astronomy for more than 20 years. This 'longevity' of the theory may be attributed to the fact that the accuracy of astronomical observations improved very slowly in the midtwentieth century (it remained at a $0.1''$ level), while the overall contribution from the errors of Woolard's theory to these observations did not exceed $0.05''$. The only exception was the highly accurate latitude observations mentioned above. A number of systematic effects in latitude observations were eliminated by taking into regard the influences of the elastic mantle and the Earth's fluid core when the theory of nutation was elaborated. It should be noted that the early theoretical investigations of nutation by Jeffreys and Vicente and Molodenskii were based on the most thorough mantle models (by Bullen) available at that time and very simplified Earth's core models.

Fedorov gives in (Fedorov, 1967) a table which demonstrates distinctions between the nutation coefficients calculated for various Earth's core models (see Table 1).

Two important inferences may be drawn from this table (in view of the above definitions of nutation):

a) the theoretical nutation coefficients calculated for a rigid Earth with a fluid core are in fair agreement with observations;

b) the accuracy of classical astrometric determinations of nutation is not sufficient for choosing the most adequate model of Earth's structure.

Table 1. Nutation coefficients for various Earth's core models

Models	Nutation			
	in obliquity		in longitude	
	principal term	half-month term	principal term	half-month term
J–V(I)	9.2015″	0.0972″	6.8260″	0.0896″
J–V(II)	9.2187	0.0971	6.8491	0.0897
M(I)	9.1963	0.0969	6.8325	0.0899
M(II)	9.1997	0.0965	6.8369	0.0895
Observations	9.1974	0.0965	6.8437	0.0934

N o t e. J–V(I) — first model by Jeffreys and Vicente: homogeneous fluid core with additional mass at the center; J–V(II) — second model by Jeffreys and Vicente: fluid core with its density varying by Roche's law; M(I) — first model by Molodenskii: fluid core with its density varying with pressure in a certain manner; M(II) — second model by Molodenskii: as distinct from M(I), it includes a rigid central core with constant density.

Advances in space research required a revision of the system of astronomical constants, and this revision was conducted on the recommendation of COSPAR. A new system of astronomical constants was approved at the XVI General Assembly of IAU in 1976 . However, a change to a new theory of nutation had not been sufficiently substantiated by that time, and it was decided to retain temporarily the Newcomb's nutation constant in the new IAU system of constants, although it was reduced to the new standard epoch J 2000 ($N_{2000} = 9.2109''$). At the same time, following the R. Atkinson's proposition, the Assembly recommended to use the Woolard's nutation expansions for the axis of the Earth's figure rather than the expansions for the instantaneous axis of rotation (Atkinson, 1973). These IAU resolutions initiated numerous thorough discussions of the nutation-related problems at various scientific meetings. By the proposal of the author made at the XVI IAU General Assembly IAU Symposium No. 78 "Nutation and the Earth's Rotation" was organized in Kyiv in 1977, its Scientific Organizing Committee was headed by E. Fedorov. New theoretical investigations of nutation and the results of determinations of the nutation coefficients from observations were reported and discussed at the Symposium. Special emphasis was placed on the choice of an axis whose motion should be described by theoretical nutation expansions. Fedorov held to the idea (with some additional substantiation and refinements) that this should be the axis of angular momentum (Fedorov, 1980). No unanimity could be achieved at this symposium in this question as well as in many other problems. A working group of nine members was organized; headed by P.K. Seidelmann, it worked fruitfully during two years and presented its report at the XVII IAU General Assembly (Montreal, 1979) . The report included a new system of nutation coefficients based on the Kinasita's theory of the rotation of a perfectly rigid Earth (it was more exact as compared to Woolard's theory); the effect on nutation of the departure of the Earth from a perfectly rigid body was also taken into account in accordance with Molodenskii's theory (model M(II)) (Molodenskii, 1961). A proposal to adopt the recently completed theory of nutation by Wahr as an international standard was also discussed at the Montreal Assembly. Wahr built the most exact theory of nutation with the use of the Earth model proposed by F. Gilbert and A.M. Dziewonski (model 1066A). Discrepancies between the nutation coefficients proposed by the Working Group and Wahr ranged up to 0.002″. Such discrepancies were insignificant for the accuracy attained at that time in astronomical and space observations. Therefore, the proposal of the IAU Working Group was accepted, as the IAU Nutation 1979.

Before long, however, the IGGU General Assembly (Canberra, 1979) suggested that the IAU should revise this nutation system so that a new theory of nutation could be based on a more modern Earth model (as compared to Molodenskii's model). This IGGU Assembly resolution gave rise to a new wave of discussions, and as a result the IAU Nutation 1980 based on Wahr's theory of nutation was adopted as a standard.

5. BRIEF DIGRESSION

Fedorov followed attentively the discussion on the problem of nutation. However, his health became worse, and he could not leave Kyiv to take an active part in this discussion. At the same time the author was fortunate to have a chance to present his ideas on the problem of nutation to Fedorov, a kind critic and an exacting teacher. The author has compared the IAU 1979 and the IAU 1980 nutation series (Yatskiv, 1982) and showed that the IAU 1980 Nutation theory offered an essential advantage over Molodenskii's theory — the Earth structure peculiarities were taken into account more exactly in it (at a 0.001″ level). In preprint "Nutation in the system of astronomical constants" the author attempted to summarize the discussion on the nutation problem as of 1980 (Yatskiv, 1980). Fedorov read the manuscript, made some useful comments, and wrote a review (see, Yatskiv, 1999). This seems to be Fedorov's last printed opinion on nutation.

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CURRENT STATUS OF THE NUTATION THEORY

V. DEHANT
Royal Observatory of Belgium
3 avenue Circulaire, B-1180 Brussels, Belgium
e-mail: v.dehant@oma.be

ABSTRACT. This paper is a summary of what has been discussed during these last years among the members and correspondents of the Joint IAU/IUGG WG on 'Non-rigid Earth Nutation Therory' and of what has been decided at the IAU Colloquium 180 (Washington, USA) and at the last IAU General Assembly (Manchester, UK).

INTRODUCTION

The WG on 'Non-rigid Earth Nutation Theory' has been working at different levels from the rheological models of the Earth's interior, the transfer function for the non-rigid Earth, the rigid nutation models, to the atmospheric and ocean effects on nutations, and further comparisons with the VLBI observations.

1. FIRST SET OF CONCLUSIONS

The first conclusions of the WG (see also Dehant et al., 1999) were that there are three rigid Earth nutation models available presently and precise at the one tenth of microarcsecond (mas) level:
(1) SMART97 of Bretagnon et al. (1997, 1998); these authors have used the torque approach and have used a truncation level of 0.01 μas; they have obtained 3910 terms at that level of precision; at 0.1 μas truncation level, they have 1590 terms.
(2) RDAN97 of Roosbeek and Dehant (1998); these authors have also used the torque approach, but they have used a truncation level of 0.1 μas, in which case they have 1507 terms.
(3) REN2000 of Souchay and Kinoshita (1996 and 1997); these authors have used the Hamiltonian approach in order to compute the rigid Earth nutations; they have worked at the 0.1 μas level and have 1325 terms.

All the series are very precise, but SMART97 is the most precise one. The arguments used for this last series are however not the classical ones. Although they are more appropriate for general secular theory, they are not widely used.

2. SECOND SET OF CONCLUSIONS

The second set of conclusions is related to the transfer functions for the non-rigid Earth. The numerical integration models inside the Earth are not yet competitive with the empirical or semi-analytical models due to the lack of dissipation (no electromagnetic torque at the core-mantle boundary). The remaining competitive models are:

(1) MHB2000 of Mathews et al. (2000, see also Mathews, 2000), a semi-analytical model, starting from a transfer function for the Earth and a rigid Earth nutation series with about 1500 waves, incorporating a frequency resonance with physical parameters fitted to VLBI observations, obeying the sum rules, incorporating atmospheric annual effect fitted to observations, including frequency dependence (partly fitted) for ocean tides due to the FCN resonance and to ocean dynamics, and finally including electromagnetic coupling of the core (conductivity);

(2) FG2000 of Getino and Ferrandiz (2000), using a global Hamiltonian approach for 106 waves fitted to VLBI data, incorporating a resonance with global parameters fitted to observations as the FCN free mode frequency, compliances, and dissipation coefficients, incorporating ocean corrections from Huang et al. (2000) with a frequency dependent resonance, incorporating fitted atmospheric corrections, and necessitating empirical corrections;

(3) SF2000 of Shirai and Fukushima (2000a and 2000b) or Herring (2000, not published), empirical models, based on a simple resonance formula fitted to the VLBI observation; although these models are very simple, they provide values very close to the observations.

Due to dissipation at the core boundaries and in the inelastic part of the Earth, the resonance formula cannot be considered as an exact formula. The resonance strengths and the mode frequencies are frequency dependent. This is the reason why MHB2000 provides a 'mean' resonance formula and additional theoretical corrections. This is not accounted for in GF2000 or in the empirical models as SF2000 or the one developed by Herring.

The physical modeling of the electromagnetic torque at the core-mantle boundary involves the presence of an induced electric current in a conductive layer at the bottom of the mantle. The high conductance (product of the conductivity and the thickness of the layer) of this layer is not only necessary to explain the nutations, but also to explain the length-of-day variations. But laboratory experiments on porosity of the perovskite in the mantle and the iron infiltration from the core did not allow such a large conductance (see Poirier and Le Mouël, 1992). The thickness of the layer was thus previously rather controversial. However, it has been explained at the last SEDI symposium (Study of the Earth Deep Interior, Exeter, UK, July 2000) by a chemical reaction between the perovskite of the mantle and the iron of the liquid core, followed by a sedimentation process (Buffett et al., 2000). Therefore, these new findings reconcile the different thicknesses found in the literature, and in particular, justify the nutation dissipation found from the VLBI data.

The advantage of GF2000 model is the global approach. This is believed to be the future for nutation computation. It has the advantage of the uniformization between the rigid and non-rigid nutations.

The advantage of the empirical models is their simplicity. But the parameters obtained in the fit may not be interpreted in terms of physics of the Earth's interior.

Consequently, it is believed that the most complete geophysics-based nutation series is MHB2000. Additionally, MHB2000 does not only give the nutations of long periods, but also diurnal and subdiurnal nutations. The IAU2000 resolution refers to that model.

On the other hand, analyses of residuals of the competitive models with respect to the observations have shown that MHB2000 was slightly better, but the differences are at a level lower than the observation precision.

3. THIRD SET OF CONCLUSIONS

Concerning the ocean and atmospheric effects, two approaches have been used for computing these contributions to nutations: (1) the torque approach and (2) the angular momentum approach. The torque approach consists in computing the fluid pressure, gravitational and friction torques acting on the solid Earth. The angular momentum approach consists in considering the Earth and the fluid ocean and atmosphere as one global system conserving its angular momentum. So, the angular momentum variations of the solid Earth can be computed from the angular momentum variations of the fluids. Presently, there are two order of magnitude difference between the results from the two approaches. This problem still needs to be solved. Additionally, the results computed from one series of data (or by one meteorological center for instance) are very different. It is certain that this field needs further investigations and that the meteorology community should help the geodesists in that frame.

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NEW DEVELOPMENT OF THE NUMERICAL THEORY OF THE EARTH ROTATION

V.V. PASHKEVICH
Central Astronomical Observatory of the Russian Academy of Sciences
Pulkovskoe Shosse 65/1, Saint Petersburg, 196140, Russia
e-mail: apeks@gao.spb.ru

ABSTRACT. The account for the geodetic perturbations in the rotational motion of the rigid Earth is discussed. The comparison of the numerical solution of the problem and the semi-analytical theory SMART97 (KINEMATICAL solution) reveals large discrepancies of the mixed periodical character attaining $300\mu as$ at the end of 200yr time interval. It is found that the reason of this disagreement is the procedure of taking into account the geodetic perturbations in the semi-analytical theory. The KINEMATICAL solution of SMART97 is constructed by means of the geodetic rotations of the DYNAMICAL solution which must not contain geodetic perturbations. But the DYNAMICAL solution of SMART97 includes these perturbations implicitly. Namely the values of the mean motions of the planets contain the value of the geodetic precession and in the result of the analytical integration the geodetic perturbations arise in the amplitudes of the nutational harmonics. In the present paper it is shown that after the removal of the geodetic precession from the mean longitudes of the planets in the nutational harmonic arguments the discrepancies between the KINEMATICAL solution and the numerical integration do not exceed $15\mu as$ at the 200 yr time interval and do not surpass $7\mu as$, after correcting the amplitudes of the principal nutational harmonics.

1. INTRODUCTION

During the last years a number of investigations appears (see (Dehant et al.,1999)) concerning the construction of high precision theories of the non-rigid Earth rotation. In order to model correctly the non-rigid effects in the Earth rotation on the microarcsecond level of accuracy it is necessary to create a high-precision numerical theory of the rigid Earth rotation.

In the paper (Pashkevich, 1999) a numerical solution of the rigid Earth rotation was performed in the Rodrigues-Hamilton parameters. A comparison of this solution with the semi-analytical solution of the Earth's rotation SMART97 (Bretagnon et al., 1998) was carried out in the standard Euler angles over 2000-2199 years time interval. The residuals did not surpass $\pm 4.5\mu as$ in the DYNAMICAL case (without taking into account the geodetic perturbations) and $\pm 25\mu as$ in the KINEMATICAL one (with account for the geodetic perturbations). The purpose of this investigation is the explanation of a good agreement between this numerical solution and the semi-analytical solution SMART97 in the DYNAMICAL case and not sufficiently good consistency between them for the KINEMATICAL case.

In the present paper the same mathematical model of the rigid Earth rotation is considered, as was used in (Pashkevich, 2000). The rotation of the Earth about its centre of masses is described as the rotation of its principal axes of inertia with respect to the non-rotating geocentric coordinate system. As the variables of the problem, four Rodrigues-Hamilton parameters are taken. In oder to generate the geodetic perturbations in the Earth rotation the same additional geodetic part of the Lagrange function is used, as in (Eroshkin, Pashkevich, 1997).

It can be noted that the method of calculating the geodetic perturbations used in the numerical solution reproduces rather well the main components of the geodetic rotation of the Earth, namely, the geodetic precession ($P_g = -1''.919...$ per century) and the main terms of the geodetic nutation ($\Delta_g\psi = -149.22\mu as \cos\lambda_3 - 34.28\mu as \sin\lambda_3$, where λ_3 is the mean longitude of the Earth).

The numerical integration of the problem is carried out over the time interval 2000 – 2199 years from the initial epoch JD 2451545.0 (January 1, 2000) by the high-precision numerical integration method HIPPI (Eroshkin, 2000). The integration is performed with one day constant step size and the 32-th degree of Chebyshev polynomials approximating uniformly the right-hand sides of the differential equations.

The initial conditions of the numerical integration are taken from the semi-analytical theory SMART97. The initial condition for the angle of proper rotation ϕ is calculated by the following formula:

$$\phi = \phi_{SMART97} + \frac{1}{2} arctg \left(\frac{S_{2,2}}{C_{2,2}} \right).$$

2. ANALYSIS OF THE DISCREPANCIES

A comparison of the results of the numerical integration with the semi-analytical solution SMART97 in Euler angles is carried out for each day over all the interval of integration. The corresponding discrepancies have systematic trends and periodic harmonics. For the mathematical description of the character of these discrepancies the least squares method is applied. Systematic trends are found in the discrepancies for all Euler angles. In these discrepancies the systematic trends are removed in the expressions for the luni-solar precession and inclination and the solutions SMART97 is correspondingly modified. The initial conditions are taken from the original theory SMART97. The process of the numerical integration of the rotational motion equations of the rigid Earth is repeated anew, with the initial conditions taken from the modified semi-analytical solution SMART97. After the removal of the systematic trends the discrepancies represent a superposition of the harmonics of a mixed-periodic character (Fig.1, 3, 5). The period of the basic harmonic is the one of the principal nutational harmonic ($\sim$ 18.6 years). The period of the comparison time is not sufficiently large for a precise determination of the argument of this harmonic. Thus the most reasonable is the determination of the coefficients of the above nutational harmonic, whose argument is considered known and taken from the semi-analytical theory SMART97. This principle is applied for the determination of the correction coefficients to the other harmonics in the SMART97 theory.

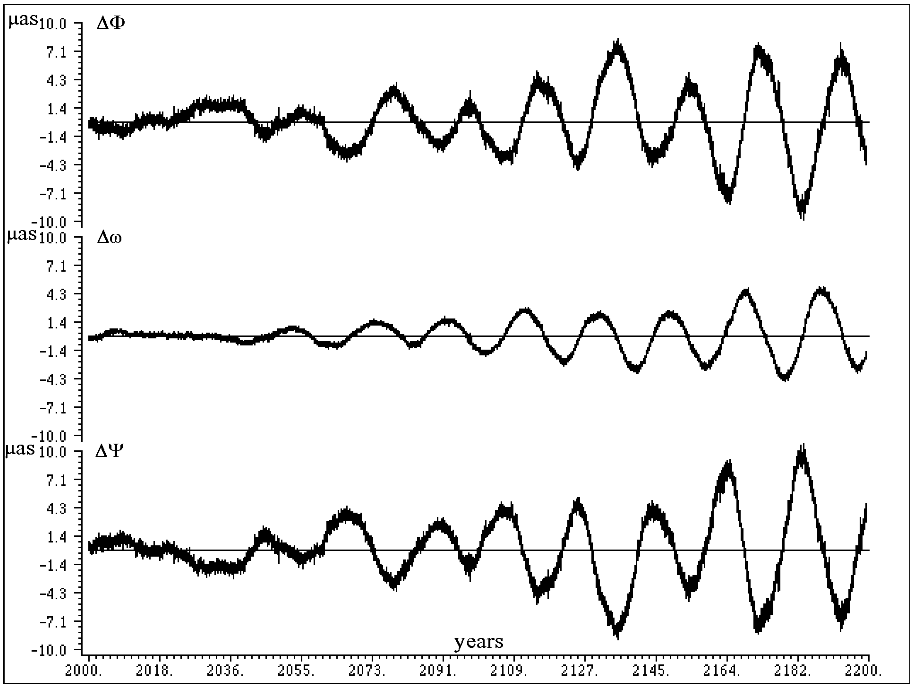

Figure 1: Simulated impact of ocean tides on polar motion.

It should be noted that the comparison of the numerical solution was carried out in the paper (Pashkevich, 1999) and a good agreement was established with the DYNAMICAL solution SMART97 (Fig. 1, 2).

A similar comparison was performed in (ibid.) with the KINEMATICAL solution SMART97 but no good agreement was revealed between the numerical and the semi-analytical theories (Fig. 3, 4).

Large mixed-periodical harmonics in the residuals between the results of the numerical integration with the KINEMATICAL solution SMART97 was discovered, as can be seen from Fig.3.

In order to study this phenomenon in more detail the analysis of the semi-analytical theory SMART97 is carried out in the present paper. The authors of SMART97 "...have added the precession to the mean longitudes of the planets in order to have the same frequencies and the same periods as in the other solutions. Thus, the mean longitudes of the planets are reckoned from the equinox of date." Therefore in the DYNAMICAL solution SMART97 the arguments of the nutational harmonics already contain the geodetic precession. Consequently, in the result of the analytical solution of the corresponding differential equations the coefficients of the nutational harmonics in the DYNAMICAL solution SMART97 also contain the geodetic perturbations. Then from of the DYNAMICAL solution the authors of SMART97 constructed the KINEMATICAL solution SMART97 by means of the geodetic rotations. However the DYNAMICAL solution should not contain the geodetic perturbations if the KINEMATICAL solution is constructed from it.

The following numerical experiment confirms this statement. In the KINEMATICAL solution SMART97 the geodetic precession is excluded from the mean longitudes of the planets. The initial conditions are calculated anew and the numerical integration is performed.

Then the least squares method is applied for comparison of the results of the numerical integration and the KINEMATICAL SMART97 solution. As a result, the discrepancies $d\psi$ and $d\omega$ are revealed in the expressions of the KINEMATICAL luni-solar precession $\psi_{smart97}$ and KINEMATICAL luni-solar inclination $\omega_{smart97}$:

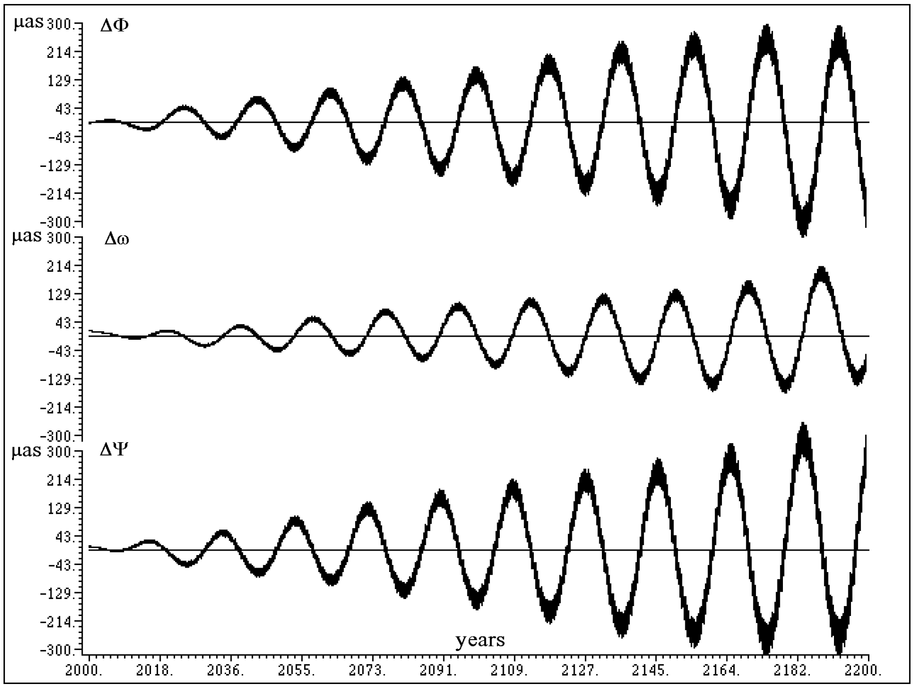

Figure 2: Numerical integration minus KINEMATICAL solution SMART97 after inserting the secular corrections.

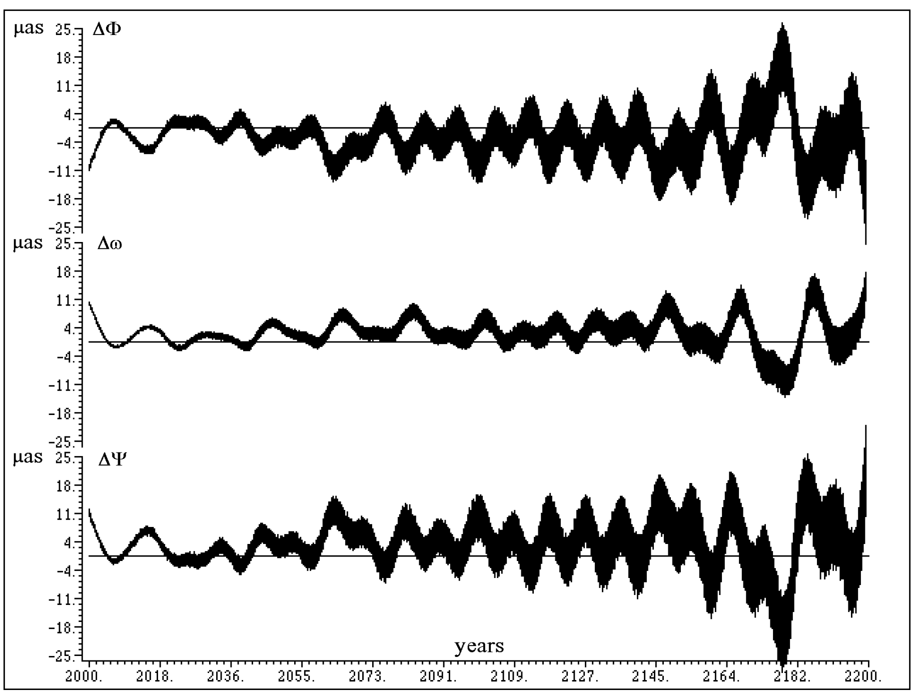

Figure 3: Numerical integration minus KINEMATICAL solution SMART97 after inserting the secular and the periodic corrections.

$$\psi_{smart97} = 50384564881.3693T - 107199953.5817T^2 - 1143646.1500T^3 + +1328317.7356T^4 - 9396.2895T^5$$

$$
\begin{aligned}
d\psi &= -0.8_{\pm 0.1} + 327_{\pm 8}T + 1900_{\pm 200}T^2 - 22000_{\pm 1000}T^3 + 31000_{\pm 3000}T^4 \\
\omega_{smart97} &= 84381409000 - 265011.2586T + 5127634.2488T^2 - 7727159.4229T^3 - \\
&\quad -4916.7335T^4 + 33292.5474T^5 \\
d\omega &= -2.34_{\pm 0.06} + 28_{\pm 4}T - 2260_{\pm 80}T^2 + 3800_{\pm 600}T^3 + 11000_{\pm 2000}T^4
\end{aligned}
$$

where T is the time expressed in Julian thousand years, counted from the fundamental epoch J2000. The values of the coefficients are expressed in microarcseconds.

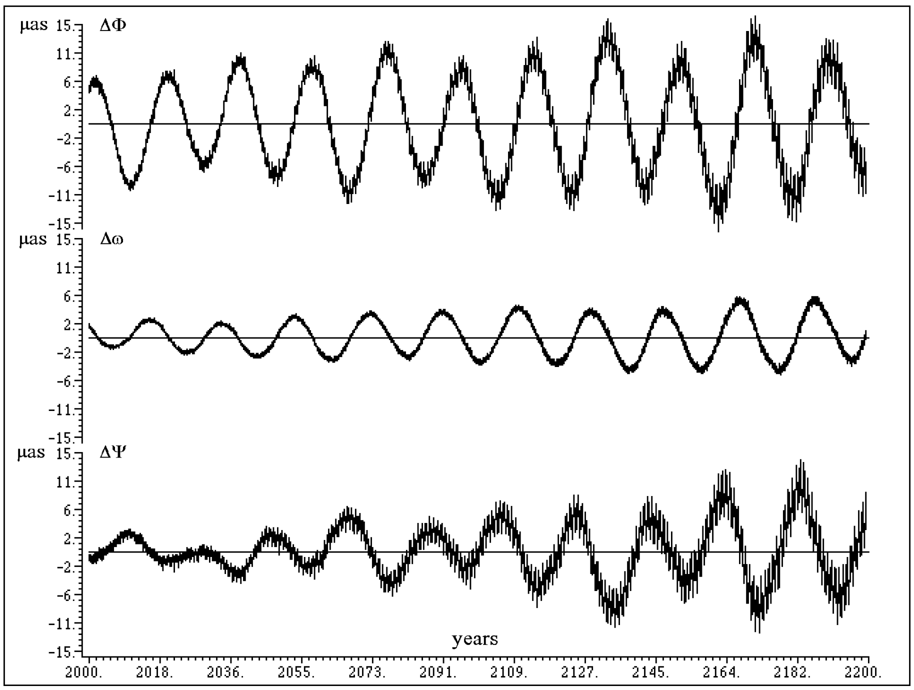

Figure 4: Numerical integration minus KINEMATICAL solution SMART97 after inserting the secular corrections.

The systematic trend in the angle ϕ is removed by selecting an initial date of the numerical integration, namely JD2451545.08366. After the corrections to the angles ψ, ω and ϕ are introduced into the semi-analytical solution SMART97 the process of the numerical integration is repeated anew with the initial conditions taken from the modified semi-analytical solution SMART97. The behaviour of the discrepancies in the longitude of the ascending node of the dynamical Earth's equator $\Delta\psi$, in the inclination of the dynamical Earth's equator to the fixed ecliptic J2000.0 $\Delta\ \omega$ and in the proper rotation angle $\Delta\ \Phi$ is presented in Fig.5. The comparison of Fig.3 and Fig.5 demonstrates the correctness of the above given statement concerning the KINEMATICAL solution.

The discrepancies $\Delta\psi$, $\Delta\omega$ and $\Delta\phi$ in the coefficients of some nutational harmonics of the Euler angles of the semi-analytical theory SMART97 are found by the least squares method:

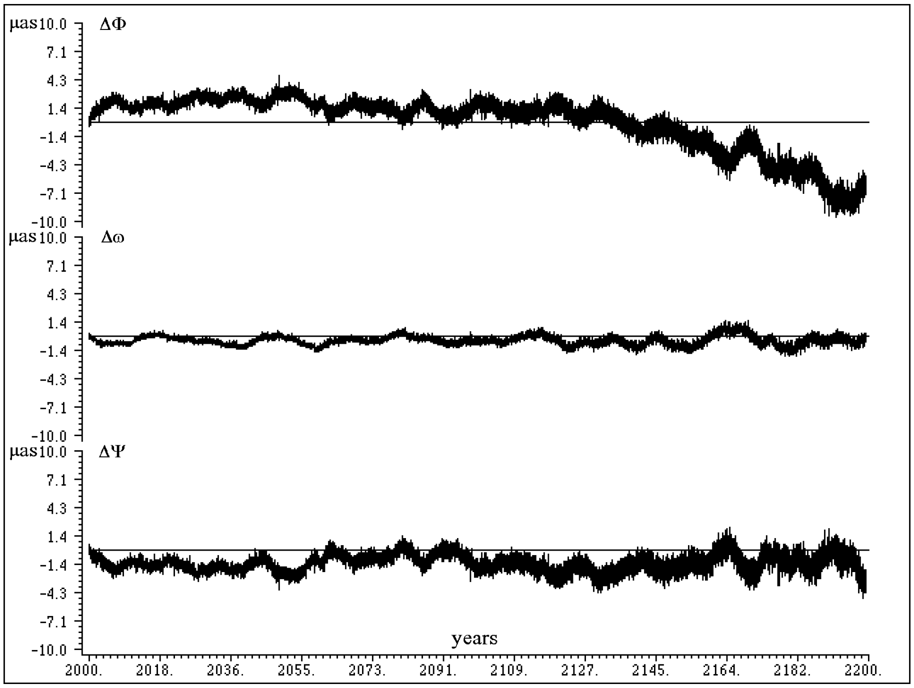

Figure 5: Numerical integration minus KINEMATICAL solution SMART97 after inserting the secular and the periodic corrections.

$$\begin{aligned}
\Delta\psi = \ & (-0.38 - 64.1T + 277T^2)\sin(\lambda_3 + D - F) + (1.33 - 18.4T - 157T^2)\cos(\lambda_3 + D - F) + \\
& +(-0.02 - 0.5T + 2T^2)\sin(2\lambda_3) + (-0.07 + 0.8T + 4T^2)\cos(2\lambda_3) + \\
& +(-0.01 + 14.9T - 9T^2)\sin(\lambda_3) + (-0.01 - 3.1T + 1T^2)\cos(\lambda_3) + \\
& +(-0.19 + 8.1T - 39T^2)\sin(3\lambda_2 - 5\lambda_3) + \\
& +(-0.16 + 5.6T - 18T^2)\cos(3\lambda_2 - 5\lambda_3) + \\
& +(0.16 - 16.6T + 48T^2)\sin(5\lambda_2 - 6\lambda_3 + 2D - 2F) + \\
& +(0.15 - 19.2T + 113T^2)\cos(5\lambda_2 - 6\lambda_3 + 2D - 2F) + \\
& +(1.04 - 10.2T + 0T^2)\sin(2\lambda_3 - 3\lambda_5 + 2D - 2l) + \\
& +(-0.59 + 24.7T - 156T^2)\cos(2\lambda_2 - 3\lambda_3 + 2D - 2l) + \\
& +(0.27 - 12.1T + 37T^2)\sin(\lambda_6) + \\
& +(0.60 + 6.6T - 42T^2)\cos(\lambda_6),
\end{aligned}$$

$$\Delta\omega = \ (0.423 - 8.9T - 77T^2)\sin(\lambda_3 + D - F) + (1.38 + 41.9T - 188T^2)\cos(\lambda_3 + D - F),$$

$$\begin{aligned}
\Delta\phi = \ & (-6.40 - 65.3T + 291T^2)\sin(\lambda_3 + D - F) + (1.31 - 16.9T - 140T^2)\cos(\lambda_3 + D - F) + \\
& +(-0.03 - 0.5T + 2T^2)\sin(2\lambda_3) + (-0.06 + 0.3T + 4T^2)\cos(2\lambda_3) + \\
& +(0.02 + 12.3T - 4T^2)\sin(\lambda_3) + (0 - 4.9T + 2T^2)\cos(\lambda_3) + \\
& +(-0.17 + 6.3T - 30T^2)\sin(3\lambda_2 - 5\lambda_3) + \\
& +(-0.35 + 6.4T - 21T^2)\cos(3\lambda_2 - 5\lambda_3) + \\
& +(0.07 - 13.8T + 39T^2)\sin(5\lambda_2 - 6\lambda_3 + 2D - 2F) + \\
& +(-0.02 - 13.0T + 79T^2)\cos(5\lambda_2 - 6\lambda_3 + 2D - 2F) + \\
& +(0.98 - 11.5T + 16T^2)\sin(2\lambda_3 - 3\lambda_5 + 2D - 2l) + \\
& +(-0.69 + 25.1T - 153T^2)\cos(2\lambda_2 - 3\lambda_3 + 2D - 2l) + \\
& +(0.22 - 11.5T + 35T^2)\sin(\lambda_6) + \\
& +(0.48 + 7.7T - 48T^2)\cos(\lambda_6).
\end{aligned}$$

Here $\lambda_3 + D - F = \Omega + 180^o$; $\lambda_2, \lambda_3, \lambda_5, \lambda_6$ are the mean longitudes of the Venus, Earth, Jupiter and Saturn respectively; D is the difference between the mean longitudes of the Moon and the Sun; Ω is the mean longitude of the ascending node of the lunar orbit; F is the mean argument of the Moon's latitude; l is the mean anomaly of the Moon. The values of the coefficients are expressed in microarcseconds.

The mean-root-square discrepancies in the above coefficients are less than the values of the corresponding coefficients by two orders.

The results of comparison of the modified semi-analytical solution SMART97 with the numerical solution in the angles $\Delta\psi$, $\Delta\ \omega$ and $\Delta\ \Phi$ demonstrate a good consistency between these theories (Fig.6.).

3. CONCLUSION

The results of the present investigation are as follows.

a) The comparison of the numerical solution of the problem and the semi-analytical theory SMART97 (KINEMATICAL solution) reveals large discrepancies of the mixed periodical character.

b) The present investigation has shown that large mixed-periodical harmonics in the residuals between the numerical theory (Pashkevich, 1999) and the KINEMATICAL solution SMART97 can be explained by the fact that the KINEMATICAL solution was derived from the DYNAMICAL one, which already contained the geodetic perturbations.

c) After the removal of the geodetic precession from the mean longitudes of the planets in the nutational harmonic arguments the discrepancies between the KINEMATICAL solution and the numerical integration do not exceed $15\mu as$ at the 200yr time interval.

d) The investigation of the discrepancies by the least squares method and the calculation of the correction coefficients to the precessional values and the main nutational harmonics of the semi-analytical theory SMART97 was carried out.

e) The modification of the theory SMART97 by means of addition of the correction terms to the corresponding coefficients of the semi-analytical expansions was carried out and the numerical integration of the rotational motion equations of the rigid Earth was repeated anew, with the initial conditions taken from the modified semi-analytical solution SMART97.

f) After such modification of the solution SMART97 the residuals over all the interval of comparison do not surpass ± 5 of microarcseconds in the longitude of the ascending node of the dynamical Earth's equator, ± 7 of microarcseconds in the proper rotation angle, and ± 4 of microarcseconds in the inclination of the dynamical Earth's equator to the fixed ecliptic J2000.0.

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CONSTRUCTION OF A NONRIGID EARTH ROTATION SOLUTION USING THE MODEL MHB2000

P. BRETAGNON
Institut de Mécanique céleste, UMR2028
Observatoire de Paris
77, avenue Denfert-Rochereau, 75014 PARIS, FRANCE

ABSTRACT. The accuracy of the rigid Earth solution SMART97 is 2 μas over the time interval (1968, 2023), accuracy showed by the comparison with a numerical integration using the positions of the Moon, the Sun, and the planets given by DE403. To obtain a nonrigid Earth solution, we use the transfer function of Mathews et al (2000) and, to keep the precision of our rigid Earth solution in the computation of the geophysical effects, we apply this transfer function to the Earth's angular velocity vector in order to avoid the inherent approximations of the classical methods. Moreover the perturbations of the third component of the angular velocity vector are taken into account. Lastly, we take into account, in an iterative process, the second order perturbations due to the geophysical effects. The results are compared with the Herring solution (1996) published in the IERS Conventions.

1. THE RIGID EARTH SOLUTION SMART97

The complete second order differential equations of the rigid Earth rotation are the following ones

$$
\begin{aligned}
\ddot{\omega} &= \frac{L}{A} + \frac{B-A}{B}\sin\tilde{\varphi}\Big(\frac{M}{A}\cos\tilde{\varphi} - \frac{L}{A}\sin\tilde{\varphi}\Big) - \dot{\psi}\dot{\varphi}\sin\omega \\
&- \frac{C-B}{A}\dot{\psi}\sin\omega\,(\dot{\varphi} + \dot{\psi}\cos\omega) \\
&+ \frac{B-A}{A}\frac{C-A-B}{B} \times (\dot{\psi}\sin\omega\sin^2\tilde{\varphi} + \frac{1}{2}\dot{\omega}\sin 2\tilde{\varphi})\,(\dot{\varphi} + \dot{\psi}\cos\omega) \\
\sin\omega\,\ddot{\psi} &= \frac{M}{B} + \frac{B-A}{A}\sin\tilde{\varphi}\Big(\frac{M}{B}\sin\tilde{\varphi} + \frac{L}{B}\cos\tilde{\varphi}\Big) + \frac{C+B-A}{B}\dot{\varphi}\dot{\omega} \\
&+ \frac{C-A-B}{B}\dot{\psi}\dot{\omega}\cos\omega \\
&+ \frac{B-A}{A}\,\frac{C-A-B}{B} \times (\dot{\omega}\sin^2\tilde{\varphi} - \frac{1}{2}\dot{\psi}\sin\omega\sin 2\tilde{\varphi})\,(\dot{\varphi} + \dot{\psi}\cos\omega) \\
\ddot{\varphi} &= \frac{N}{C} - \ddot{\psi}\,\cos\omega + \dot{\psi}\,\dot{\omega}\,\sin\omega \\
&+ \frac{B-A}{C}\left(\frac{1}{2}\dot{\omega}^2\sin 2\tilde{\varphi} - \dot{\psi}\dot{\omega}\cos 2\tilde{\varphi}\sin\omega - \frac{1}{4}\dot{\psi}^2\sin 2\tilde{\varphi}(1-\cos 2\omega)\right).
\end{aligned}
$$

The torque (L, M, N) has to be globally computed that is to say by using simultaneously the zonal and the tesseral harmonics because the first and the second derivatives of the diurnal and the semidiurnal terms are very important with respect to the long period terms. Table 1 gives the amplitudes of the semidiurnal term (coming from $C_{2,2}$ and $S_{2,2}$), of the 18.6 year term and of the 13.66 day term.

Table 1: Amplitude of the 18.6 year, 13.66 day, 12 hour terms and of their first and second derivatives

period	ψ(in $''$)	$\dot{\psi}$(in $''$/yr)	$\ddot{\psi}$(in $''/\mathrm{yr}^2$)
18.6 years	17.280776	5.838	1.96
13.66 days	0.221507	37.212	6251.37
12 hours	0.000036	0.132	762.56

Integrated in this way, the rigid Earth solution SMART97 (Bretagnon et al, 1998) can reach a high accuracy. It has been compared with a numerical integration using DE403 (Standish et al, 1995) for the positions of the Moon, the Sun and the planets. The accuracy is 2 μas over 1968–2023. The figure 1 gives the differences for ψ, ω and for the Earth rotation angle φ.

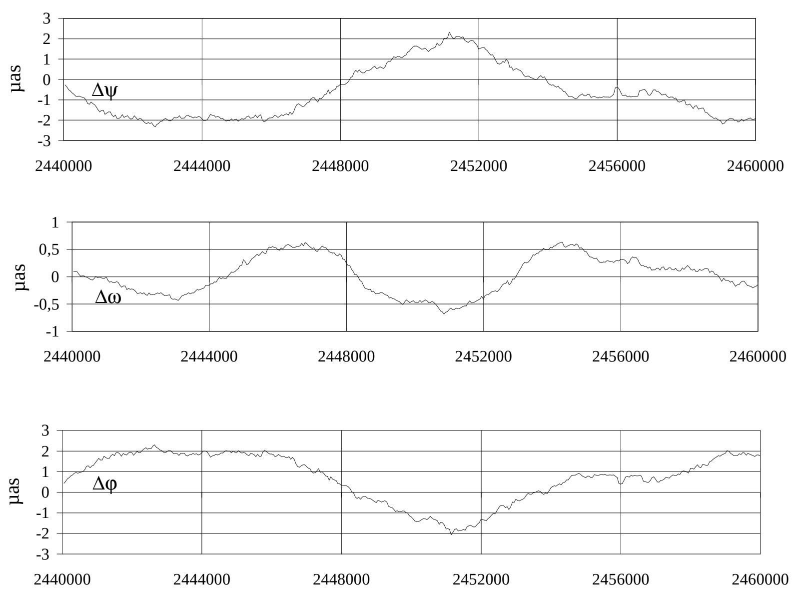

Figure 1: Theory SMART97 – numerical integration using DE403 over 1968-2023

2. THE INFLUENCE OF THE TRUNCATION LEVEL

The table 2 gives the number of the periodic terms greater than σ (for $\sigma = 0.01\mu$as, $\sigma = 0.1\mu$as and $\sigma = 1\mu$as) and the number of Poisson terms greater than σ over [J2000.0−100 yrs, J2000.0+100 yrs], for the rigid Earth solution SMART97. The table gives also the accuracy of the solution for the different levels of truncation. Data are given for the variable p ($\Delta\psi$ in the IERS Conventions 1996) of nutation in longitude reckoned from the equinox of date. The number of terms of the Herring solution for a nonrigid Earth (IERS Conventions 1996) is given

Table 2: Number of terms of the nutation in longitude p in SMART97 for different truncation levels σ

Solution and σ	P	P×t	P×t^2	P×t^3	P×t^4	Total	Accuracy
SMART (0.01μas)	3910	815	183	14	2	4924	2.2 μas
SMART (0.10μas)	1586	266	61	4	0	1917	8 μas
SMART (1.00μas)	642	87	17	1	0	747	40 μas
Herring (1.00μas)	375	109	–	–	–	490	

Table 3: Number of terms of the nutation in longitude of the nonrigid Earth solution at the 0.1 μas truncation level.

Periodic	P×t	P×t^2	P×t^3	P×t^4	Total
1581	264	60	4	0	1909

for comparison.

2.1 The periodic terms

Many terms with similar periods which appear in the other solutions are represented by only one term in SMART97. So, for the same truncation level, our solution must be more compact than the other ones. But SMART97 keeps many hundred terms missing in the other solutions. That reason explains that for a truncation of 0.1 μas, our solution contains 1586 terms and the others less than 1500; it explains also the better accuracy of our solution.

2.2 The Poisson terms

In SMART97, the P×t Poisson terms (Poisson terms of degree 1) represent 13.6% of the periodic terms with a truncation of 1μas, 16.8% with a truncation of 0.1μas and 20.8% with a truncation of 0.01μas. That must be compared to the Herring solution in which the P×t Poisson terms represent 29.1% of the periodic terms with a truncation of 1μas.

2.3 The truncation

In the construction of the SMART solution for a nonrigid Earth using the transfer function of Mathews, the amplitudes of some terms in resonance must be considerably increased. So, to obtain all the terms greater than 0.1 μas in the nonrigid Earth solution it is necessary to keep a level of truncation of 0.01 μas for the rigid Earth solution. We give the number of terms of the SMART solution for a nonrigid Earth in table 3.

3. NONRIGID EARTH SOLUTION

3.1 The transfer function of Mathews, Herring, Buffet (2000)

To compute the nonrigid Earth solution, we use the transfer functions in a strict process. For instance, the transfer function MHB2000 of Mathews et al (2000) is

$$T(\sigma; e|e_R) = \frac{(e_R - \sigma)}{(e_R + 1)} N_0 \left[1 + (1+\sigma) \left(Q_0 + \sum_\alpha \frac{Q_\alpha}{\sigma - s_\alpha} \right) \right]$$

with $e_R = 0.003284507$, $(1+\sigma)$ is the nutation frequency in space. The complex frequencies s_α have the following values

$$\begin{aligned}
s_1 &= (0.00315746, \ 0.00041782) \\
s_2 &= (-1.00231816, \ 0.00002603) \\
s_3 &= (-0.99893880, \ 0.00070674) \\
s_4 &= (0.00041351, \ 0.00000029)
\end{aligned}$$

with the complex coefficients

$$\begin{aligned}
N_0 &= \ \ 1.00001011 \\
Q_0 &= (-0.181007, \ 0.0343671) \\
Q_1 &= (-0.961751, \ 0.0727335) \\
Q_2 &= (0.0489773, \ 0.0016520) \\
Q_3 &= (0.0002915, \ -0.0000835) \\
Q_4 &= (-0.0000107, \ -0.0000013)
\end{aligned}$$

3.2 Application to the Earth's angular velocity vector

We apply the new transfer function not to the quantities $\sin\varepsilon_0\Delta\psi$ and $\Delta\varepsilon$ but to the Earth's angular velocity vector $(p,\,q,\,r)$ of the rigid case expressed in function of the three Euler angles

$$\begin{aligned}
p_R &= \dot\psi_R \sin\omega_R \sin\varphi_R + \dot\omega_R \cos\varphi_R \\
q_R &= \dot\psi_R \sin\omega_R \cos\varphi_R - \dot\omega_R \sin\varphi_R \\
r_R &= \dot\psi_R \cos\omega_R + \dot\varphi_R
\end{aligned} \tag{1}$$

For each prograde or retrograde argument, the angular velocity vector of the nonrigid Earth is obtained by multiplication by the transfer function

$$(p_{NR} + i\,q_{NR}) = (p_R + i\,q_R) \times T(\sigma; e|e_R)$$

From the quantities $(p_{NR},\,q_{NR},\,r_{NR})$, we can compute the derivatives of ψ, ω and φ in the nonrigid case by the reciprocal system of (1)

$$\begin{aligned}
\dot\psi_{NR} &= (p_{NR} \sin\varphi_{NR} + q_{NR} \cos\varphi_{NR}) / \sin\omega_{NR} \\
\dot\omega_{NR} &= (p_{NR} \cos\varphi_{NR} - q_{NR} \sin\varphi_{NR}) \\
\dot\varphi_{NR} &= r_{NR} - \dot\psi_{NR} \cos\omega_{NR}
\end{aligned} \tag{2}$$

These equations are strict but we see that the right hand members depend on the solutions by ω and φ and by the derivative of ψ and we have to proceed by iterations to solve this system. The process converges without difficulties. The precision of the computation of $\dot\psi_{NR}$ is better than $3.5 \times 10^{-9\prime\prime}$/year which yields an accuracy of 0.01 μas for the 18.6 year term and 0.50 μas for the 883 year term.

3.2.1 The classical method

This method assumes moreover than $r_{NR} = r_R$. One obtains in this case the results illustrated by table 4. We note SM97M2000 the solution obtained by SMART97 + Mathews et al (2000). We can see the results coming from the Mathews's function are very close to the series of Herring (1996).

Table 4: Nonrigid solution computed with the help of the transfer function MHB2000.

Solution	Argument	p (sin)	p (cos)	ε (sin)	ε (cos)
SM97M2000	$\lambda_3 + D - F$	17 206 664	−3 357	−1 488	−9 205 156
Herring		17 206 394	−3 702	−1 523	−9 205 474
SM97M2000	$2\lambda_3$	−1 318 625	−662	−486	573 040
Herring		−1 318 526	−670	−471	573 046
SM97M2000	$2\lambda_3 + 2D$	−227 663	309	150	97 854
Herring		−227 720	269	136	97 864
SM97M2000	λ_3	−36 674	−123 068	16 616	684
Herring		−36 777	−123 010	16 590	698

3.2.2 The full method

In fact, the rigorous method is to take into account the tidal variations in the Earth's rotation. We introduced the series $(\omega - \omega_S)$ and $(\omega - \omega_D)$ (IERS Conventions 1996) in order to obtain r_{NR} from r_R

$$r_{NR} = r_R + (\omega - \omega_S) + (\omega - \omega_D).$$

The modifications are very important not only for the third Euler angle φ but also for the first two ones. For instance, we obtain

$$\begin{aligned}\Delta p &= 42\,242\,t + 69\sin(2\lambda_3 + 2D - 2F) - 4\sin(\lambda_3 + D - F) + \cdots \\ \Delta\varepsilon &= -15\,t + 712\cos(\lambda_3 + D - F) - 19\cos(2\lambda_3 + 2D - 2F) + \cdots\end{aligned}$$

where the amplitudes are in μas and the time in thousands of Julian years from J2000. Therefore, for ε, the 18.6 year term becomes

$$\varepsilon = -1\,488\sin(\lambda_3 + D - F) - 9\,204\,444\cos(\lambda_3 + D - F).$$

By comparison with the previous table we see the necessity to determine again the transfer function. The modifications Δp and $\Delta\varepsilon$ are plotted on the figure 2; they reach, over 1968–2023, about 1200 μas in p and 800 μas in ε.

3.3 Iterative method

Now, with this nonrigid solution thus obtained, we have to iterate the process of computation of the rigid Earth solution completed by the transfer function in order to determine the second order perturbations due to the geophysical effects.

The introduction of these effects in the rotation of the Earth modifies the position of the Earth about its mass center and, therefore, the perturbations due to the Moon, the Sun and the planets.

So, we have computed again the whole of the perturbations of the rigid case and of the geophysical effects from the last result. This iterative process converges and produces the following modifications in the nutations

$$\begin{aligned}\Delta p &= -49\,t - 7.4\sin(\lambda_3 + D - F) + 5.5\sin(2\lambda_3 + 2D - 2F) + \cdots \\ \Delta\varepsilon &= -147\,t - 0.93\cos(2\lambda_3 + 2D - 2F) - 0.24\cos(\lambda_3 + D - F) + \cdots\end{aligned}$$

where the amplitudes are in μas and the time t in thousands of Julian years from J2000.

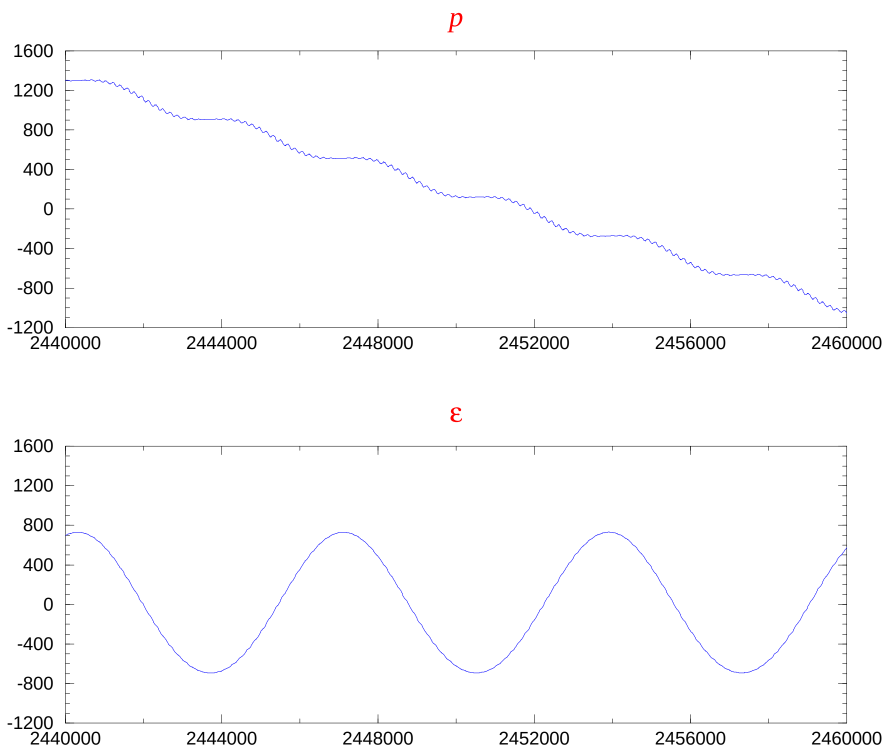

Figure 2: Influences of $(\omega - \omega_S)$ and $(\omega - \omega_D)$ on p and ε over1968–2023. Unit is the μas.

These perturbations reach, over (1968-2023), about 15 μas in p.

3.4 Geodetic precession-nutation

The solution is completed by the geodetic precession-nutation of Brumberg (1997) for the three Euler angles ψ, ω and φ, as well as the variables p and ε.

4. CONCLUSION

We built a theory of the rotation of the nonrigid Earth by using in a rigorous way the transfer function of Mathews et al. (2000) and by introducing the perturbations of the third component of the angular velocity vector. We show that the approximations inherent in the classical methods involve significant differences and that it is necessary to recompute the transfer function starting from our results.

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EFFECT OF INNER CORE VISCOSITY ON SPATIAL NUTATIONS INDUCED BY LUNI-SOLAR TIDES

M. GREFF [a], H. LEGROS [b] and V. DEHANT [c]
[a] I.P.G.P., 4 place Jussieu, 75252 Paris, France
[b] E.O.S.T., 5 rue R. Descartes, 67084 Strasbourg, France
[c] Observatoire Royal de Belgique, 3 av Circulaire, B-1180 Bruxelles, Belgique

ABSTRACT. The equations governing the rotation of the Earth, of the fluid outer core and of the inner core are generalized to include electromagnetic torques at the core-mantle and inner core boundary (ICB) and to take into account the viscous relaxation of the inner core. The four rotational eigenmodes (especially the eigenperiods and their quality factor) are investigated. Using a simple Maxwell model of rheology for the inner core with an effective viscosity ranging from 10^{12} up to 10^{16} Pa.s, we have found that, the spatial period of the FICN varies from 75 days (quasi-fluid inner core) up to some thousand days (quasi-elastic inner core with strong magnetic field), and that the inner core wobble disappears for a quasi-fluid rheological behaviour of the inner core. Finally, the influence of the inner core viscosity and of the magnetic friction at the ICB on the perturbations induced by the nearly-diurnal tidal potential in the spatial nutations are investigated.

1. ROTATIONAL EIGENMODES FOR AN ELASTIC INNER CORE

1.1 Theoretical approach

The equations governing the variation of the rotations for a planet having a liquid core and a solid inner core with rigid differential rotations with respect to the solid mantle, are the equations for conservation of angular momentum (noted $\vec{H}$, $\vec{H}^c$ and $\vec{H}^s$ respectively for the global Earth, for the fluid outer core and for the inner core), written in the Tisserand frame of the non-deformed initial mantle. If the Earth is submitted to a tidal volumic potential, noted W_2, (involving an external torque $\vec{L}$), we have, in absence of topographic torque at the CMB and at the ICB:

$$\left\{ \begin{array}{lcl} \frac{d\vec{H}}{dt} + \vec{\omega} \wedge \vec{H} & = & \vec{L} \\ \frac{d\vec{H}^c}{dt} - \vec{\omega}^c \wedge \vec{H}^c & = & \vec{T}^c \\ \frac{d\vec{H}^s}{dt} + \vec{\omega} \wedge \vec{H}^s & = & \vec{\Gamma}^s + \vec{T}^s \end{array} \right. \tag{1}$$

where $\vec{\omega}$ is the angular velocity of the mantle, $\vec{\omega}^c$ and $\vec{\omega}^s$ the differential rotation of the fluid core and of the inner core, respectively, with respect to the mantle; $\vec{T}^c$ and $\vec{T}^s$ are frictional torques which may appear at the CMB and at the ICB; $\vec{\Gamma}^s$ is the gravitational torque and the pressure torque acting on the inner core.

The angular momentum of a deformable body is expressible as the product of its angular rotation and its mass redistribution described by the inertial tensor for the entire Earth, for the core and for the inner core. The Earth, the fluid core and the inner core are deformed by the tidal potential but also by the rotation itself (rotational potential of the Earth, of the fluid core and of the inner core) and by the load associated with the tilt of the inner core. The inertia products perturbations are computed using generalized Love numbers.

The gravitational and pressure torque acting on the inner core may be written, noting $\vec{n}^s$ the outer normal of the inner core:

$$\vec{\Gamma}^s = \int_{\text{inner core}} \vec{r} \wedge \left[\rho^s \vec{\nabla}\left(\Phi^s(r) + \frac{r^2}{a^2} W\right)\right] dv - \int_{ICB} \vec{r} \wedge \vec{n}^s P^c ds \tag{2}$$

where Φ^s is the inner core gravitational potential and P^c the fluid pressure acting at the ICB. This torque takes into account the figure-figure gravitational coupling of the inner core to the mantle and the tidal torque on the inner core.

The tesseral degree two tidal potential W acts on the Earth's equatorial bulge (noted α) involving an equatorial torque such as: $L_1 + iL_2 = -\dfrac{3i\alpha A}{a^2} W_2$, where A is the inertia moment of the Earth and a the surface radius.

Because of the conductivity of the lower mantle and of the inner core, there is a frictional magnetic torque which appears at the CMB and at the ICB. These equatorial torques acting on the fluid outer core T^c and on the inner core T^s have been computed from Buffett (1992).

$$T^c = -A^c \Omega K_c (1+i) \omega^c + A^s \Omega K_s (1+i) \omega^s; \qquad T^s = A^s \Omega K_s (1+i)[\omega^c - \omega^s] \tag{3}$$

where K_c and K_s are frictional constants depending on the conductivity model and on the amplitudes of the magnetic field.

We have substituted the inertia products and these torques by their form into the equatorial angular momentum equations, in the frequency domain (for details see Greff et al. 2000). The determinant of the system vanishes for 4 eigenfrequencies:

the Chandler wobble (motion of $\vec{\omega}$ with respect to the figure axis of the mantle) noted λ_{CW}, the inner-core wobble (motion of $\vec{\omega} + \vec{\omega}^s$ with respect to the figure axis of the inner core) noted λ_{ICW}, and two nearly-diurnal modes, the Free-Core -Nutation (FCN) noted λ_{FCN}, and the Free-Inner-Core-Nutation (FICN) noted λ_{FICN}. For the PREM model, assuming that there is no visco-electromagnetic torque at the CMB and ICB, we obtain the following elastic rotational eigenmodes, noting Ω the sidereal rotation: $\lambda_{FCN} = -\Omega\left[1 + \dfrac{1}{458.6}\right]$; $\lambda_{FICN} = -\Omega\left[1 - \dfrac{1}{484.9}\right]$; $\lambda_{CW} = \dfrac{\Omega}{397.3}$; $\lambda_{ICW} = \dfrac{\Omega}{2319.3}$.

1.2 Influence of electromagnetic coupling at the CMB and ICB on the nearly-diurnal rotational eigenmodes for an elastic inner core

The electromagnetic coupling on the core does not perturb significantly the spatial period of the FCN and induces a quality factor large in comparison with the quality factor of the FCN observed using gravimeters and VLBI stacking which is about 50000 with large uncertainties.
The influence on the FICN of the electromagnetic torque acting on the elastic inner core and the associated perturbations in the observed nutations have first been investigated by Buffett (1992) and then by Mathews et al. (1998).
From the analytical form of the frequency, the FICN may be simply written (in the first order of approximation):

$\lambda_{FICN} = \lambda^{el}_{FICN} - \Omega K_s + i\Omega K_s$

There is a damping effect with a quality factor $\frac{1}{2K_s}$ and the spatial period of the FICN is increased by the magnetic friction at the ICB (Figure 1). K_s is proportional to the square of the amplitude of the radial component of the geomagnetic field at the ICB (B_r^{ICB}). This value is not well known. Assuming that B_r^{ICB} has the same order of magnitude than B_r^{CMB} , the

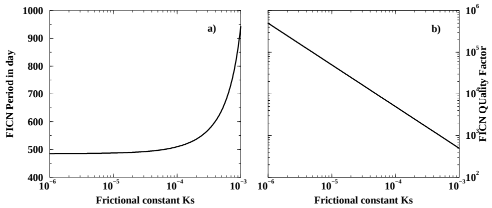

Figure 1: Spatial period (in sidereal day) and quality factor of the FICN as a function of the friction constant at the ICB

magnetic frictional constant K_s is of about 10^{-5}. Note that for $K_s = 10^{-3}$, we obtain a spatial period of about 945 days. With this kind of consideration, Mathews et al. (1998), from the amplitudes of the nutations obtained by analysis of VLBI data, estimate the strengths of the magnetic coupling at the CMB and ICB. Estimate obtained for the real part of the magnetic torque is $K_s = (100 \pm 8)10^{-5}$; it corresponds to a magnetic strength field at the ICB of about 23 Gauss, that is to say 5 times larger than the field at the CMB. The FICN spatial period obtained for such a frictional constant is about 920 days.
But this study does not take into account the viscosity of the inner core.

2. ROTATIONAL EIGENMODES FOR A VISCOELASTIC INNER CORE

We want to investigate the influence of the rheological behaviour of the inner core on the rotational eigenperiods and their damping.

2.1 Without electromagnetic torque

For simplicity, we first assume that there is no electromagnetic torque at the CMB and at the ICB.

Before to assume viscoelastic model of rheology, let us suppose that the inner core is an inviscid fluid. If the inner core is an inviscid fluid, the equipotential, equipressure and equidensity within the inner core are identical, and consequently, there are some relations between Love numbers. We have analytically shown that the wobble of the inner core disappears: when the inner core is an inviscid fluid, its axis of rotation is identical to its axis of figure. On the contrary, we have found that for the FICN, when there is a density jump at the ICB, this eigenmode exists even if the inner core is fluid. For the PREM model (with an homogeneous inner-core), we obtain a spatial period of the FICN $\Omega/(\Omega + \lambda^f_{FICN}) \simeq 74$ sidereal days, that is to say a spatial period 6 times smaller than the one obtained for an elastic inner core. The rheological behaviour of the inner core seems to play a critical role in the rotation of the inner core.

Now, we assume for the homogeneous incompressible inner core a Maxwell model of rheology. The elastic mantle and the inviscid fluid core are radially stratified in density, compressibility and rigidity following the PREM model. The viscoelastic deformations of the Earth are dependent on the viscoelastic relaxation time of the inner core, noted τ_G. This time is proportional to the viscosity of the inner core and to the inverse of the density jump at the ICB, and its value is plotted in Figure 2, in day as a function of the inner core viscosity.

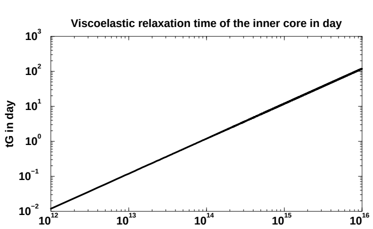

Figure 2: Viscoelastic relaxation time of the inner core (in day) as a function of the inner core viscosity. Note that for viscosity smaller than 10^{14} Pa.s, the relaxation time of the inner core is smaller than one day and thus for excitation source with a characteristic time of about one day, there is important relaxation effect.

We have computed the complex eigenfrequencies, for different viscosities of the inner core varying from 10^{12} Pa.s up to 10^{16} Pa.s. These frequencies are now damped because of the viscosity of the inner core and we have computed their quality factors. We have plotted in Figure 3 the periods and the quality factors of these rotational eigenmodes.

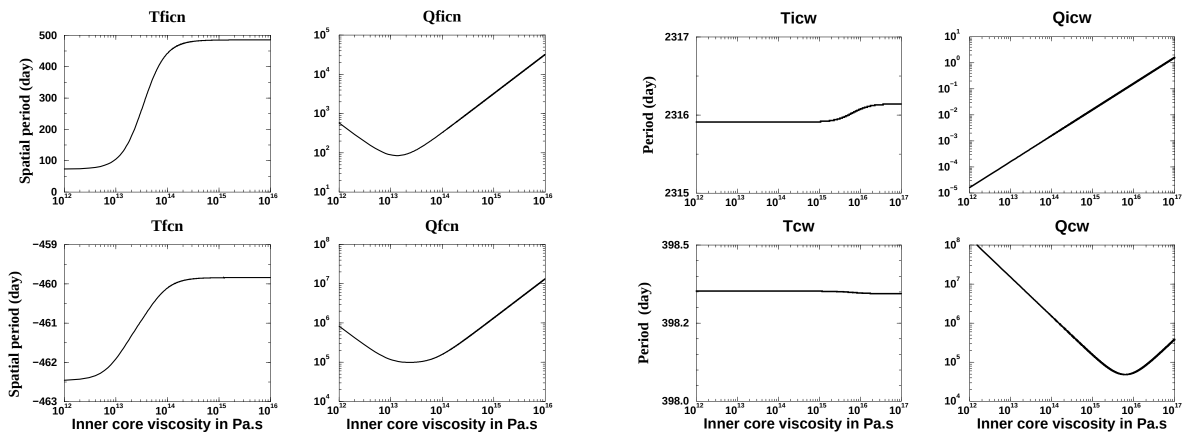

Figure 3: Periods and quality factors of the rotational eigenmodes

Note that the spatial period of the FICN (T_{ficn}) is strongly dependent on the inner core viscosity. For value $\nu_s > 2 \times 10^{14}$ Pa.s, that is to say for viscoelastic relaxation times of the inner core τ_G greater than 2 days, the period does not vary and has the value obtained for an elastic inner core ($\simeq$ 485 days). For value $\nu_s < 10^{13}$ Pa.s, the spatial period reaches the limit obtained for a fluid inner core ($\simeq$ 75 days). The damping factor (Q_{ficn}) is infinite for a fluid inner core and for an elastic inner core (in these two cases, the deformations are instantaneous and there is no dissipation) and is varying from 100 up to 1000 for viscosity between 10^{13} - 2×10^{14} Pa.s. The value of the FCN's spatial period is not significantly perturb by the presence of an elastic inner core with respect to a model with a fluid inner core (about 3 days). A surprising result is that there is a non-negligible damping factor of the FCN (Q_{fcn} about 10^5) induced by the viscosity of the inner core for ν_s between 10^{13} - 2×10^{14} Pa.s.
For viscosity larger than 10^{17} Pa.s, the quality factor associated with the inner core wobble (Q_{icw}) is larger than one and consequently the characteristic time of the damping of the wobble (which has a period around 6.3 years) is greater than 10 years. For viscosity lower than 10^{17} Pa.s, this damping is very fast so that the motion of the rotation axis of the inner core with respect to its inertia axis vanishes.
The Chandler period (T_{cw}) is not affected by the viscosity of the inner core but this wobble is damped with a quality factor which may reach 5×10^4 for an inner core viscosity between 5×10^{15} - 10^{16} Pa.s. (such viscosities are associated with relaxation times τ_G of about some hundreds days, i.e. comparable with the characteristic time of the Chandler wobble). Its value is larger than the observed one which has an order of magnitude around one hundred.

2.2 With electromagenetic torque

Here, we take into account simultaneously the influence of the viscosity of the inner core and the influence of the magnetic friction at the ICB on the spatial periods and quality factors of the nearly diurnal rotational eigenmodes, especially on the FICN. We have plotted the spatial period and the quality factor of the FICN as a function of the viscosity of the inner core ν_s and the frictional constant K_s at the ICB.

Note that if the observed spatial period of the FICN is less than 400 days, the inner core is not elastic, whatever the magnetic frictional constant, and has an effective viscosity less than 10^{14} Pa.s. For spatial period larger then 500 days, it seems difficult to obtain independent observations on K_s and ν_s. The observed value of 920 days observed by Mathews et al. (1998) may be interpreted in 2 ways: on one hand, by a frictional magnetic constant of about 10^{-3}, i.e. a

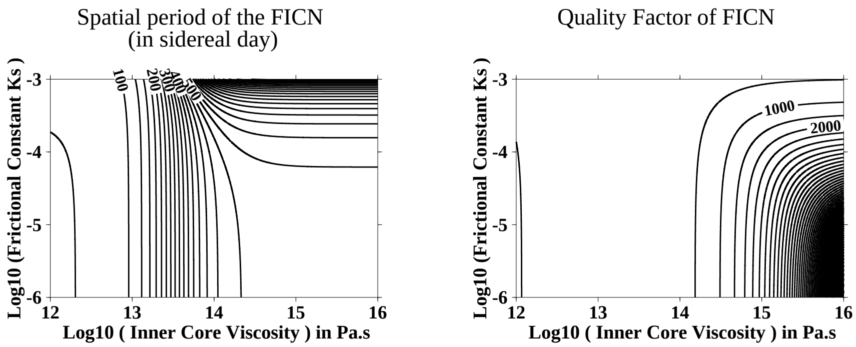

Figure 4: Spatial period and quality factor of the FICN

radial magnetic field at the ICB of about 23 gauss if the viscosity of the inner core is larger than 10^{14} Pa.s , that is to say if the inner core has a quasi-elastic behaviour at the diurnal time-scale, and on another hand, by a magnetic strength field at the ICB much larger than 23 gauss if the effective viscosity of inner core is smaller then 10^{14} Pa.s, that is to say if the inner core has a viscous rheological behaviour at the diurnal time-scale The quality factor associated with the FICN varies with several order of magnitude. It is less than 1000 if the viscosity of the inner core is less than 10^{14} Pa.s, whatever the frictional constant and it tends to reach 200000 when the inner core is elastic and the frictional constant K_s of about 10^{-6}.
Theoretically, the geodetical or gravimetric observations of the period of the FICN simultaneously with its quality factor will permit to determine both the viscosity of the inner core and the frictional constant at the ICB.

3. EFFECTS OF INNER CORE VISCOSITY ON SPATIAL NUTATIONS

The influence of the inner core viscosity and of the electromagnetic friction on the perturbations induced by the nearly-diurnal solar tidal potential in the spatial nutations are investigated. The spatial period of the FICN varying from 75 days up to some thousand days, it could have been in resonance with a tidal wave. For example, an effective viscosity of the inner core of about 6×10^{13} Pa.s. and a magnetic field at the ICB with the same order of magnitude as at the CMB leads to a spatial period of the FICN closed to 366.25 days which is exactly the spatial period of the S_1 tidal wave. By this resonance effect, we may have, on one hand an increase in the amplitude of the observed surface perturbations and on another hand, a phase shift, due to the viscous deformation of the inner core, between the excitation source (tidal potential) and the Earth's response in rotation.

The tesseral degree two tidal potential associated with a nearly diurnal tidal wave n_0 with frequency λ_x may be written (Roosbeek 1996): $W = \dfrac{3}{2} W_o(n_o) \sin 2\theta \sin(-\lambda_x t + \varphi)$, with $\lambda_x = -\Omega(1+x)$ and $x \ll 1$. φ and θ are respectively the east longitude and the colatitude, $W_o(n_0)$ the amplitude of the potential for each wave n_o. The frequency in the celestial frame of the solar tidal waves are such as $x = \frac{k}{366.25}$ with $k = \pm\ 1, 2, 3$.
Using the angular momentum equations, we can compute the perturbations of the spatial nutations $\omega e^{i\Omega t}$ in the celestial frame. In the frame co-rotating with the mantle, the equatorial component of the rotation $\omega = \omega_1 + i\omega_2$ may be written: $\omega = \omega_o e^{i\lambda_x t}$. In a celestial frame, we have: $\omega' = \omega e^{i\Omega t} = \omega_o e^{-i\Omega x t}$. In this inertial frame, we may compute the associated perturbations in the obliquity ϵ and precession Ψ from

$$\dot{\epsilon} + i \sin \epsilon_o \dot{\Psi} = \Big(\omega_{o1} + i\omega_{o2}\Big) \mathrm{e}^{-i\Omega x t} \tag{4}$$

The variations in obliquity and longitude have a spatial frequency equal to $-\Omega x$:

$$\delta\epsilon = -\frac{\omega_{o1}}{\Omega x} \sin(-\Omega x t) - \frac{\omega_{o2}}{\Omega x} \cos(-\Omega x t); \qquad \delta\Psi \sin \epsilon_o = -\frac{\omega_{o2}}{\Omega x} \sin(-\Omega x t) + \frac{\omega_{o1}}{\Omega x} \cos(-\Omega x t) \tag{5}$$

There is a component in phase with respect to the forcing tidal potential $(-\frac{\omega_{o2}}{\Omega x})$ and a component out of phase $(\frac{\omega_{o1}}{\Omega x})$ which exists because of the magnetic friction and of the inner core viscosity. We have computed this out of phase component for the prograde and retrograde semi-annual and annual nutations as well as for the prograde and retrograde Bradley nutations. The out of phase components have orders of magnitude which may be detectable using VLBI (Herring et al., 1991).

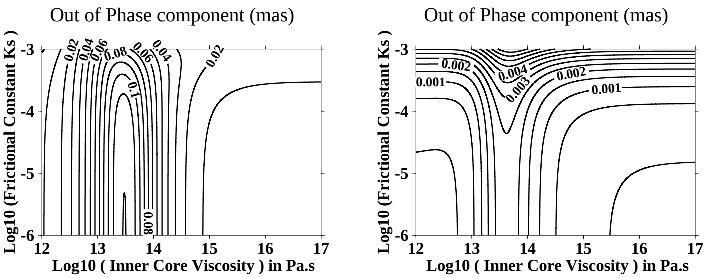

Figure 5: Out of Phase component of the prograde and retrograde semi-annual nutation

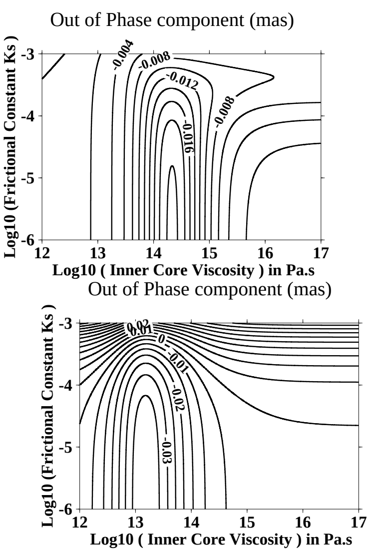

Figure 6: Out of Phase component of the prograde and retrograde annual nutation

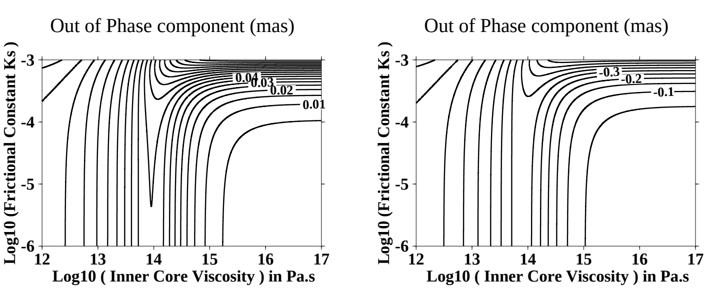

Figure 7: Out of Phase component of the prograde and retrograde 18.6 yr nutation

4. CONCLUSION

Not only magnetic friction but also the viscosity of the inner core can pertub the FICN and the nutations. Very precise observations of the components in phase and out of phase of the nutations can give informations on the Earth's deep interior, especially on the density jump at the ICB, on the effective viscosity of the inner core and on the amplitude of the radial component of the magnetic field at the ICB.

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THE POST-NEWTONIAN TREATMENT OF AN ELASTIC DEFORMABLE EARTH

C. XU, X. WU and M. SOFFEL
Lohrmann Observatory,
Dresden Technical University,
D-01062 Dresden, Germany

ABSTRACT. Here we deal with the problem of rotational motion of some deformable astronomical body such as the Earth. The Newtonian derivation of the dynamical equation of the displacement field is reviewed. Then the formalism of the displacement field in Einstein's theory of gravity is discussed. We have derived the post-Newtonian equations for the displacement field. Some important steps in the derivation are indicated. More details will be published elsewhere.

1. THE NEWTONIAN THEORY OF THE DISPLACEMENT FIELD

To describe the global motion of an elastic deformable Earth in space basically two approaches have been employed in the literature. In the theories of layers the Earth is devided into several parts, usually into elastic mantle, fluid outer core and a solid inner core and defines angular momenta for the various layers that can be used to derive certain equations of motion (Poincaré 1910, Molodensky 1961, Sasao-Okubo-Saito 1980, Mathews et al., 2000, Getino-Ferrándiz, 1999). In contrast to this a local theory is usually based on a displacement field

$$\mathbf{s}(t, \mathbf{x})$$

that can e.g., be used to derive certain parameters like Love numbers of layer theories (Wahr 1981, Dehant-Defraigne 1997, Schastok 1997). Here we will focus upon the second approach. In the *Newtonian theory* for the displacement field s^i one considers i) one single undeformed body (the elastomechanical ground state) and ii) a corresponding deformed body with remote astronomical bodies that excert tidal forces. The basic equation of motion for the displacement field in some global, inertial coordinate system simply reads

$$\rho \ddot{s}^i = \rho\, f^i + \frac{\partial p_{ij}}{\partial x^j}$$

where ρ is the matter density, f^i the force per unit mass that is given by $\mathbf{f} = \nabla U$ with

$$\Delta U = -4\pi G\, \rho$$

and p_{ij} is the socalled stress tensor. This stress tensor can be written as

$$p_{ij} = \lambda\theta\, \delta_{ij} + 2\mu\, e_{ij}$$

where λ and μ are the two Lamé parameters describing the elastic properties of matter. μ is called the shear-modulus, $\kappa \equiv \lambda + 2\mu/3$ the compression-modulus. e_{ij} is the strain tensor:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial s^i}{\partial x^j} + \frac{\partial s^j}{\partial x^i}\right)$$

and

$$\theta = e_{kk} = \frac{\partial s^k}{\partial x^k}$$

the volume dilatation. Usually one introduces a nutation frame $\Sigma_{\rm nut}$ with coordinates

$$\overline{x}^i = R^{ij} x^j; \qquad \frac{dR^{ij}}{dt} = \epsilon_{jkl}\Omega^k R^{il}$$

with $\Omega^k = \Omega(0,0,1)^T$, i.e., $\Sigma_{\rm nut}$ rotates with constant angular velocity Ω with respect to the space-fixed coordinate system. The basic equation for the displacement field in $\Sigma_{\rm nut}$ then takes the form

$$\rho(\ddot{\mathbf{s}} + 2\,\mathbf{\Omega} \times \dot{\mathbf{s}}) = \rho \nabla W_{\rm geo} + {\rm div}\,\mathbf{P}$$

where the geopotential $W_{\rm geo}$ is given by

$$W_{\rm geo} = U + \frac{1}{2}\mathbf{v}^2$$

where $\mathbf{v} \equiv \mathbf{\Omega} \times \mathbf{x}$. Usually one considers some stationary relaxed hydrostatic equilibrium configuration as reference state (undeformed body) for which

$$p_{,i} = \rho\, W_{{\rm geo},i}\,.$$

For the perturbed state one also considers effects from the tidal forces with potential

$$U_{\rm ext}\,.$$

Then with respect to the reference state one finds the desired equation of motion to first order in $\mathbf{s}$. In the Newtonian framework it is of the form

$$\rho(\ddot{\mathbf{s}} + 2\,\mathbf{\Omega} \times \dot{\mathbf{s}})^i = R^i$$

with

$$R^i = -\rho \frac{\partial s^k}{\partial x^k}\frac{\partial W_{\rm geo}}{\partial x^i} + \rho \frac{\partial s^k}{\partial x^i}\frac{\partial W_{\rm geo}}{\partial x^k} + \rho s^k \frac{\partial^2 W_{\rm geo}}{\partial x^i \partial x^k} + + \rho \frac{\partial}{\partial x^i}(U_{\rm ext} + \delta U) + \frac{\partial p_{ij}}{\partial x^j}$$

and

$$\Delta(\delta U) = 4\pi G\ {\rm div}(\rho \mathbf{s})\,.$$

Here $\delta U$is the Eulerian variation of U (Jeffreys and Vicente, 1957).

2. THE POST-NEWTONIAN THEORY OF THE DISPLACEMENT FIELD

The main message of this contribution is that we have succeeded to formulate a relativistic theory for the displacement field at the first post-Newtonian approximation of Einstein's theory of gravity. Here we indicate only some major steps in the derivation of some post-Newtonian equation of motion for the displacement field. More details can be found in Xu et al., 2001.

Also in relativity one considers two configurations of the deformable body: some unperturbed and some perturbed state. For the unpterturbed state one considers a single isolated body first in some global (non-rotating) coordinate system (ct, x^i). Here the metric is written in the form

$$\begin{aligned} g_{00} &= -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + \mathcal{O}(c^{-5}) \\ g_{0i} &= -4\frac{w^i}{c^3} \\ g_{ij} &= \delta_{ij}\left(1 + \frac{2w}{c^2}\right) + \mathcal{O}(c^{-4}) \end{aligned} \tag{1}$$

where the metric potentials w and w^i satisfy the harmonic field equations in the form (Damour et al., 1991)

$$\Delta w - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} = -4\pi G\,\sigma; \qquad \Delta w^i = -4\pi G\,\sigma^i\,.$$

Here σ and σ^i are the gravitational mass density and mass current respectively (Damour et al., 1991). Finally the local equations of motion can be derived from

$$T^{\mu\nu}{}_{;\nu} = 0$$

where $T^{\mu\nu}$ is the energy-momentum tensor. The introduction of the nutation frame is equivalent to the one in Newton's theory, i.e., the transformation from global coordinates $x^\mu = (ct, x^i)$ to rotating coordinates $\overline{x}^\mu = (c\overline{t}, \overline{x}^i)$ is given by

$$\overline{t} = t; \qquad \overline{x}^i = R^{ij}x^j; \qquad \frac{dR^{ij}}{dt} = \epsilon_{jkl}\Omega^k R^{il}\,.$$

Transformation of the metric tensor into $\Sigma_{\rm nut}$ is straightforward. E.g.

$$\overline{g}_{00} = -\exp\left(-\frac{2W_{\rm geo}}{c^2}\right) + \mathcal{O}(c^{-6})$$

with

$$W_{\rm geo} = w + \frac{1}{2}\mathbf{v}^2 + \frac{1}{c^2}\left(2w\mathbf{v}^2 + \frac{1}{4}\mathbf{v}^4 - 4w^i v^i\right)\,.$$

Note that $W_{\rm geo}$ generalizes the Newtonian geopotential and the last equation leads to a post-Newtonian definition of the GEOID. One then considers the local equations of motion in the nutation frame and formulates the post-Newtonian conditon for for hydrostatic equilibrium.

One then introduces the concepts of Eulerian- and Lagrangian variations of tensorial objects. Details can be found in Carter, 1973. E.g., the Eulerian variation of the metric tensor is denoted by $h_{\mu\nu}$

$$\delta g_{\mu\nu} = h_{\mu\nu}\,(\delta w, \delta w^i)$$

With a certain trick one can identify the various material elements of the body. This is achieved with a mapping of the four dimensional space-time into a three dimensional manifold of material elements. One takes a canonical coordinate system for both the undisturbed and the perturbed configuration. Then one can consider the coordinate position of some material element, z^μ in the ground state and $z^\mu + \Delta z^\mu$ in the perturbed state. The quantity Δz^μ is the Lagrangian displacement of our material element. Taking its temporal component to be zero the three dimensional displacement field s^i is defined by

$$\Delta z^\mu = (0, s^i)\,.$$

In the next step on considers the perturbed local equations of motion in $\Sigma_{\rm nut}$

$$\delta\left(\overline{T}^{\nu}_{\mu;\nu}\right) = 0\,.$$

Writing these equations explicitly one encounters a lot Eularian variations of tensorial objects that can be derived from the expressions given in Carter, 1973. The $\mu = 0$ equation reads

$$\delta\rho = -\rho_{,i}s^i - \rho s^i_{,i} + c^{-2} - \text{terms}$$

where the $1/c^2$-terms have been derived explicitly. Finally the $\mu = i$ equation yields the desired post-Newtonian equation for the diplacement field, i.e., the post-Newtonian Jeffreys-Vicente equation. The explicit version can be found in Xu et al., 2001. This equation might serve as a basis for future considerations of global geodynamics in the framework of Einstein's theory of gravity.

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INFLUENCE OF THE GRAVITATIONAL LUNISOLAR TORQUE ON THE ATMOSPHERIC ANGULAR MOMENTUM BUDGET

C. BIZOUARD and S. LAMBERT
DANOF/UMR8630-CNRS
Observatoire de Paris, 61, avenue de l'Observatoire 75014 Paris, France
e-mail: Christian.Bizouard@obspm.fr

1. ATMOSPHERIC ANGULAR MOMENTUM BUDGET

Athmospheric effects on the Earth rotation are usually estimated by assuming that the {Earth+atmosphere} system is isolated. Hence, any change in the atmospheric angular momentum (AAM) is totally governed by the interaction torque between the solid Earth and the atmosphere, and is considered to be fully transmitted to that of the solid Earth. This would no more be the case if external forces were perturbing this system (an external torque).

So, it seems necessary to check to wich extent the usual approach is a satisfying approach. We have considered two independant time series of the AAM (computed by the NCEP/NCAR meteorological data assimilation model), and of the interaction torque between the solid Earth and the atmosphere (computed by O. de Viron). The two series stretch from 1968 to 2000. The time derivative of the AAM has been computed numerically (interpolation every two hours, with Lagrange polynomials, and derivation over three points), and compared with the interaction torque (on figure 1, the x components are displayed). We have shown that the correlation between the two series is excellent for periods longer than two days (about 0.95), but it drops to 0.3 for the diurnal and subdiurnal periods. A comparison of the diurnal equatorial circular spectral components, estimated by least square fit, shows that retrograde diurnal components are larger in the interaction torque, and prograde diurnal components are larger in the time

We conclude that the AAM budget seems to be violated for the diurnal and subdiurnal fluctuation within the atmosphere. Two hypothesis have been proposed : the AAM or the interaction torque are not well estimated for the shorter periods, or another external torque perturbs the budget in these low periods frequency bands. A candidate is the lunisolar torque on the atmosphere, which is mainly diurnal and semidiurnal.

2. THE LUNISOLAR TORQUE ON THE ATMOSPHERE

The lunisolar torque on the atmosphere has been computed by considering the effect of the tidal potential on the mean pressure field (figure 2). A non negligible contribution has been found (10^{17}N.m) at several diurnal frequencies, emphasizing the necessity of taking this effect into account for computing the atmospheric effects on the Earth rotation.

We have also found that the lunisolar torque on the atmosphere compensates slightly the equatorial interaction torque, but it can not explain fully the discrepancy between the time derivative of the AAM and the interaction torque.

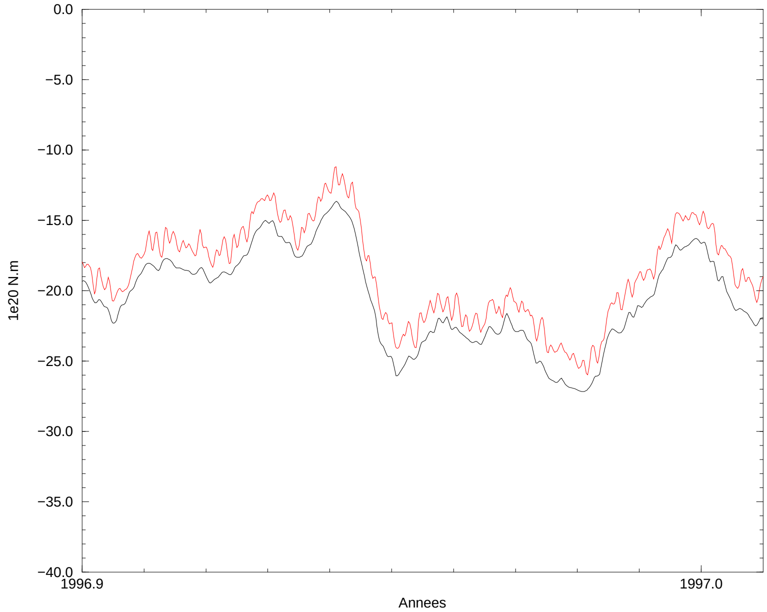

Figure 1: Time derivative of the AAM and interaction torque, component x in the terrestrial frame, between 1996.9 and 1997.0

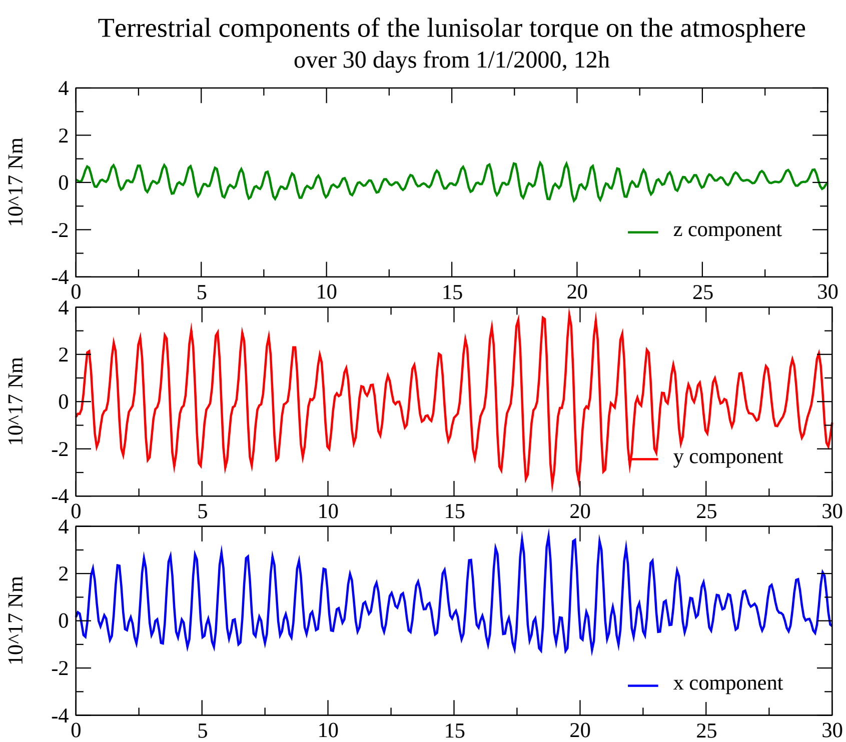

Figure 2: Lunisolar torque on the atmosphere

THE NETLANDER IONOSPHERE AND GEODESY EXPERIMENT

J.-P. BARRIOT[1], V. DEHANT[2], W FOLKNER[3], J.-C. CERISIER[4], A. RIBES[5], J. BENOIST[5], R. PRESTON[3], T. VAN HOOLST[2], and R. WARNANT[2]

[1] Observatoire Midi-Pyrénées/CNES(GRGS)
14, Av. E. Belin,31400 Toulouse, France

[2] Royal Observatory of Belgium
3 avenue Circulaire, B-1180 Brussels, Belgium
e-mail: v.dehant@oma.be

[3] Jet Propulsion Laboratory
MS 238-600, 4800 Oak Grove Drive, Pasadena, CA 91109, USA
e-mail: william.folkner@jpl.nasa.gov

[4] Centre d'Etude des Environnements Terrestres et Planétaires
4, Av. de Neptune, F-94107 St Maur des Fossés Cedex, France
e-mail: Jean-Claude.Cerisier@cetp.ipsl.fr

[5] Centre National d'Etudes Spatiales
8, Av. E. Belin, F-31401 Toulouse Cedex, France
e-mail: jeremie.benoist@cnes.fr

1. DESCRIPTION OF THE EXPERIMENT

The NEIGE experiment is one of the nine experiments under study in the framework of the Netlander mission to Mars. The basic idea underlying NEIGE is that the network of the Netlander microstations on the Martian surface will define a frame linked to the planetary crust (four landers are currently planned, with separations up to several thousand kilometers). By monitoring the orientation of this frame with respect to the Earth's reference frame of "known" motion, we will be able to measure the variations in the angular orientation of Mars in space, and then to derive information about the interior of Mars (e.g. physical state and size of the core). As a correction needed for accurate rotation state measurements, NEIGE will also perform precise measurements of the total electron content (TEC) of the Martian ionosphere. NEIGE data will also be used to improve our knowledge of the gravity field of Mars.

Because of their small size (roughly 25 kg), the Netlander microstations will be unable to communicate directly with the Earth. A Martian orbiter will be used as a data relay, and its orbit, suitably "averaged" over a period of at least one Martian year, will play the role of an "intermediate frame", between the Mars frame and the Earth frame. However, this "intermediate frame" will induce additional uncertainties related to the measurement and modeling of the motion of the orbiter. We will reconstruct the motion of the orbiter by combining the Doppler shifts on the radio link orbiter and the Earth with measurements of the Doppler shift between the Netlanders. For the link between the Netlanders and the orbiter, the limited mass and power for the lander payloads requires an innovative design. We will use a two-way coherent link at UHF band between the landers and the orbiter, and an associated one-way (Netlander to orbiter) coherent X-band (or perhaps S-band) pure carrier with omnidirectional antennas at both ends. The Doppler accuracy will be of the order of 0.1 mm/s for a 20 s Doppler-counting window. At least one tracking pass per week per Netlander will suffice to reach the geodetic and ionosphere objectives; two tracking passes per week or even one tracking pass per day would do better. The tracking of the orbiter from the Earth will be as continuous as possible to insure a good modeling of the orbit.

2. OBJECTIVES OF THE EXPERIMENT

The objectives of the NEIGE experiments are twofold: one part is related to geodesy and the other to the ionosphere.

The geodetic observations will give data on polar motion, precession-nutation and variations of the length-of-day (LOD) of Mars. ¿From this, information can be derived about the interior of the planet and about the atmosphere and the seasonal ice cap variations related to the sublimation/condensation process.

The knowledge of the interior of Mars is presently very limited. We know that Mars has undergone a differentiation process at its formation as did the Earth, and that, as a result, Mars has a silicate mantle and an iron core. The past measurements of Viking and Pathfinder landers have allowed to determine the principal polar moments of inertia with a precision of 0.5%, and the dimension of the core has been estimated with a precison of 200 km (Folkner et al., 1997, Yoder and Standish, 1997). The NEIGE experiment will allow us to refine those numbers and answer the questions whether the core is liquid or solid, and whether there is a solid inner core in the potential liquid core. From recent magnetic observations (Acuña et al., 1998), it is believed that Mars has only a remanent magnetic field. The absence of a dipole magnetic field implies that no dynamo is presently active in Mars, a fact that can most easily be explained if the Martian core is entirely liquid or solid. The larger sulfur concentration on Mars than on Earth, as inferred from SNC meteorites, is usuallly invoked to favour an entirely liquid core (Schubert and Spohn, 1990). However, in principle, an inner solid core of arbitrary dimension inside a liquid core seems not to be ruled out.

The examination of the nutation amplitudes and in particular of the nutation amplitudes near the Free Core Nutation (FCN) and Free Inner Core Nutation (FICN) will allow us to study that question and to provide information about Mars' deep interior (see Dehant et al., 2000a, 2000b and Van Hoolst et al., 2000a, 2000b).

The observation of polar motion will lead to the characterisation of the three main components which are believed to contribute: the annual and semi-annual components and the Chandler Wobble (CW). Their amplitudes are determined by the atmosphere and the sublimation/condensation process of the polar ice caps. The Chandler wobble amplitude additionally depends on the quality factor characterizing the mantle rheology. The observation of polar motion will thus constrain the mantle inelasticity and the global atmospheric and ice cap angular-momentum components in the direction perpendicular to the rotation axis.

Similarly to the seasonal polar motion components, the observation of the seasonal changes in LOD variations will allow us to derive the component of the atmospheric and ice cap angular momentum parallel to the rotation axis (see Defraigne et al., 2000).

The ionosphere of Mars is characterized by spatially and temporally varying local changes. The observation of NEIGE using two frequencies will allow to determine the Total Electron Content (TEC) of the ionosphere. From this observation, it will be possible to determine the plasma escape process, to analyse the large-scale heterogeneities and the scintillations (small scale heterogeneities).

3. PRESENT SITUATION

The Netlander project is currently in phase A+, during which we are performing feasibility studies. Simulations have been done at JPL (Barriot et al., 2000) using the GYPSY/Oasis software and extended simulations are presently being performed jointly by the Royal Observatory of Belgium (ROB) and the Groupe de Recherche de Geodesie Spatiale (GRGS) at CNES using GINS software. These simulations are being performed for the geodesy part of the experiment. Simulations on the ionospheric part of the experiment are also underway; these simulations are a joint effort of the ROB and the Centre d'Etude des Environnements Terrestres et Planétaires (CETP).

The Netlander launch is planned for 2007 on an Ariane 5 rocket. The four Netlanders will be attached to a CNES orbiter, which will be used to deliver the Netlanders to their individual landing sites. The landing sites of the four landers will be determined from the solar energy and Entry-Descend-Landing (EDL) constraints, as well as from the scientific constraints of all the experiments participating in the network.

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A NUMERICAL MODEL OF THE TORQUE-FREE MOTION

M. FOLGUEIRA[1] and J. SOUCHAY
DANOF/UMR8630-CNRS
Observatoire de Paris, 61 Avenue de l'Observatoire. F-75014, Paris. France.
e-mail: Souchay@obspm.fr
[1] *On leave from:* Instituto de Astronomía y Geodesia (UCM-CSIC)
Facultad de Ciencias Matemáticas. Universidad Complutense de Madrid.
28040, Madrid. Spain.
e-mail: martafl@iagmat1.mat.ucm.es

ABSTRACT.

We present here the preliminary results obtained by applying the numerical Runge-Kutta-Fehlberg method to the study of the torque-free motion in terms of the rectangular components of the angular velocity vector, Eulerïs angles and Andoyer's variables, depending on the different values of the principal moments of inertia.

1. THE TORQUE-FREE MOTION EQUATIONS.

The problem of the torque-free motion (the force function for the external forces, U, is equal to 0) has already studied analytically by several authors: Deprit (1967), Jupp (1974) and Kinoshita (1992). Here, we apply the numerical integration to the torque-free motion equations and we consider two different treatments, that is Eulerian method and Hamiltonian method. For this purpose, we have taken into account:

- the classical Euler's kinematical and dynamical equations for $U = 0$ to explain the problem from the point of view Eulerian, and
- the Hamiltonian equations for the free-torque motion to represent the problem from the point of view of Hamiltonian theory.

2. THE NUMERICAL INTEGRATION.

We have used a fifth-order adaptive stepsize Runge-Kutta-Fehlberg (RKF) algorithm to integrate the initial value problem represented by the two systems of first order differential equations, described in the previous section:

$$\underline{\text{Eulerian method}}: \quad \frac{dy}{dt} = f(y,t) \quad ; \quad y(0) = (\omega_1^0, \omega_2^0, \omega_3^0, \psi_0, \theta_0, \varphi_0)$$

$$\underline{\text{Hamiltonian method}}: \quad \frac{dz}{dt} = g(z,t) \quad ; \quad z(0) = (L_0, G_0, H_0, l_0, g_0, h_0)$$

where, $(\omega_1, \omega_2, \omega_3)$ are the rectangular components of the angular velocity vector along the principal axes, (ψ, θ, φ), the Euler's angles and (L, G, H, l, g, h) Andoyer variables and their canonically conjugate variables. The terms $\frac{dy}{dt}$, $\frac{dz}{dt}$, y, z, f and g are vectors. Note that the two above systems of six differential equations have the same form and express the change of y and z in terms of the principal moments of inertia, the components of angular vector of rotation, Euler's angles, Andoyer's variables and their corresponding canonically conjugate variables.

The algorithm yields the solution, y and z, of the initial value problem where the independent variable t has been incremented by the adaptive step h. *RKF* method predicts the size of h for the subsequent step with monitoring of local truncation error to ensure accuracy and adjust stepsize.

3. CONCLUSIONS AND APLICATIONS.

- The comparison of the results of the numerical integration in Euler's angles and in Andoyer's variables revealed a good agreement.
- The principal advantage of the numerical integration with respect to an analytical method is that the integration of the equations by the last one is quite complicated than by numerical integration since its exact solution involved elliptic functions and elliptic integrals.
- The results of this survey can be applied to the torque-free motions of any celestial body in the solar system (Souchay and Folgueira, 2000).

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ON THE CELESTIAL POLE OFFSETS FROM OPTICAL ASTROMETRY IN 1899 – 1992

C. RON and J. VONDRÁK
Astronomical Institute of the Academy of Sciences of the Czech Republic
Boční II 1401, 141 31 Praha 4, Czech Republic
e-mail: ron@ig.cas.cz, vondrak@ig.cas.cz

1. INTRODUCTION

Several solutions of the Earth orientation parameters (EOP), i.e. polar motion x, y, Universal Time UT1–TAI and celestial pole offsets (CPO) $\Delta\psi$, $\Delta\varepsilon$ from optical astrometry covering the period 1899.7–1992.0 has been described in Vondrák *et al.* (1998). We shall discuss here the CPO of the final solution in Vondrák *et al.* (1998) further referred to as *OA97* and the newer one of the same authors presented at IAU Colloquium 178 in Cagliari (Vondrák *et al.*, 2000) referred to as *OA99*. The latter solution includes two new instruments and more observations than *OA97*. Moreover, in case two or more similar instruments were used at the same observatory, their results were merged into a single series, with the steps due to different locations of the instruments removed, and further treated as a single instrument. These were Pulkovo ZT before and after the second world war, Washington PZT's, Richmond PZT's, Mizusawa ZT's and others. The solution *OA99* is referred to a slightly different reference frame due to proper motions corrections.

Yaya *et al.* (2000) noted the differences between these two CPO series that were seen especially in the annual term and that could not be explained by the slight changes of the number of observations and instruments used. To explain the differences we shall simulate the ideal observations and the solution will be compared to the CPO obtained from real data of optical astrometry. In Fig. 1 are shown the filtered CPO by band-pass filter with the cut-off periods 0.3 and 5 years to see the differences in their annual signals. We present only the component $\Delta\psi \sin\varepsilon$ due to the lack of place. Note the differences before 1930 marked by arrows.

2. SOLUTION OF THE SIMULATED DATA

To understand the behavior of the adjusted CPO, we have prepared simulated observed data at three stations regularly distributed along the parallel 40° (with longitudes 0°, 120°, −120°). The observations have been simulated on the interval of 10000 days every three days; the observations start three hours before and terminate three hours after the local midnight, every 15 minutes. The observed latitude variations were always equal to zero. The expected results of the adjustement is equal to zero for polar motion as well as for CPO. The next simulation is done on the same data but the shift of 0.2" has been added to the observed latitude variation at one of the stations ($\lambda = 120°$), after day 5000. As you can see in Fig. 2, the shift in the station latitude has been projected also into the CPO due to the constraints tying together the station parameters, see Vondrák *et al.* (1998).

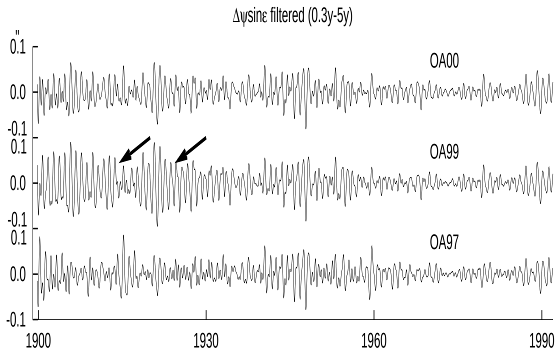

Fig 1. $\Delta\psi$ component of the CPO from the solutions *OA97, OA99, OA00*.

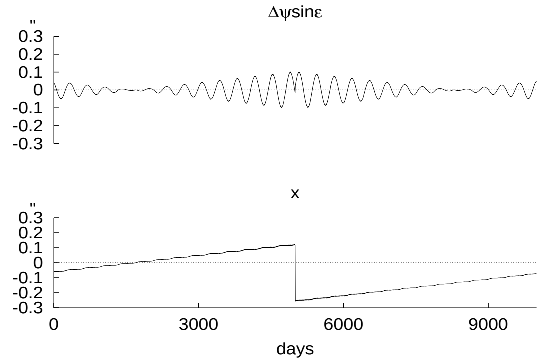

Fig 2. The EOP component x and $\Delta\psi$ from the solution of simulated observations.

3. NEW SOLUTION OF REAL DATA AND CONCLUSIONS

The results of the simulations lead us to the inspection of the merged instruments in the solution *OA99*. We treated again separately the Pulkovo zenith telescope (whose observations were interrupted during the second world war) and the Washington PZT's (that were found to have quite different trend before and after 1955). We have obtained the new solution referred to as *OA00* (Vondrák & Ron, 2000). The filtered CPO are compared to that of *OA97* in Fig. 1. One can see that the unreal annual amplitudes of the solution *OA99* before the second world war almost disappeared. One can conclude by the following:

- The annual signal of CPO is highly sensitive to sudden steps in the data of individual instruments, in combination with relatively short nightly intervals of observation;
- The difference in annual signal of CPO between the solutions *OA97* and *OA99* was explained by merging some of the instruments into a single instrument;
- It remains to explain the increasing of the annual signal in the CPO before 1920;
- The big amplitude of annual signal of the CPO around 1940 is caused by lack of observations in Europe during the second world war.

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NUMERICAL DETERMINATION OF DYNAMICAL ELLIPTICITY FROM VLBI DATA

T. SHIRAI
University of Tokyo, Graduate School of Science, Department of Astronomy
3-8-1, Hongoh, Bunkyo-ku, Tokyo 113-0033, Japan
e-mail: t.shirai@nao.ac.jp

T. FUKUSHIMA
National Astronomical Observatory
2-21-1, Ohsawa, Mitaka, Tokyo 181-8588, Japan
e-mail: toshio@nao.ac.jp

ABSTRACT. We determined the dynamical ellipticity from the VLBI data through the numerical theory of rigid Earth nutaiton. We integrated full Euler equation, which means that we solved the precession and nutation simultaneously. About ephemeris and astronomical constants, we used DE405 which is the contemporary numerical ephemeris of NASA/JPL. And we also used the contemporary VLBI data complied by USNO from 1979 to 2000. Our determined value of the dynamical ellipticity is 0.0032737914(4). We also estimated the offsets at J2000.0, namely -45.95 ± 0.08 mas in longitude and -3.69 ± 0.01 mas in obliquity.

1. METHOD

Some researchers determined the dynamical ellipticity from the approximation formula and the given precession values when they developed analytical theories of rigid Earth nutation (Souchay and Kinoshita 1995, Roosbeek and Dehant 1998, Bretagnon et al. 1997, Hartmann et al. 1999). However we have to solved the nutation and precession simultaneously to determine the dynamical ellipticity precisely. So we did full integration of Euler equation by using the DE405 which is the contemporary numerical of NASA/JPL for the ephemeris and astronomical constants. And Fitting the contemporary VLBI data which complied by USNO from 1979 to 2000, we estimated the initial values and dynamical ellipticity by the weighted least square method. We subtracted the geodesic prcession and nutation from the VLBI data (Fukushima 1991). Note that we also subtracted the analytical correction of the non-rigidy of the Earth. This is because that the period of VLBI data is not enough long to distinguish the 18.6 yrs nutation from the precession completely. We used the 4th Runge-Kutta method for the integrator whose step size is 1/100 day. And the integration time span is 21 years, namely from 1979 to 2000. We considered the Sun, Moon and all Planets for the perturbations and J2, J3 and J4 for the the geopotential.

2. RESULTS

Our determined value of the dynamical ellipticity is 0.0032737914(4). This value is little lager than analytical ones. See Table 1. We also estimated the offsets at J2000.0, namely -45.95 ± 0.08 mas in longitude and -3.69 ± 0.01 mas in obliquity. See Table 2. The long period integration to determine precession constants and offsets and the direct introduction of the non-rigidy of the Earth are reserved for the future research.

Acknowledgements. The authors wish to thank D.D. McCarthy for providing the VLBI observational data compiled by USNO.

Table 1: Dynamical Ellipticity

	Series name	Dynamical ellipticity
Souchay and Kinoshita	REN2000	0.0032737 548
Roosbeek and Dehant	RDAN98	0.0032737 674
Bretagnon et al.	SMART97	0.0032737 671
Hartman et al.	HS97	0.0032737 92489
Shirai and Fukushima	NEW	0.0032737 914(4)

Table 2: Offsets of Celestial Ephemeris Pole at J2000.0

Method & Reference		$\Delta\psi_0 \sin\varepsilon_0$ (mas)		$\Delta\varepsilon_0$ (mas)	
		Value	σ	Value	σ
VLBI	Herring (1995)	−17.3	0.2	−5.1	0.2
LLR	Chapront *et al.* (1999)	−18.3	0.4	−5.6	0.2
Optical	Vondrak & Ron (2000)	−12.3	0.7	−9.2	0.6
VLBI	Vondrak & Ron (2000)	−17.10	0.05	−4.95	0.05
VLBI	Mathews *et al.* (2000)	−16.18		−4.53	
VLBI	Shirai & Fukushima (2000)	−16.889	0.013	−5.186	0.013
VLBI	Shirai & Fukushima (NEW)	−18.28	0.03	−3.69	0.01

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ESTIMATION OF NUTATION USING THE GPS

R. WEBER
Dep. of Advanced Geodesy, TU-Vienna
Gusshausstr. 27-29, A-1040 Vienna
e-mail: rweber@luna.tuwien.ac.at

1. INTRODUCTION

GPS measurements obtained from the global IGS network are undoubtedly a valuable source for the determination of ERP series. This series are at the one hand of high quality ($\sigma_{xp,yp} \approx 0.1 mas$; $\sigma_{LOD} \approx 0.02 msec/day$) because of the more or less regular distribution of the active IGS tracking sites (curently about 140 stations). Moreover the uninterrupted coverage of almost 10 years with daily estimates is also of importance for detailed studies.

In February 1994 CODE, one of the seven Analysis centers of the IGS started to derive nutation rates in addition to polar motion and the routinely estimated UT1-UTC rates. In 1999, a set of nutation amplitude corrections for 34 periods with respect to the IERS96 model as well as the IAU1980 model was presented by (Rothacher et al.,1999) based on 3.5 years of data. It has been demonstrated that GPS is especially sensitive to periods up to about 20 days. This overview deals with preliminary results of the most recent update of the published nutation amplitude corrections by taking into account 6 years of observation data and focusing on the 13.66 days term.

2. ESTIMATION OF NUTATION RATES

Contrary to polar motion, offsets in the three remaining earth orientation components ($\Delta\varepsilon$, $\Delta\psi$, UT1-UTC) are fully correlated with the orbital parameters describing the orientation of the orbital planes of the satellites (ascending node, inclination, and argument of latitude). Nevertheless, an almost 'perfect' force model would describe the satellite orbits accurately enough, at least over an interval of a few days (3 days arcs at CODE) to allow for the determination of EOP-drift parameters. In reality major biases in the rate estimates show up at the satellites revolution period or at annual and semi-annual periods due to solar radiation pressure.

Daily nutation rates in longitude and obliquity estimated from GPS data over the past 6 years are available now. All rates are corrections to the IAU 1980 Theory of Nutation. The series seems to become somewhat noisier at the end of 1996, caused by a change of the orbit model in October 1996. The formal errors of the estimates grow by a factor of about 2-3 because of correlations to the newly added orbit parameters. Nevertheless, (Rothacher et al.,1999) have shown in a thorough discussion, that nutation corrections of similar quality can be obtained with both orbit models.

3. ESTIMATION OF NUTATION AMPLITUDES

To decribe amplitude corrections A_{ij}, A_{oj} to nutation terms from a series of nutation rates $\delta\Delta\varepsilon'$ and $\delta\Delta\psi'$ we use the following formulation (e.g. Weber, 1999)

$$\delta\Delta\varepsilon(t)' = -\sum_{j=1}^{n} A^{\varepsilon}_{ij} \sin\theta_j(t)\frac{\partial\theta_j}{\partial t} + \sum_{j=1}^{n} A^{\varepsilon}_{oj} \cos\theta_j(t)\frac{\partial\theta_j}{\partial t} \tag{1}$$

$$\delta\Delta\psi(t)' = -\sum_{j=1}^{n} A^{\psi}_{oj} \sin\theta_j(t)\frac{\partial\theta_j}{\partial t} + \sum_{j=1}^{n} A^{\psi}_{ij} \cos\theta_j(t)\frac{\partial\theta_j}{\partial t} \tag{2}$$

The angles θ_j denote a linear combination of the Delauny variables and n is the number of nutation terms taken into account. The A_{ij}, A_{oj} are usually called in-phase and out -of-phase components, which can be obtained by a least squares adjustment using the GPS nutation rates as pseudo-observations.

The diagram presents the first results in evaluating this new enlarged nutation rate series. A comparison of the nutation in-phase and out-of-phase components for the 13.66 days period given in various well-known models with the most recent model by Souchay/Kinoshita (1997.2;SKV972) shows the excellent agreement and the increasing convergence of the GPS solutions with SKV972.

We may conclude that even now GPS allows an independent check of present-day nutation models and VLBI results at the high frequency end of the spectrum. In future a combination of the VLBI and the GPS series at the postprocessing or even at the observation level might be extremely promising.

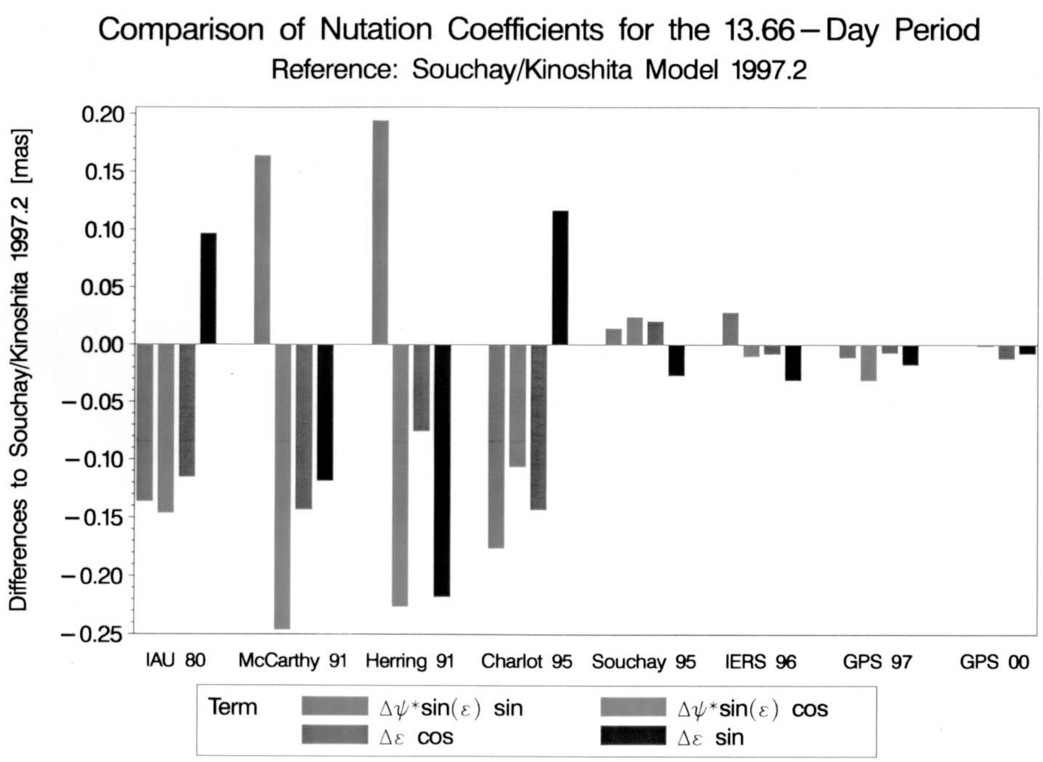

Fig. 1. Overview of the distribution of the IVS components

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TIME SYSTEMS AND TIME FRAMES, THE EPOCHS

B. GUINOT
Observatoire de Paris
61, avenue de l'Observatoire
e-mail: guinot.bernard@wanadoo.fr

ABSTRACT. The relativistic definition of time scales requires that their origins be defined as a conventional date for a specified four dimensional event. In 1991, the IAU specified this event at the geocentre and in terms of TAI; all coordinate times received the same date for this event. The consequence is that event J2000.0 has dates which are different in the various coordinate times. The paper includes comments on the distinction between system and frame and points out some ambiguities resulting from the IAU decisions.

1. INTRODUCTION

The main purpose of this paper is to recall how the epoch J2000.0 is defined and realized. We shall see that this involves an interaction between realized time scales and their theoretical counterpart, that is, adopting the terminology set up by Kovalevsky and Mueller (1981, 1989), between conventional time frame and conventional time system. For this reason, I will begin by a general reflection on the various steps in the realization of a conventional reference frame as described by these authors. Then, we shall see how the definition of J2000.0 is related to the origin of time scales in 1977. The influence of the IAU resolutions adopted in 2000 and some ambiguities will be considered.

2. SYSTEMS AND FRAMES

Four main steps from the most abstracted form of a reference system to its realization are described by Kovalevsky and Mueller.

- The concept of an *ideal reference system.* This is purely theoretical and linked to the model for space-time and gravitation. The choice of the system obeys a criterion of simplicity for theoretical developments (e.g. no acceleration terms, no Coriolis terms). An interesting discussion on this simplicity criterion, for time, was published by Poincaré, (1906).

- The *reference system* (proper) appears when the above model is applied to a specified structure (e.g. the solar system).

- The *conventional reference system* incorporates additional information: the units, conventional values of constants...

- The *conventional reference frame* is a realization of the conventional reference system which is available for the user: an ephemeris, a catalogue of positions of celestial objects, a disseminated time scale allowing to date events.

This hierarchy is well adapted for reference frames based on the dynamics, where ephemerides of celestial bodies provide a realization of both the orientation of a celestial reference frame and a time scale (Ephemeris Time). But difficulties are met with what I call 'empirical' or 'natural' references, now in use, where we postulate that physical phenomena have some properties of ideal reference systems. Presently, the basic postulates are:

- the more distant is an object, the less its direction rotates, as seen from the barycentre of the solar system (and the rotation is negligible for the most distant objects we can observe, at least on an average over many of them),

- the atomic transitions provide a realization of the proper time of relativistic theories.

Consider, for example, the International Celestial Reference System (ICRS), the adjective 'international' replacing 'conventional'. The theory does not privilege a particular orientation. It would not have been wise to fix arbitrarily three coordinates of specified objects. In fact, it is the ensemble of coordinates adopted for the International Celestial Reference Frame (ICRF) which defines in a statistical manner, i.e. not rigorously, the orientation of the ICRS. Thus the system appears more as some sort of convenient cover name for all the versions of the frame, than a scientific object. The same can be said, of course for the terrestrial system and frame, ITRS and ITRF.
A similar situation is observed for fixing the origin of time scales, because, even if a sufficiently well defined event could fix the origin of a time scale by adopting for it a conventional date, the access to this event can only be provided by a realized time scale. This will be explained in the following.

3. THE 1977 ORIGIN OF TIME SYSTEMS

The scale unit and origin of all coordinate times is defined by the IAU Resolution A4 (1991), Recommendation III, in the following terms :

- the unit of measurement of the coordinate times of all coordinate systems centered at the ensembles of masses be chosen so that they are consistent with the proper unit of time, the SI second,

- the reading of these coordinate times be 1977 January 1, 0h 0m 32.184s exactly, on 1977 January 1, 0h 0m 0s TAI exactly (JD = 2443144.5 TAI), at the geocentre.

The conventional coordinate time systems are only distinguished from their general theoretical counterparts by use of capitals, such as in 'Barycentric Coordinate Time' (TCB), and the idea that they are conventional is implicit. Moreover, no special name is proposed for their realization, i.e. the coordinate time frames; a notation is suggested, such as TCB(TAI) for a realization of TCB based on the International Atomic Time (TAI), but it is not widely used.
The origin of all coordinate times is defined by assigning a conventional date, identical for all of them, to an event whose four coordinates are provided in the Geocentric Reference System and are (t = 1977 January 1 0h TAI, 0, 0, 0). This event has not been given a name. Let us call it here the '1977 Origin'. The IAU Recommendation may raise a number of questions.

(a) Why 1977? Because on 1977 January 1, a 'steering' of the TAI frequency based on the data of the best primary frequency standards was initiated. It was thus expected that the departure between TAI and its theoretical form, Terrestrial Time (TT) (apart from the conventional time offset), would increase much slower than previously.

(b) Why this offset of 32.184 s? To have TT and the Dynamical Barycentric Time (TDB) in quasi continuity with Ephemeris Time (TAI was set to be equal to UT2 on 1958 January 1).

(c) Why the 1977 Origin is specified in the Geocentric Reference System? Because TAI is defined in this system.

(d) Is the 1977 Origin perfectly defined? No, because TAI is the result of computations and is made available through corrections to physical clocks. As a consequence of uncertainties of clock comparisons, the various clocks used to materialize TAI lead to slightly different events for the 1977 Origin, in the range $\pm$ 1 microsecond. However, at the 1977 Origin, the dates in the various coordinate times (time systems) strictly coincide.

(e) Would not have been better to set the origin in 2000? In 1991, nobody suggested to do it. The general feeling was that the origin should be in the past. However, when considering the problems raised by the definition of J2000.0, setting the origins in J2000.0 might have been advisable. This point is discussed section 5.

4. J2000.0

To express time varying quantities, it is convenient to reckon time from a recent date, the epoch. Until recently, the epoch was traditionally expressed in terms of Besselian date with the tropical year (slightly variable in duration) as unit. This was marked by prefix B, such as B1950.0. This definition was given in the Newtonian absolute time and could be associated with the use of Ephemeris Time. It was then found better to define the epoch in the system of Julian Days and to use the Julian year of exactly 365,25 days as a unit. This is marked by suffix J: we are concerned here with J2000.0. The time in which J2000.0 is used has to be mentioned.
The definition of J2000.0 had to be consistent with the model of general relativity adopted in 1991, with the coordinates specified in 1991 and, more precisely, in 2000. This definition was given by IAU Resolution C7(1994) which recommends:

1. *the event (epoch) J2000.0 is defined at the geocentre and at the date 2000 January 1.5 TT = Julian Date 2451545.0 TT,*

2. the Julian century is defined as 36525 days of TT.

We discuss first recommendation 1. Let us insist on the fact that, as the 1977 Origin, J2000.0 is defined by an event, also at the geocentre. The coincidence of the dates in all times (conventional coordinate time systems) at the 1977 Origin has the consequence that J2000.0 receives dates which are different in the various times.
Some important differences in the definitions of the 1977 Origin and J2000.0 are worth discussing. The 1977 Origin has its time coordinate (date) expressed in TAI, a 'frame'. In contrast, the date of J2000.0 is expressed in TT, a 'system'. Let us explain why. At the 1977 Origin, the date is the same in all time systems. Thus dating J2000.0 in one of these time systems makes possible to express the date of J2000.0 in any other time system with full theoretical rigor, the practical uncertainty being only due to approximations in theoretical developments and uncertainties of the involved constants. If we had expressed the date of J2000.0 in TAI, we would have added the time uncertainty (integral over the frequency uncertainty) of TAI. In the present situation, only the date of J2000.0 in TAI, noted $(TAI)_0$, suffers from an uncertainty. Table I gives under the designation $(T...)_0$ the date of J2000.0 in usual time systems and frame, obtained as explained below.

$(TAI)_0$ is obtained by adding to 1977 January 1, 0h 0m 0s (exactly) the difference in TT days between 2000 January 1, 12h 0m 0s and 1977 January 1, 0h 0m 32.184s. The size of the error of $(TAI)_0$ is difficult to estimate because the frequency uncertainty of frequency standards generates a strongly correlated noise in time. Some tests of recomputation of atomic time scales led to a possible size of 20 to 30 microseconds.

$(TCG)_0$ has now a rigorous value since IAU Resolution B1-9 (2000) transformed the rate of TT with respect to TCG into a defining constant for TT.

$(TCB)_0$ is obtained from $(TCG)_0$ by use of a full 4-D transformation, here applied at the geocentre. The formulation of IAU Resolution B1-5 (2000) guarantees a rate uncertainty smaller than 5×10^{-18}, then an uncertainty of less than 4 nanoseconds on $(TCB)_0$. But this resolution does not introduce a constant term arising from the value of the periodic component of $TCB - TCG$ at the 1977 Origin, $P_{1977} = -65.5\mu$s. Therefore, a contradiction with the IAU 1991 resolution appears.

Let us consider now the second part of the 1994 resolution on the Julian century. It is puzzling that the use of the Julian century is restricted to TT. I would have thought that this scale unit could have been used in all coordinate times providing that they are clearly mentioned. Unfortunately, these recommendations were voted in great haste and confusion and nobody had really an opportunity to look at them seriously beforehand.

5. A REFLECTION ON THE 1977 ORIGIN. CONCLUSION

Although it may seem a bit queer, I believe now that, in 1991, we could have defined J2000.0 in terms of (future) TAI and set the coincidence of all theoretical times at this epoch. This would have simplified the use of J2000.0 (but not of future epochs, e.g. J2050.0...). But the main advantage would have resulted from the steady improvement of TAI: from 1977 to 2000 its long term stability and accuracy have been increased by a factor ten. The coincidence of all theoretical coordinate times at J2000 with TAI + 32.184s would have much reduced the future departure of their realization. This would not have generated difficulties, since, whichever be the origin of time scales, TT is realized by TAI + 32.184s and the realization of other coordinate times is obtained from that of TT by purely theoretical relations.

Those who participated to the discussions on the definition of the reference systems and the IAU General Assemblies since 1991 may remember how it was difficult to agree on the essential. Details on secondary points were sometimes voluntarily left aside in order to avoid a total failure. Nevertheless, even if some aspects of the definition of the origin of time scale and of J2000.0 may appear somewhat clumsy, these subtle problems have been treated rigorously. The only serious difficulty is the inconsistency in the use of the periodic terms of the transformation between geocentric and barycentric coordinate times. A decision is required.

I thank P. Bretagnon for helpful discussions and for providing the values of TCB - TCG.

Table I. Dates of J2000.0 (at the geocentre)

$(TT)_0 = 2451545.0$ JD	exactly, by definition of J2000.0
$(TAI)_0 = (2451544.9996275000 \pm 3 \times 10^{-10})$JD	uncertainty arising from TAI
$(TCG)_0 = 2451545.0000058545519...$ JD	rigorous (IAU Resolution, 2000)
$(TT)_0 - (TAI)_0 = 32.184\,\mathrm{s} \pm 30\mu\mathrm{s}$	
$(TCG)_0 - (TT)_0 = 0,50583328...$ s	rigorous (IAU Resolution,2000)
$(TCB)_0 - (TCG)_0 = 10,747888...$ s $+P_0\ (-P_{1977}\ ?)$	
$(TCB)_0 - (TT)_0 = 11.253721...$ s $+P_0\ (-P_{1977}\ ?)$	
$(TDB)_0 - (TT)_0 = P_0\ (-P_{1977}\ ?)$	

$P_0 = -99.3\mu$s $\qquad P_{1977} = -65.5\mu$s

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DEFINITION AND REALIZATION OF TAI

E. F. ARIAS
Bureau International des Poids et Mesures
Pavillon de Breteuil - 92312 Sèvres Cedex, France
e-mail: farias@bipm.org
Astronome Associé au DANOF, Observatoire de Paris, France

ABSTRACT

International Atomic Time (TAI) is the uniform time scale computed at the Time Section of the Bureau International des Poids et Mesures (BIPM). The scale unit of TAI is kept as close as possible to the second of the International System of units (SI) by using data from primary frequency standards operated in some national laboratories. TAI is calculated through clock comparison data carried out using Global Positioning System (GPS) satellites and two-way satellite time and frequency transfer (TWSTFT). The two conventional time scales calculated at the BIPM, TAI and UTC (Coordinated Universal Time) are evaluated and published on a monthly basis. Work is continuously done at the BIPM to improve the stability of the atomic time scale and the accuracy of TAI.

1. INTRODUCTION

International Atomic Time is the uniform time scale calculated at the Time Section of the Bureau International des Poids et Mesures (BIPM) since 1988. The unit of TAI is the second of the International System of units (SI), defined as a fraction of the period of an atomic transition; it is realized on the basis of the measurements reported by laboratories operating primary frequency standards.

More than 200 clocks installed at about 50 national laboratories participate in TAI. The algorithm which serves to the calculation of TAI is based on clock differences, so it has been necessary to implement methods of comparison of distant clocks. In the last decade the time community has largely exploited the Global Positioning System (GPS) satellites for time transfer using receivers adapted for time comparisons. Research conducted at various laboratories proved that equal or best results could be achieved by comparing remote clocks by using the two-way time transfer via telecommunication satellites. The network of clock comparisons at the BIPM is at present supported mostly by common views on GPS setellites and by some links using TWSTFT.

Being a uniform and stable scale, TAI does not keep in step with the irregularities of the Earth's rotation. For practical reasons, Coordinated Universal Time (UTC) has been defined as an atomic scale which follows the earth's rotation behavior; whenever necessary leap seconds are introduced in UTC to avoid the atomic scale to diverge indefinitely from the rotational scale. The moment of introduction of the leap second is announced by the International Earth Rotation Service (IERS).

The BIPM Time Section calculates the two uniform time scales TAI and UTC and publishes them monthly in the BIPM Circular T in the form of their differences from local representations. Research work related to algorithms for time scales and to the improvement of time transfer methods is developed.

2. TAI ACCURACY AND EAL STABILITY

The Comité Consultatif de Temps et Fréquences (CCTF), at his 14th meeting (1999), issued recommendations concerning the comparison of primary frequency standards (PFS) and the way of stating their uncertainties.

The International Astronomical Union (IAU) defined Terrestrial Time (1991) TT as the coordinate time for the geocentric reference system, such that its scale interval agrees with the SI second on the geoid. The intricacies and changes in the definition and realization of the geoid affected directly the definition and realization of TT. To avoid that this become the major source of uncertainty in the realization of TT from atomic clocks, the $XXIV^{th}$ IAU General Assembly (2000) adopted a new definition of TT rendering it independent from the geoid realizations.

New procedures have been developed following the CCTF and the IAU recommendations for using PFS to ensure the accuracy of TAI and for reporting the results (Petit, 2000). Since May 2000 we evaluate the fractional deviation d of the scale interval of TAI from that of TT from measuments of primary frequency standards which report on their uncertainties. The uncertainty of the estimation d is the resultant of several components: the combined uncertainty from systematic effects (the only one previously stated), the uncertainty originated in the instability of the primary standard, and the uncertainties coming from the link between the PFS and the clock participating in TAI and from the link to TAI.

Ten PFS contribute at present to the BIPM, some of them operating continously as clocks participating to TAI. BIPM estimations of d and of its uncertainty are done over 30 day periods on the basis of PFS measuments. The global treatment of individual measurements led to a relative departure of the duration of the TAI scale unit from that of TT ranging, since January 2000, from +0.2 x 10^{-14} to +0.7 x 10^{-14}, with an uncertainty of 0.3 x 10^{-14} at most.

In the process of calculation of TAI an intermediate scale is evaluated; it is the atomic free scale EAL (échelle atomique libre). While steerings are applied to the TAI frequency whenever necessary to adjust its unit to the second of SI, EAL suffers no constraints at all. The medium-term stability of EAL, expressed in terms of the Allan deviation, is estimated to be 0.6 x 10^{-15} for averaging times of 20 to 40 days over the last two years. This improves the predictability of UTC for averaging times of between one and two months.

3. TIME LINKS

The establishement of TAI and UTC is based on atomic clock data coming from worldwide distrubuted laboratories, and therefore it is strongly dependent on the means of time comparison. Very accurate methods of time transfer are necessary such that the comparison does not degrade the accuracy of the atomic clocks. The present organization of the international time links at the BIPM is based on two time comparison techniques: common-views on GPS satellites and TWSTFT.

The introduction of GPS into time transfer in the 90's led to an improvement of one order of magnitude or more with respect to other methods used at that moment for time comparison. GPS allows to reach an accuracy of a few nanoseconds for short baselines, and of 10 to 20 ns when the baselines are intercontinental. These accuracies permit the comparison of the best standards for integration times over 10 days.

The GPS common-view method proposed by Allan and Weiss (1980) is used in the calculation of time links at the BIPM. Two clocks at remote stations are compared with the clock on board the same GPS satellite at the same time; when making the difference the contribution of the satellite clock vanishes, not biasing with its errors the time comparison.

GPS observations are dependent on the effects of the propagation medium on the signal. One channel C/A-code GPS time receivers track one satellite at a time. As the C/A-code is transmitted in only one frequency, there is no posibility to determine the delay introduced by the propagation of the signal along the ionosphere, and it must be modeled. A few dual frequency receivers exist, leading to measuments of the ionospheric delay along the line of sight of satellites with uncertainties at the 1 ns level. The use of ionosphere measurements rather than a model improves the time comparisons over long baselines. The delays introduced by the troposphere are treated differently; the troposphere is not a dispersive medium at radio frequencies, its effect cannot be estimated from dual frequency measuments as is done for the ionosphere. Models are used generally for the estimation of the tropospheric delay without introducing major uncertainties in time transfer when satellites are not at very low elevations.

Much progress has been done by using the International GPS Service (IGS) products to correct GPS data. All the GPS common-views in TAI are at present corrected by using the IGS total electron content (TEC) maps and satellite ephemerides. In general there is an improvement in stability when using TEC maps instead of modeling the tropospheric delay; when comparing the stability for on-site measurements to the one for TEC maps, they result to be almost equivalent (Wolf and Petit, 2000).

The development of multi-channel multi-code receivers permits to increase in an order of magnitude the number of observations with respect to the single-channel mode, and to eliminate the ionospheric delay. Some receivers are designed not only for GPS but also for the observation of the Russian Global Navigation System (GLONASS) satellites.

The BIPM establishes and distributes to the time laboratories schedules for the GPS and GLONASS satellites pursuits permitting to organize the common-view links.

The two-way time transfer via telecommunication satellites (TWSTFT) could achieve accuracies at the subnanosecond level whith daily comparisons. At present, some time laboratories are equipped with receiver/transmitting stations that conduct two-way sessions several times a week. Even under these conditions, TWSTFT is comparable to GPS time transfer, and provides a means of controlling the GPS links.

GPS C/A-code single-channel observations have been for one decade the only method of time comparison in TAI. Since the 1999 new techniques have been incorporated, reinforcing the stability of TAI. Three links are regularly calculated by using GPS multi-channel common-views, and five by the two-way method.

Monthly BIPM TWSTFT are published giving all relevant information about the evolution of the two-way links and their comparison with the respective back-up GPS links.

4. CONCLUSION

The stability of the atomic time scale has been improved due to the replacement in most laboratories of the old caesium clocks by the more performant new-type ones. The algorithm used in the calculation of TAI assignes relative weights to participating clocks, thus assuring that the stability is highly dominated by the best clocks. PFS frequency measuments are used to evaluate the departure of the scale unit of TAI relative to the SI second; they are either excellent commercial caesium clocks or clocks developed at some laboratories which have long-term stability. Time transfer techniques need to be accurate enough for the comparison of precise clocks. GPS single-channel and multi-channel common views plus TWSTFT are used to compare the clocks participating in TAI, leading to a stability of about 2 parts in 10^{15} for periods of about one month.

Research work on algorithms for time scales and on time transfer techniques are permanently conducted at the BIPM. Clock comparison by using the GLONASS satellites could be used in TAI in the future provided that the russian satellite constellation be complete and stable (Azoubib and Lewandowski, 2000). In a joint effort, the BIPM and the IGS organized a pilot project to investigate the potentialities of time and frequency transfer by using geodetic-type GPS receivers (Jiang et al., 2000). Work on calibration of this type of reveivers is organized and conducted at the BIPM (Petit et al., 2000).

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TIME FOR THE EPHEMERIDES AND THE OBSERVATIONS

J. KOVALEVSKY
Observatoire de la Côte d'Azur - CERGA
Avenue Copernic
06130 Grasse, France
email: jean.kovalevsky@obs-azur.fr

ABSTRACT. In a first part, the historical evolution of time scales is sketched. In particular the reasons for the short lifetime of the Ephemeris time (ET) are explained: it was introduced when atomic clocks were invented and which gave a much better reading precision. Therefore, it was soon replaced by the international atomic time scale (TAI). Then, the terrestrial and geocentric times (TDT, TT, and TCG) are presented in the framework of General Relativity and in the light of the decisions of the UAI General Assembly in 2000. Finally, a similar presentation of the barycentric times (TDB and TCB) is made, including the relations between their secular trends.

1. HISTORICAL INTRODUCTION

Historically, the need for an accurate and uniform time scale is a rather new requirement. In every day life, human activities were ruled by the apparent motion of the Sun and by the succession of days and nights. Therefore, it was sufficient to have a continuous count of hours without any need for them to be strictly equal. One needed a time scale with a sufficient number of reference points and not a precise unit of time.

Even when world-wide navigation became an important activity, in the 17-th and 18-th centuries, precise clocks were not a requirement *per se*. They were used as an instrument that represented the rotation of the Earth with respect to the stars, a rotation that was considered as being uniform. The actual real need for an intrinsic uniform time scale resulted from the progressive development of the theory of the motion of planets and of the Moon. After Newton and his followers in the 18-th and 19-th centuries, uniform time was recognised as the inescapable parameter of Celestial Mechanics. The postulate of an uniform rotation of the Earth was used to define the uniform time scale. Since it was found that this postulate is not true, a series of changes of the reference time scales occurred.

At the end of the 19-th century, some discrepancies between the observations and the theories of motion of the Moon and planets seemed to show that there could be some inconsistency between the independent variable of Celestial Mechanics and the time defined by the rotation of the Earth. The most conspicuous effect was the secular acceleration of the Moon, already recognised by Delaunay, and evaluated by Brown (1919) who found that tidal torques could not account entirely for the observed acceleration and that there remained irregular terms that he represented empirically. Newcomb (1912) had already assumed that a part could be explained by irregularities of the rotation of the Earth, but he was aware that, in order to prove this hypothesis, one must find similar fluctuations in the motions of the planets, with an amplitude proportional to their respective mean motions.

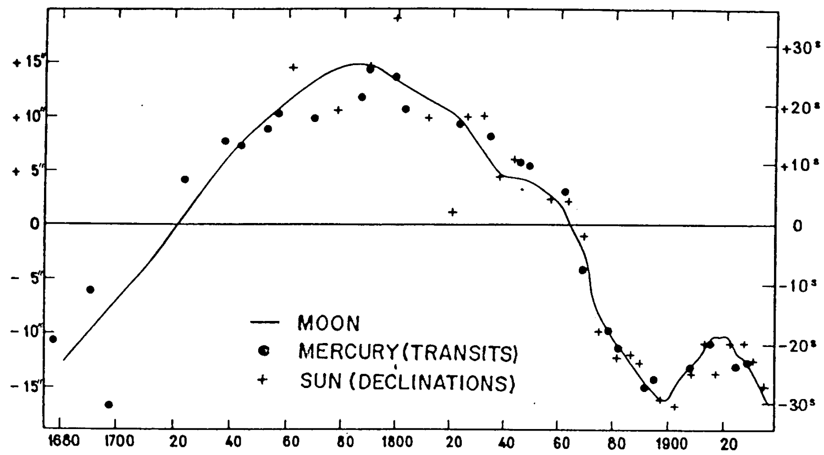

Figure 1: Fluctuations of the longitudes of the Moon, the Sun, and Mercury as given by Spencer Jones (1939). The left scale is in seconds of arc, the right scale represents the effect on Universal Time

The actual proof of the reality of this effect is due to Spencer Jones (1939) who compared the observations of the Sun, the Moon and Mercury (Figure 1). A little earlier, Stoyko (1937) had discovered periodic seasonal variations in the rotation of the Earth. Both results were in contradiction with the assumption that the rotation of the Earth was unifom. But, in the meantime, the universal time UT1, based on the rotation of the Earth, was already adopted for the world-wide time system as well as for the definition of the second. The introduction of UT2, which was UT1 corrected by a conventional formula for the seasonal irregularities, did not change significantly the problem.

2. A HECTIC DECADE: 1957-1967

At that epoch, only one solution was available to built a uniform time scale: to use the independent variable of Celestial Mechanics that describes the motions in the Solar system. Actually, would one have waited a few more years, with the advent of atomic clocks, a much better solution would have been decided right away, avoiding the most hectic decade in the history of time.

Ephemeris time (ET) was introduced by the IAU in 1958. It was based upon Newcomb's theory of the motion of the Earth around the Sun. It was the best available theory, although it was 70 years old. Actually, it was not even consistent because Newcomb did not correct the observations for the irregularities of the rotation of the Earth which were not yet known. Other inconsistencies exist in this theory (see Kovalevsky, 1965) so that one must consider that the adopted definition is purely conventional. It reads as follows: Ephemeris time τ, expressed in units of 36 525 Ephemeris days, is defined by the following expression of the celestial longitude of the Sun referred to the mean equinox of date:

$$L_s = 279^\circ 41' 48''.04 + 129\,602\,768''.13\tau + 1''.089\tau^2 + \sum(per). \tag{1}$$

The origin $\tau = 0$ corresponds conventionally to the date 1900, January 0, 12 hours ET exactly. The periodic terms were those of Newcomb's theory which did not represent exactly the real motion. As a consequence, periodic and even secular errors of the theory were bound to pollute the interpretation of observations in terms of ET.

From this definition, the Confrence gnrale des poids et mesures (CGPM) resolved in 1960 that the unit of time, the second of the international system of units (SI), was equal to the fraction 1/31 556 925.9747 of the tropical year for 1900, January 0, 12 hours ET. The origin had to be fixed since, following equation (1), the length of the tropical year varies with time.

The decisions concerning ET were not yet enforced, that atomic clocks using cæsium frequency standards changed completely the situation. The first cæsium clock was built in the National Physical Laboratory by Essen and Parry (1955) and was continuously in operation since then. Several others were built, in particular in the US Naval Observatory, so that first results on the variations of the rotation of the Earth using these clocks were available three years later (Essen et al., 1958). Already in 1960, the first time signals based upon atomic clocks were emitted and, in 1961, the Bureau international de l'heure (BIH) constructed an experimental atomic time scale based on sevaral such clocks. In 1964, this time scale was regularly run and recognised world-wide as the BIH time scale.

The great disadvantage of ET was that it was in practise not accessible with the required accuracy. The primary "clock" was the Sun and one had to determine its longitude with transit instruments and then obtain ET using formula (1). A secondary "clock" was the Moon observed with the Markowitz lunar camera (Markowitz, 1954). The measurement of the position of the Moon provided ET based upon the theoretical relation between the longitudes of the Sun and of the Moon. In both cases, the actual readings of ET were not better than 10^{-7} for the mean of one month observations This is to be compared with dayly precisions of a few 10^{-9} for UT1 and 10^{-11} for the new atomic time scale. So, for practical reasons, it was not possible to keep Ephemeris time as the reference and atomic time quickly supplanted ET.

Already in 1967, the CGPM drew the consequences of this situation and redefined the second of the SI as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cæsium atom 133. The same year, IAU approved the International Atomic Time (TAI) as a coordinate time scale defined in a geocentric reference frame with an SI second realised on the rotating geoid. This was exactly the BIH time scale, and the BIH was appointed to determine TAI on the basis of the readings of atomic clocks operated in various establishments in accordance with the definition of the second of the SI. Since 1987, this task is performed by the BIPM.

Ephemeris time, despite its drawbacks had a great advantage: it could be extended towards the past using old observations of the Moon and planets. One can evaluate to 1 second the possible drift of the ET during the 19-th century after correction of the irregularities of the Earth rotation recovered from past astronomical observations. This is of course not the case for atomic time, but shifting TAI by a constant quantity (32.184 s), one obtains a parallel time scale called TDT (Terrestrial Dynamic Time), then renamed in 1991, TT (Terrestrial time). The shift of the origin of TAI was chosen in such a way that ET can be considered as representing the extension of TAI before 1957.

3. GEOCENTRIC AND TERRESTRIAL TIMES

The change of denomination just mentioned, together with a few others, shows the difficulties encountered by astronomers in front of two major changes in their scientific paradigms. First, the time was no more astronomically defined, and was in the hands of physicists so that it was not always clear how to use it for astronomical purposes. Second, General Relativity was to become the basic theory on which to rely not only for the dynamics of the Solar system but, even more so, for the construction of time scales.

The dynamics of bodies revolving around the Earth, (Moon and artificial satellites) must be studied in a geocentric reference frame. In terms of General Relativity, this means that the space-time in which one works has its origin at the centre of mass G of the Earth. In this context, the space coordinates of the origin are:

$$x_G = y_G = z_G = 0,$$

and the coordinate time t_G is identical to its proper time τ. The problem is that one does not observe from G, but from the surface of the Earth. Now, to extend the geocentric space-time to another event (a point with coordinates x, y, z and its time t), one must use the metric of this space-time as defined by the IAU in 1991 and then as completed by the IAU in August 2000. Neglecting the perturbing Lense-Thirring acceleration, it reads:

$$ds^2 = -c^2 d\tau^2 = (-1 + 2U_G/c^2 - 2U_G{}^2/c^4)c^2dt^2 + (1 + 2U_G/c^2)(dx^2 + dy^2 + dz^2), \qquad (2)$$

Where U_G is the gravitational potential. From this equation, one can derive the relation between the proper time τ and the coordinate time t, noting that the coordinate velocity due to the rotation of the Earth is $dz/dt = 0$ and

$$v = \sqrt{(dx^2 + dy^2)/dt^2}$$

$$(d\tau/dt)^2 = 1 - 2U_G/c^2 + 2U_G{}^2/c^4 - v^2/c^2 - 2U_G v^2/c^4. \qquad (3)$$

Let us now introduce the Coriolis potential due to the rotation of the Earth

$$U_R = \omega^2(x^2 + y^2)/2 = v^2/2$$

and let U be the total potential, $U = U_G + U_R$, then one obtains finally:

$$d\tau/dt = 1 - 2U/c^2 + 2U_G(U_G - 2U_R)/c^4. \qquad (4)$$

This is usually written as:

$$d\tau/dt = 1 - L_G. \qquad (5)$$

In this expression, τ represents the proper time of a clock on the geoid. This is exactly what is done when constructeing TAI, and consequently TT. The coordinate time t is the time coordinate t_G of the geocentric space-time. It is called Geocentric Coordinate Time (TCG). Its relation with TT is provided by equation (5). L_G is a function of the potential of the Earth at the geoid. Its determination improves with time, and additional terms (as here in c^{-4}) can also change its value. More important even is the fact that there are temporal changes inherent to the definition and the realisation of the geoid. These are sources of uncertainties in the definition and the realisation of TT. So, the IAU General Assembly of 2000 decided that the relation between TCG and TT will be fixed by convention using a defining constant linking TT to TCG. In other terms, we have now a new definition of TT:

The terrestrial time scale (TT) is a time scale differing from TCG uniquely by a constant rate conventionally fixed so that

$$d(\mathrm{TT})/d(\mathrm{TCG}) = 1 - L_G = 1 - 6.969\,290\,134 \times 10^{-10} \qquad (6)$$

This value of L_G is consistent with the best current estimate of the geopotential at the geoid (Groten, 2000).

In practise, observations of an artificial satellite or of the Moon are made in UTC from which TT is obtained by subtracting the cumulated leap seconds and adding 32.184 seconds. The reduction of observations and the dynamical interpretation must be made in TCG. The transformation from TT to TCG yields from the integration of equation (6) taking into account that the common origin is January 1, $0^h0^m0^s$ TAI exactly. This gives, in seconds:

$$\mathrm{TCG} - \mathrm{TT} = L_G \times (JD - 2\,443\,144.5) \times 86\,400, \qquad (7)$$

where JD is the Julian date of the observation. This new definition of TT is particularly well suited for space clocks for which the actual geoid is not accessible in contrast with the local gravitational field.

In summary, one can use indifferently UTC, TAI or TT for recording the observations. Normally, one should use TCG for ephemerides and Celestial Mechanics. It is however possible to use TT, but one must take into account that the units are different and are linked by equation (6).

4. BARYCENTRIC TIMES

The time coordinate of the barycentric reference frame defined by its metric is what should be used in studying the dynamics of the solar system. The correct method to reach it is to start from the metric of the geocentric reference frame (2) and to determine its relation with the metric ds'^2 of the barycentric reference frame in which we have again neglected the Lense-Thirring acceleration:

$$ds'^2 = -c^2 d\tau'^2 = (-1 + 2U_B/c^2 - 2{U_B}^2/c^4)c^2 dt'^2 + (1 + 2U_B/c^2)(dx'^2 + dy'^2 + dz'^2), \qquad (8)$$

where U_B is the generalised Newtonian gravitational potential. To get the required relation, one must apply the general formula of transformation between two space-time systems in General Relativity. This is a complicated set of linear equations between the coefficients $g_{\alpha\beta}$ of the first metric and the g'_{ij} of the second. The equations involve partial derivatives of the coordinates (x', y', z', t') of the second system with respect to the coordinates of the first (x, y, z, t). Let us restrict ourselves to the transformation between the time coordinates t which is TCG and t' which is the barycentric coordinate time (TCB). The result, limited to the second order terms in $1/c$ is:

$$\mathrm{TCB} - \mathrm{TCG} = \mathrm{c}^{-2}\left[\int_{\mathrm{t_0}}^{\mathrm{t}} \left(\mathbf{v}_\mathbf{e}^2/2 + \mathrm{U}_{ext}(\mathrm{x_e})\right) dt + \mathbf{v}_\mathbf{e}.(\mathbf{x} - \mathbf{x}_\mathbf{e})\right], \qquad (9)$$

where $\mathbf{x}_\mathbf{e}$ and $\mathbf{v}_\mathbf{e}$ denote the barycentric position and velocity of the geocentre and $\mathbf{x}$ is the barycentric position of the observer. the time t_0 corresponds to the origin of TT. The potential $U_{\mathrm{ext}}$is the potential of all solar system bodies except Earth. This potential varies with time because of the motion of planets so that there periodic as well as secular terms when this formula is integrated (see Irwin and Fukushima, 1999).

The estimated accuracy of this formula is of the order of 10^{-14}. In order to get an uncertainty of 5×10^{-18} in rate and rate amplitude of the quasi- periodic terms and 0.2 ps in phase amplitude for locations farther than a few solar radii from the Sun, one must add terms in c^{-4} that are given in recommendation 5 of the resolution adopted by the IAU in August 2000 (Andersen 2001).

In 1976, when a barycentric time was introduced, It was decided that its difference with TT should have no secular term, a condition that is not compatible with equation (9). This particular time scale was called Barycentric Dynamical Time (TBD). This definition was not consistent with the proper unit of time, the SI second, but corresponded to the general use of the terrestrial time for ephemerides and provided continuity with Ephemeris time. This inconsistency was removed in 1991 by the introduction of TCB, but TDT is still used in planetary navigation. The difference between TDB and TCB is the secular term in equation (9). Its value is (Irwin and Fukushima 1999):

$$< \mathrm{TCB} - \mathrm{TCG} >= 1 - \mathrm{L_C}$$

with

$$L_C = 1.480\,826\,86741 \times 10^{-8}.$$

The uncertainty on L_C is $\pm 2 \times 10^{-17}$. One can also define

$$< \mathrm{TCB} - \mathrm{TT} >= 1 - \mathrm{L_B}$$

with

$$L_B = 1.550\,519\,76772 \times 10^{-8}.$$

From equation (6) and these definitions, on infers:

$$1 - L_B = (1 - L_C)(1 - L_G).$$

Contrarily to the geocentric case, it is not advisable to give to L_C a conventional value as for L_G because its value depends on the theory used to describe the periodic terms in the motion of the Earth with respect to the barycentre of the Solar system. It is, at a smaller scale, the same problem as in the case of ET.

5. CONCLUSION

As shown in this presentation, the various time scales in use have undergone many innovations during the last half century. Figure 2 sketches these changes and shows the filiation between some of them. Considering the care with which the time scales that exist at present are defined, one may hope that the concepts will stabilise in the decades to come. However, this may not be the case for their realisation, in particular if atomic clocks in the optical domain are developped in connection with the new femtosecond lasers. It is therefor possible that, in the future,the cæsium atom transition might be replaced, in the definition of the second, by some transition in the optical wavelengths.

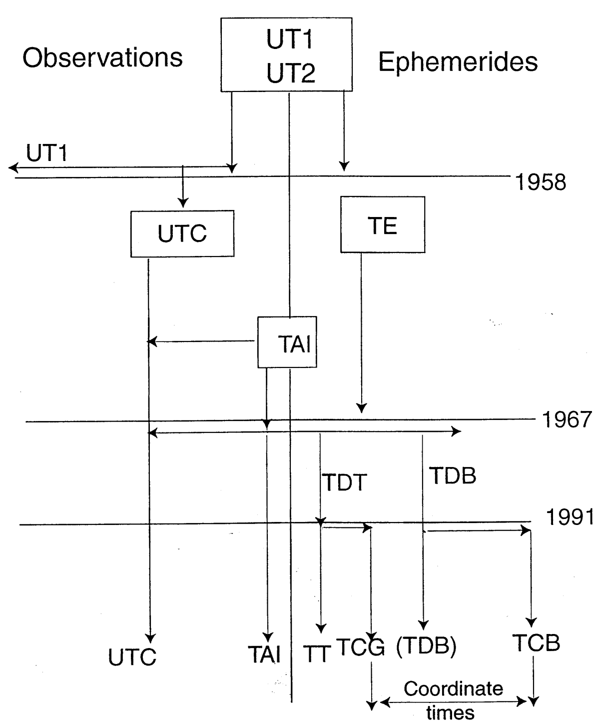

Figure 2: History and filiation of various time scales since 1950

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RECENT PROGRESS ON THE COLD ATOMS CLOCKS AT BNM-LPTF

M. ABGRALL, P. LEMONDE, S. BIZE, Y. SORTAIS, S. ZHANG,
G. SANTARELLI, P. LAURENT, A. CLAIRON, and C. SALOMON*
BNM-LPTF, Observatoire de Paris, 61 Av. de l'Observatoire, 75014 Paris, France
* Laboratoire Kastler Brossel, département de physique de l'Ecole Normale Supérieure, 24 rue Lhomond 75231, Paris, France
e-mail: michel.abgrall@obspm.fr

ABSTRACT. We present recent results on microwave frequency standards using cold atoms. Two cesium fountains have been built and exhibit a frequency accuracy of 1 10^{-15}. Though quite different in their design, both fountains are found to give the same frequency within the error bars of the measurements. One of the fountains is transportable. It was moved to Germany and used as a reference for a phase coherent measurement of the 1S-2S transition of Hydrogen with a 2 10^{-14} accuracy. When using a cryogenic sapphire oscillator as an interrogation oscillator, the frequency stability reaches the fundamental limit set by the quantum projection noise. A short term stability of 4 10^{-14} $\tau^{-1/2}$ has been obtained. One limitation to the performances of cesium fountains is the frequency shift due to collisions between cold atoms. We show that with rubidium atoms, this effect can be decreased by two orders of magnitude. This feature should allow to vastly improve both the stability and accuracy of microwave fountains. Finally by tracking the frequency between rubidium and cesium fountains, we test the stability of the fine structure constant α with a few 10^{-15} resolution. We also present the status of the ACES space project.

1. INTRODUCTION

As shown in Fig. 1, the BNM-LPTF is operating two cold atom Cs fountains, a ^{87}Rb fountain, and an optically pumped Cs thermal beam[1]. In this paper, we present recent advances on the cold atom devices. With a hydrogen maser as a flywheel oscillator, the frequencies of these clocks are compared with a fractional resolution of about 10^{-15}, limited by the flicker floor of the maser. First we show that by reaching the fundamental quantum limit set by the measurement process, a cesium fountain can operate with a record frequency stability of 4 10^{-14} $\tau^{-1/2}$ where τ is the integration time in seconds[2]. Second, the ^{87}Rb ground-state hyperfine splitting has been measured with a relative accuracy of 2.4 10^{-15}, an improvement of 4 orders of magnitude over previous measurements[3]. Third, we describe a transportable cold cesium clock, PHARAO[4]. Finally we present the science objectives of the ACES mission (Atomic Clock Ensemble in Space), which will demonstrate the operation of a laser cooled atomic clock in space.

2. OBSERVATION OF THE QUANTUM PROJECTION NOISE LIMIT IN A CESIUM FOUNTAIN

Until recently, the frequency stability of atomic fountains was limited by the phase noise of

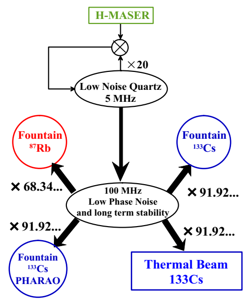


Figure 1: The BNM-LPTF clock ensemble. The interrogating microwave frequency of each clock is generated from the same local oscillator, which delivers a 100 MHz frequency. This frequency is synthesized from a 5 MHz BVA quartz weakly locked to a H-maser. The Cs PHARAO fountain is a transportable device able to operate autonomously.

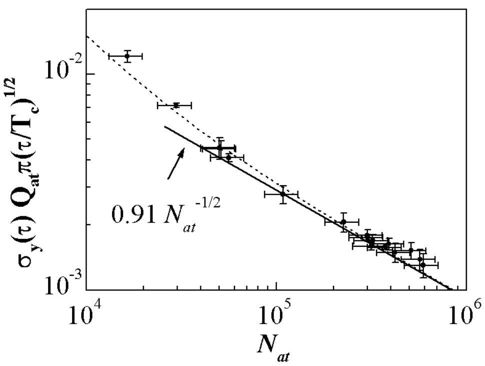


Figure 2: the normalized frequency fluctuations as a function of the number of detected atoms N_{at}. The expected quantum projection noise law is $y = N_{at}^{-1/2}$. The thick line $y = 0.91(0.1)N_{at}^{-1/2}$ is a least square fit to the experimental points for $N_{at} > 4\ 10^4$. The dashed line is the quadratic sum of the detection noise and quantum projection noise.

the quartz oscillator used for the atom interrogation[5]. With state-of-the-art quartz oscillators, this limit is above $10^{-13}\,\tau^{-1/2}$, with τ the averaging time in seconds. With such a stability, a 10^{-16} accuracy goal is challenging since a single measurement with this resolution implies an averaging time of more than one week. By using a cryogenic sapphire oscillator (SCO)[6], we have been able to overcome this limitation and to reach the quantum projection noise limit[7, 8].

Before being detected, the atomic internal state is a linear superposition of the two clock transition states $|e\rangle$ and $|g\rangle$: $|\psi\rangle = \sqrt{p}|e\rangle + \sqrt{1-p}|e\rangle$. The detection process consists of determining whether each atom is in state $|e\rangle$ or $|g\rangle$. The probability of finding an atom in $|e\rangle$ is $p = \langle\psi|P_e|\psi\rangle$, with P_e the projection operator onto state $|e\rangle$. The measurement exhibits quantum fluctuations of standard deviation $\sigma = \left(\langle\psi|P_e^2|\psi\rangle - (\langle\psi|P_e|\psi\rangle)^2\right)^{1/2} = \sqrt{p(1-p)}$. For a set of N_{at} uncorrelated atoms, σ scales as $N_{at}^{-1/2}$.

The first crucial element in our experiments is the use of an ultralow frequency noise cryogenic sapphire oscillator[6] as the interrogation oscillator in the fountain. The SCO, developed at University of Western Australia, possesses a mode which is only 1.3 MHz away from the Cs hyperfine splitting frequency ν_0. Oscillating on this mode, its frequency stability is below 10^{-14} from 0.1 to 10 s. The excess noise is then negligible compared to the projection noise in our atomic fountain. The second key feature of the experiment is an efficient method to measure the populations of the two states of the cesium clock transition.

The Allan standard deviation of the relative frequency fluctuations $y(t)$ of an atomic fountain can be expressed as:

$$\sigma_y(\tau) = \frac{1}{\pi Q_{at}}\sqrt{\frac{T_c}{\tau}}\left(\frac{1}{N_{at}} + \frac{1}{N_{at}n_{ph}} + \frac{2\sigma_{\delta N}^2}{N_{at}^2} + \gamma\right)^{1/2}. \tag{1}$$

In (1) τ is the measurement time in seconds, T_c is the fountain cycle duration ($\sim$ 1 s) and $\tau > T_c$. $Q_{at} = \nu_0/\Delta\nu$ is the atomic quality factor. The first term in parentheses is the atomic projection noise $\propto N_{at}^{-1/2}$ [8]. The second term is due to the photon shot-noise of the detection fluorescence pulses and is less than 1% of the projection noise. The third term represents the effect of the noise of the detection system. $\sigma_{\delta N}$, the uncorrelated rms fluctuations of the atom number per detection channel, is about 85 atoms per fountain cycle. This noise contribution becomes less than the projection noise when $N_{at} > 2\ 10^4$. γ is the contribution of the frequency noise of the interrogation oscillator[9, 5]. With the SCO, this contribution is at most $10^{-14}\tau^{-1/2}$ and can be neglected. As an example, for $N_{at} \sim 6\ 10^5$ detected atoms, $\Delta\nu = 0.8\,\mathrm{Hz}$, $Q_{at} = 1.2\ 10^{10}$ and $T_c = 1.1\,\mathrm{s}$, the expected frequency stability is $\sigma_y(\tau) = 4\ 10^{-14}\tau^{-1/2}$.

To observe the quantum projection noise, we vary the number of atoms in the fountain and measure the frequency stability $\sigma_y(\tau)$ by comparison with the free running sapphire oscillator which is used as a very stable reference up to $10-20$ seconds. A plot of the normalized Allan standard deviation as a function of atom number is presented in Fig. 2. Since we explored several values of Q_{at} and of the cycle duration T_c, we plot the quantity $\sigma_y(\tau)\pi Q_{at}\sqrt{\tau/T_c}$ for $\tau = 4\,\mathrm{s}$. This quantity should simply be equal to $N_{at}^{-1/2}$ when the detection noise is negligible. At low atom numbers, the $1/N_{at}$ slope indicates that the stability is limited by the noise of the detection system (third term in eq. 1). For $N_{at} > 4\ 10^4$, the experimental points are in good agreement with the relation $y = aN_{at}^{-1/2}$ with $a = 0.91(0.1)$, close to the expected value of 1 (Fig. 2).

With the largest number of detected atoms, $N_{at} = 6\ 10^5$, the stability is $4\ 10^{-14}\tau^{-1/2}$ for $Q_{at} = 1.2\ 10^{10}$, $T_c = 1.1\,\mathrm{s}$. This is an improvement by a factor of five for primary atomic frequency standards[10, 11] and is comparable to the best short-term stability achieved with microwave ion clocks using uncooled $^{199}Hg^+$ and $^{171}Yb^+$ samples[12, 13].

Table 1 presents the current accuracy budget of the Cs fountain. The fountain has the best accuracy ever reported for frequency standards. This evaluation was performed by amplifying the systematic effects when possible as described in[11] and with a resolution for each measurement of 10^{-15}. With the current set of cold atom devices, a resolution of $1-2\ 10^{-16}$ will become accessible. An accuracy of $1\ 10^{-16}$ is thus a realistic target.

Effect	Correction $[10^{-15}]$	Uncertainty $[10^{-15}]$
First Order Doppler	0.1	$\leq$ 0.5
Second order Doppler and gravitation	0.1	0.01
Cold collisions	1	$\leq$ 0.5
Background gas collisions	0	$\leq$ 0.5
Blackbody radiation	+17.6	$\leq$ 0.5
Zeeman shift	−133	$\leq$ 0.1
Pulling by other lines	0	0.4
Microwave leaks	0	0.2
Microwave spectrum	0	0.2
Total 1σ uncertainty		**1.1**

Table 1: Present Accuracy budget of the ^{133}Cs fountain. The device operates with a stability of $1.5\ 10^{-13}\tau^{-1/2}$, a number of detected atoms of 10^5. The MOT is disabled and atoms are captured in optical molasses. The gravitational redshift is not intrinsic to the clock and is not included in the table.

3. RUBIDIUM FOUNTAIN

The construction of a ^{87}Rb fountain has been motivated by two reasons. First, the collisional shift has been predicted to be 15 times lower for ^{87}Rb than for ^{133}Cs[16]. Since this frequency shift is one of the dominant terms in the uncertainty budget of Cs fountains, this prediction makes Rb attractive for pushing the limits of cold atom frequency standards. Second, by comparing the hyperfine frequency of different atoms, one can search for possible variations of the fine structure constant $\alpha = e^2/\hbar c$ with time[17]. With a potential accuracy close to 10^{-16}, Cs and Rb fountains would provide a laboratory test within the $10^{-16} - 10^{-17}\,\mathrm{yr}^{-1}$ range, two orders of magnitude beyond present laboratory measurements.

The experimental set-up is described in[3]. With 25 mW per cooling beam, we trap in a MOT up to 8 10^8 atoms spread among the $F = 2$ Zeeman sublevels. The density of atoms selected in $m_F = 0$, and averaged along the flight above the cavity, reaches $\sim 10^8$ at cm^{-3}. The short term stability of the device is limited by the phase noise of the quartz oscillator and is 1.5 $10^{-13}\tau^{-1/2}$. It averages down to 1 10^{-15} after a 2 10^4 s integration. By comparison with the cesium fountain, we obtain a new value for the ^{87}Rb ground state hyperfine splitting: 6 834 682 610.904 333(17) Hz. The accuracy of the measurement is 2.4 10^{-15} and is 10^4 times more precise than previous atomic beam measurements[18]. This illustrates the potential of cold fountains for high precision measurements.

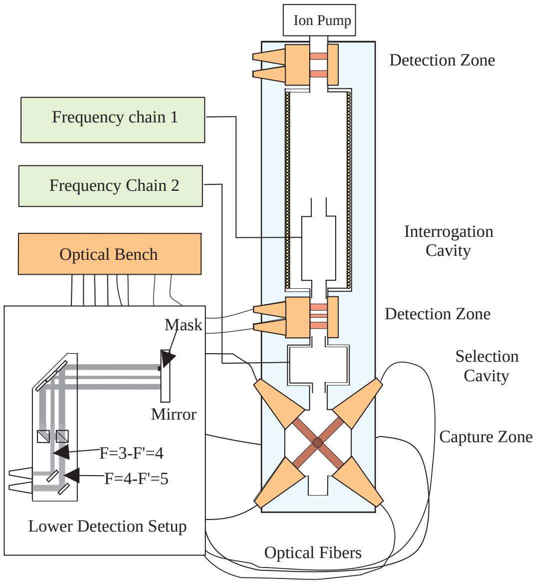
We have searched for the shift induced by cold atom collisions in Rb. By changing the loading duration of the MOT, we varied the average atomic density from 2 10^6 to 1 $10^8\,\mathrm{cm}^{-3}$. The atomic density was deduced from the time of flight signals and from the size of the atomic cloud just after launch, measured on a CCD camera. We find that the Rb density dependent shift is between 40 and 150 times smaller than in Cs[14, 15]. Such a result leads us to reevaluate the ultimate performance of cold atom fountain standards. Such a strong reduction of the collisional shift would allow vast improvement of both the short and long term stability of the standard as well as its accuracy. The challenge is to reach the quantum projection noise limit with up to $10^7 - 10^8$ atoms per cycle. The long term stability would in principle get into the 10^{-17} range.

Over a period of 13 months, we have performed two measurements of the Rb ground state hyperfine frequency by comparison with Cs. The uncertainties on these measurements are respectively 1.3 10^{-14} and 2.5 10^{-15}. We then obtain an upper bound for the rate of change of α, $\dot{\alpha}/\alpha \leq 3\ 10^{-14}\,\mathrm{yr}^{-1}$. The accuracy of this test is comparable to the best previous laboratory test[17]. With our improved accuracy, this test should soon improve by one order of magnitude. Furthermore, comparisons with other alkali-like atoms, such as Hg^+ and Yb^+, would give a clear signature of any variations in α[17, 19].

4. A TRANSPORTABLE COLD ATOM CLOCK

The cold cesium PHARAO clock is a transportable primary frequency standard. It is shown in Fig.3. It was designed for operation in the absence of gravity and was tested in aircraft parabolic flights in May 1997[4]. It also serves as an engineering model for the future space-qualified cold atom clock for the ACES mission (see section 5). As shown in Fig. 3, it can operate either in single pass through the 65 mm long TE_{011} microwave cavity with the upper detection zone or in a fountain geometry with the lower detection zone. The single pass mode mimics the operation of the space clock, whereas the fountain mode is optimal in presence of gravity. Comparisons between the two modes will give access to the first order residual Doppler correction (table 1).

A preliminary accuracy evaluation of PHARAO operating in the fountain mode has been performed. Both the quadratic Zeeman effect and the cold collision shift have been measured. For the first time, the shift due to collisions has been measured with atoms from an optical molasses. For comparison with the other cesium fountain we also account for the shift due to blackbody radiation. All other systematic effects are expected to be small compared to the present 10^{-15} resolution of the measurements (Table 1). The frequencies of the two cesium fountains are indeed found to agree within the 10^{-15} error bars. This is the case both when the devices are operated with the same interrogation



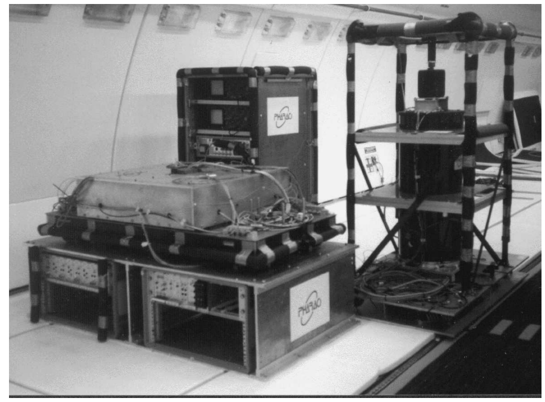


Figure 3: Left: schematic of the PHARAO clock setup. Insert: lower detection laser system. Right: The transportable PHARAO cold cesium clock in the *ZERO g* aircraft. The tube on the right part of the photograph is the 1 m high vacuum chamber. On the left part of the photograph is the optical bench, of dimensions $65 \times 65 \times 15$ cm, and control electronics.

oscillator, such as depicted in Fig. 1, or when a separate quartz crystal oscillator is locked to the atomic signal of PHARAO. Since the two devices are quite different in their design (launch geometry, microwave synthesis and cavity, detection...), this agreement nicely confirms the current accuracy of 1.1 10^{-15} quoted in Table 1. We believe that the accuracy of this transportable device will also be in the low 10^{-16} range.

A new generation of atomic frequency standards with such potential accuracy is under development worldwide[20]. It includes cold atom and trapped ion standards, using microwave or optical transitions. We plan to transport the PHARAO fountain to various laboratories to compare these clocks. Indeed, alternate methods[20] such as two way time transfer or GPS-phase have not done yet frequency comparisons below 10^{-15}.

5. ACES: ATOMIC CLOCK ENSEMBLE IN SPACE

ACES is a space mission which has been selected by the European Space Agency (ESA) to fly on the international space station (ISS) in 2005. ACES consists of two clocks, a cold atom clock (PHARAO) and a hydrogen maser (SHM, Neuchâtel observatory), together with microwave and optical links for time and frequency transfer to ground users. These equipments will fit on a nadir oriented express pallet of dimensions $863 \times 1168 \times 1240$ mm. The total mass will not exceed 225 kg and the electrical power 500 Watts.

Because micro-gravity allows long interaction times between the atoms and the microwave field and because the atomic velocity is smaller than in earth fountains and constant, we expect an excellent accuracy for a cold atom space clock. The specified frequency stability of PHARAO for ACES is better than 1 $10^{-13}\tau^{-1/2}$. Averaged over one day, this stability should be 3 10^{-16}. Its projected accuracy is 1 10^{-16}.

Scientific objectives of ACES also include measurement of the gravitational red-shift with a 25-fold improvement over the Gravity Probe A mission of Vessot et al. [21], a better test of the isotropy of speed of light, a search for a possible drift of the fine structure constant α. Finally, with the time transfer equipments, ACES will allow synchronization of time scales of distant ground laboratories with 30 ps accuracy and frequency comparisons with 10^{-16} accuracy.

6. ACKNOWLEDGMENTS

We acknowledge A. MANN, A. LUITEN and S. CHANG from University Western Australia, who provided us the cryogenic sapphire oscillator. This work is supported in part by CNES. BNM-LPTF is included in an unité associée au CNRS (UMR 8630) of Paris Observatory. Laboratoire Kastler Brossel is unité associée au CNRS (UMR 8552) et à l'Université Paris 6.

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REVISION OF THE TIDAL ACCELERATION OF THE MOON AND THE TIDAL DECELERATION OF THE EARTH'S ROTATION FROM HISTORICAL OPTICAL OBSERVATIONS OF PLANETS

Y.B. KOLESNIK
Institute of Astronomy of the Russian Academy of Sciences
48 Piatnitskaya str., 109017 Moscow, Russia
e-mail: kolesnik@inasan.rssi.ru

1.INTRODUCTION

The tidal acceleration of the Moon was first estimated by Clemence (1948) who used Spencer Jones (1939) results on the apparent accelerations of longitudes of the Sun, Mercury and Venus observed with respect to Universal time. Clemence proposed to introduce an empirical term into a purely gravitational lunar theory to account for the tidal acceleration and to conform the origin of longitudes and the time-like argument of ephemerides of planets to the ephemeris of the Moon. Morrison (1979) amended the Jones-Clemence empirical correction, and this was applied to the ILE j=2 to form the ET-UT differences by Stephenson & Morrison (1984) which are now accepted by all astronomical almanacs. Since Spencer-Jones study a large amount of optical observations of the Sun and planets have been accumulated, the non-rotating Hipparcos reference frame and the more precise integrated ephemerides of the planets have become available. With these achievements it is reasonable to revise Jones-Clemence result to compare it with the respective estimates supplied by modern techniques.

2. OBSERVATIONS AND REDUCTION

A mass of 244960 observations of the Sun, Mercury and Venus accumulated during historical period of astronomy from 1750 to 2000 have been used. In addition to the data used by Spencer Jones we incorporated all meridian observations of Mercury, observations of the Sun and Venus in the interval 1750-1835 and observations produced from 1935 up to 2000.

In the transformation procedure a set of corrections were formed by direct comparison of the standard star catalogue with the ICRS-based catalogue rotated from the epoch J2000.0 to the respective epochs of standard catalogues by use of modern precession constant and Hipparcos-based proper motions. The systematic differences were interpolated onto observed positions of planets and applied. Other corrections account for differences in modern and historical astronomical constants. The N70E catalogue (Kolesnik 1997) rigidly rotated onto Hipparcos frame was used as a reference catalogue. Observations were compared with DE405 ephemeris.

The conditional equations for the Sun are the same as those applied by Newcomb. For Mercury and Venus they are the same as given in Kolesnik (1995). Secular variation of corrections to the longitudes were determined separately from right ascension and declination residuals. In the conditional equations for right ascensions the equinox correction is omitted assuming that it will be absorbed in solution by corrections to the longitude of the Earth. Corrections to the longitudes were derived in relatively short bins, and evaluation of secular variations of longitudes was based on a set of stepwise individual solutions in bins. Corrections to the mean longitude of the Earth ΔL_0 were determined from right ascension and declination residuals of all objects. Corrections to the mean longitudes of Mercury and Venus ΔL result from residuals in right ascension only.

3. ANALYSIS AND RESULTS

Analysis was based on the Spencer-Jones approach, which accounts for fluctuations of the Earth's rotation B supplied by occultations before determining the systematic parts of the secular variations of corrections to the longitude of the Sun and planets. The starting point was the elimination of the empirical Jones-Clemence term with Morrison's correction from the series of ET-UT based on ILE j=2. The modified series (ELT-UT)=(ET-UT) +(-10.26″-24.41″T-13.00″T^2)×1.821 has been applied in comparison of observations of the Sun and planets to the ephemeris, and the 2^{nd} order approximation of corrections ΔL, ΔL_0 is interpreted as a revised empirical correction which fits the purely gravitational theory of the Moon to the numerical ephemeris DE405. This correction also calibrates the time-like argument of the lunar theory to the one of DE405. The constant term has the meaning of an offset in the origin of longitudes in ILE j=2 theory with respect to ICRS at a certain epoch. The linear term must be interpreted as a combination of corrections to mean motion of the Moon, the motions of respective planets and the residual rotation of its origin with respect to ICRS. The quadratic term represents the revised tidal semi-acceleration $\dot{n}/2$ of the Moon. The quadratic terms as resulted from observations of different objects separately in right ascension and declination are presented in Table 1.

Table 1. Determination of the tidal acceleration of the Moon from secular variation of longitudes of the Sun and planets at the interval 1750-2000. All longitude corrections are converted to the basis of the motion of the Moon by respective ratios of the mean motions.

	$\dot{n}/2$
Right ascensions	
Sun (ΔL_0)	$8.99'' \pm 1.19''$
Mercury (ΔL_0)	$9.89'' \pm 0.60''$
Mercury (ΔL)	$9.42'' \pm 0.30''$
Venus (ΔL_0)	$9.67'' \pm 0.72''$
Venus (ΔL)	$8.76'' \pm 0.40''$
RA mean	$9.35'' \pm 0.63''$
Declinations	
Sun (ΔL_0)	$13.53'' \pm 0.54''$
Mercury (ΔL_0)	$7.26'' \pm 1.37''$
Venus (ΔL_0)	$11.35'' \pm 0.91''$

The 2^{nd} order longitude approximations derived from right ascensions of Mercury and Venus are plotted in Fig.1 in comparison with the one derived from declinations of the Sun and converted to a common basis by the respective ratios of the mean motions. It is seen from the table that observations of all objects in right ascension which provide rather consistent quadratic terms of the longitude corrections are in sharp discordance with the respective terms based on declinations. This discordance is graphically demonstrated in Fig. 1. It is seen that the curvature of parabolas fitted to longitude corrections of Mercury and Venus are significantly smaller then the one based on declinations of the Sun. It should be mentionned that the quadratic term resulted from declinations of the Sun is close to the presently accepted value of the tidal acceleration of the Moon. The analysis of declination observations in the 18^{th} and 19^{th} centuries which will be presented elsewhere has lead us to the conclusion that the origin of discrepancy should be ascribed to systematic errors in declination rather then in right ascension. We decided to accept the results derived from right ascensions as more reliable.

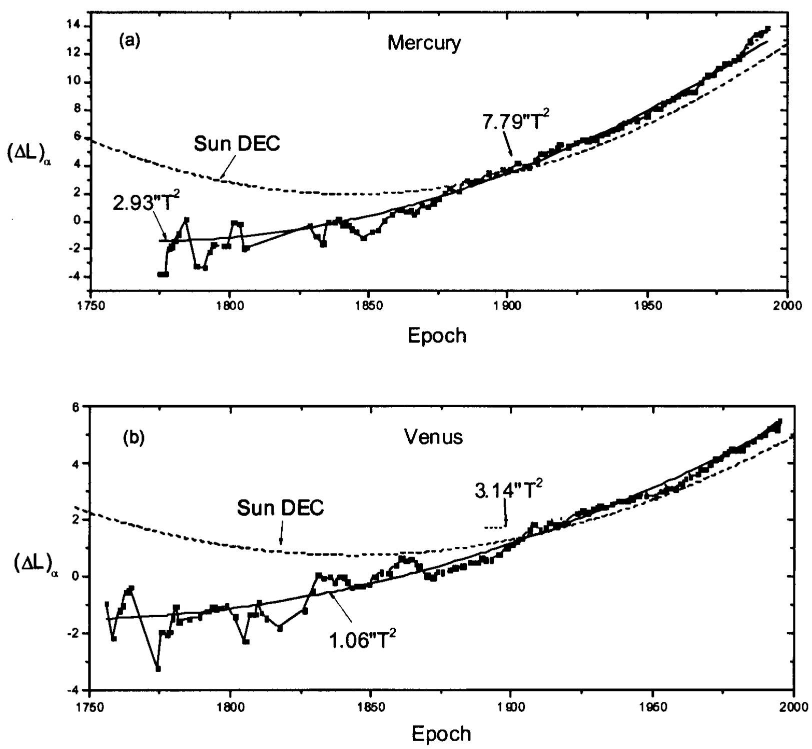

Figure 1: The 2^{nd} order approximations of corrections to the longitudes of Mercury (a) and Venus (b) at the intervals 1750-2000 and 1910-2000 as compared with the converted approximation derived from declinations of the Sun

Therefore a new empirical relation equivalent to that of Clemence has been formed as an estimated mean value of all right ascension results: $\Delta L_t = (-9.43'' - 27.33''T - 9.35''T^2)$. The formal errors of individual results seen in table are nearly equivalent to their dispersion and the most conservative value of the error of the mean is about $1''/cy^2$. The estimated revised tidal acceleration of the Moon is therefore $-18.7'' \pm 2''/cy^2$ that according to a relation from Lambeck (1980) corresponds to acceleration factor of the ET-UT differences about $32t^2$ s and to the change in the length of day -1.75 ms/cy. Such result is in sharp discordance with the estimates of the tidal acceleration of the Moon derived from LLR measurements, see e.g. (Chapront et. al. 2000).

It was outlined in (Kolesnik 2000) that in the 20^{th} century the fit to longitude corrections of the Sun yields an abnormal quadratic term which cannot be explained as simply tidal acceleration of the Moon not accounted for in the ET-UT series. The same effect is detected here in the longitudes of Mercury and Venus. The quadratic terms resulting from the fit in the interval 1910-2000 are shown in Fig.1. The excess of the apparent semi-acceleration of the longitude of Mercury is $4.9''/\mathrm{cy}^2$, and of the longitude of Venus is $2.1''/\mathrm{cy}^2$ which combined with excess of the apparent semi-acceleration of the longitude of the Sun $1.5''/\mathrm{cy}^2$ gives evidence of some unmodelled acceleration factor affecting 20^{th} century results nearly proportionally to the mean motions of planets. One possible explanation for accelerations of the planets is proposed in Masreliez (2000).

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DEVELOPMENT OF THE NEW REAL-TIME VLBI TECHNIQUE USING THE INTERNET PROTOCOL

T. KONDO, M. SEKIDO, J. NAKAJIMA, H. OKUBO, H. OSAKI
Kashima Space Research Center, Communications Research Laboratory
893-1 Hirai, Kashima, Ibaraki 314-0012, Japan
e-mail: kondo@crl.go.jp

S. NAKAGAWA
Communications Research Laboratory
4-2-1 Nukui-kita, Koganei, Tokyo 184-8795, Japan

Y. ICHIKAWA
Nihon Tsushinki Co.Ltd.

ABSTRACT. In 1997, a real-time VLBI system for practical use was developed for the first time in the Key Stone Project (KSP). In this system VLBI data are transmitted in real-time through the high-speed Asynchronous Transfer Mode (ATM) network. As the next step, we started the development of new real-time VLBI system adopting the Internet Protocol (IP) instead of ATM. Because IP technology is already well-generalized, we can expect further spreading of the real-time VLBI technique.

1. INTRODUCTION

Communications Research Laboratory has been developing the new real-time VLBI system adopting the Internet Protocol (IP) to real-time data transmission from observation sites to correlation processing site(s). In conventional VLBI observations, signals from a radio star received at two or more antennas are recorded on magnetic tapes, then the tapes are shipped to a processing center, and correlation processing is carried out. In the Key Stone Project (KSP), which is dedicated to monitoring crustal deformation around the Tokyo metropolitan area, a real-time VLBI system for practical use was developed for the first time in the world [*Koyama, et al.,* 1998; *Kiuchi, et al.,* 2000]. Radio signals are converted to digital signals with 256 Mbps data rate and they are transmitted to a correlation center in real time through the high-speed (2.4 Gbps) Asynchronous Transfer Mode (ATM) network instead of recording them on a magnetic tape. At the correlation center a dedicated correlator connected to the ATM network processes the data in real-time. We call this real-time VLBI system "ATM-VLBI" or "VLBI over ATM".

However, network-cost is still expensive and connection sites are still limited, so that the ATM-VLBI is not yet well-generalized. Hence, it has been aimed to develop new real-time VLBI system using IP technology that has already spread widely, to reduce network-cost and to expand connection sites of network. We call this system “IP-VLBI” or “VLBI over IP”.

Two kinds of system are under the consideration as a VLBI system applying IP technology for data transmission. One is the substitution of protocol from ATM to IP. In this system, serial high-speed data stream is directly sent by using IP instead of ATM (ATM may be used at the networking physical layer, but we do not care about that). We refer to the system as “High-speed IP-VLBI”. Although the data are consisted from several numbers of physical channel data, no channel-distinction is made in the transmission process.

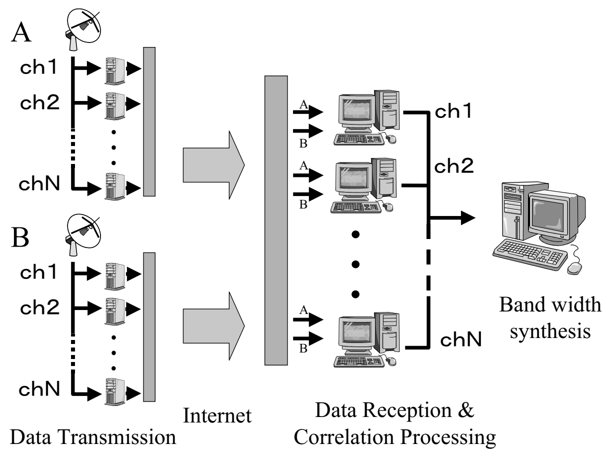

Figure 1: Concept of the multi-channel IP-VLBI system.

The other one is on the basis of channel data. A geodetic VLBI system usually receives 14 to 16 frequency channels at S and X bands. Each channel data are transmitted independently by using the IP. We refer to this system as “Multi-channel IP-VLBI” (Figure 1), because if we can establish the system for one channel, we can easily expand it to multi-channel system. Only the network speed limits the number of channels and sampling frequency. We consider that the latter system is more suitable for geodetic VLBI. In this report, we use the term “IP-VLBI” for representing the latter system. In the mean time, fringe test using data transmitted by FTP has been already realized. We call this “FTP-VLBI” distinguished from “IP-VLBI” being developed.

A sampler board for the multi-channel IP-VLBI under the development has a maximum sampling frequency of 16MHz. Various kinds of evaluation of the sampler board have been carried out, and results are reported here.

2. A PCI SAMPLER BOARD

A sampler board under the development is a PCI-bus board (Figure 2). Table 1 summarizes specifications. The board has two reference signal inputs (10 MHz and 1 PPS signals) and is designed to have four analog signal inputs. A sampling frequency of

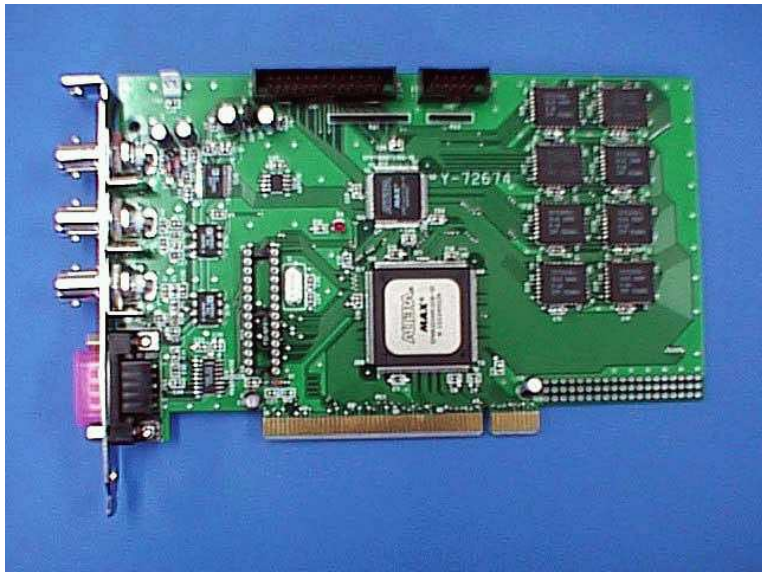

Figure 2: A PCI sampler board for the IP-VLBI system.

Table 1: Specifications of PCI sampler board.

Reference signals	10MHz +10dBm, 1PPS
Number of input channels	max 4 ch
A/D resolutions	1, 2, 4, 8 bits
Sampling frequencies	40kHz, 100kHz, 200kHz, 1MHz, 2MHz, 4MHz, 8MHz, 16MHz

from 40 kHz up to 16 MHz is selectable for a 1 bit A/D conversion. In the future an 8-bit A/D conversion will be possible for the general purpose. Video signals output from a VLBI back-end are converted to digital signals by the sampler board then time tag generated from the reference 1 PPS signal is inserted into the data stream every second. Thereafter an IP packet is formed and sent out to the Internet. Maximum bandwidth of video frequency signals is 8 MHz because maximum sampling frequency is 16 MHz. It corresponds to the case of the KSP-VLBI system.

3. SYSTEM EVALUATION

To keep coherence of signals, which is an essential for VLBI, signals must be sampled using a very stable frequency standard which must be an external reference signal. Moreover to realize a real-time VLBI, sampled data must be transmitted to correlation center in real-time and must be correlated immediately. Time elapsed for correlation processing must be shorter than observation period. If observation is made for 10 seconds, correlation processing must be finished in less than 10 seconds. Thus items should be satisfied

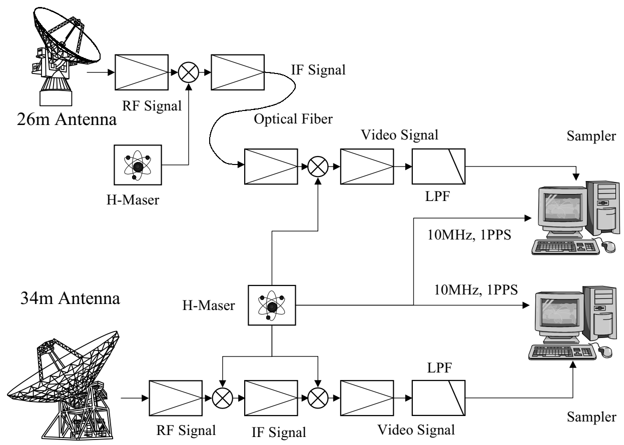

Figure 3: System configuration for a sampler board evaluation.

to realize a real-time VLBI system are as follows; 1) coherent sampling, 2) real-time data transmission, and 3) real-time correlation processing. In this report, we have evaluated these items independently.

The "coherent sampling" can be evaluated by cross-correlating two data stream simultaneously sampled by two PCI sampler boards. We first carried out test observations using two antennas at Kashima. Signals from a radio source received by 34 m antenna and 26 m antenna (distance is about 300 m) at Kashima Space Research Center were acquired by two personal computers (PC) equipped with a PCI sampler board (Figure 3).

Common noise signals were also fed to both PCI boards and sampled. Sampled data were stored in the hard disk in each PC, then they were processed using an off-line program of cross-correlation. Data acquisition was carried out with changing sampling frequency from 40 kHz to 16 MHz. Up to 8 MHz sampling, we could successfully detect a good correlation (fringe) between two data stream. Figure 4 shows fringes of 3C273B detected for 40 kHz sampling data on the Kashima 34m - Kashima 26m baseline for the first time. Fringes detected at 4 MHz sampling are represented in Figure 5. As for 16 MHz sampling, we have not yet succeeded in the detection of correlation. However, we believe that the problem will be settled by improving software that acquires data from a sampler board through the PCI-bus.

The "real-time data transmission" test through local area network (LAN) has been also carried out. So far 16 MHz sampling data seem to be successfully transmitted through the LAN. However, since sampled data are inperfect as described, we have not yet detected fringes for 16 MHz sampling data. Up to 8 MHz sampling, we have confirmed a good performance both in the "real-time data transmission" and in the "coherent sampling".

Reagrding the "real-time correlation processing", we are developing a PC program correlating two data streams without any use of dedicated hardware as long as possible. We consider that the software-correlation processing is a key technology to generalize the IP-VLBI whole over the world. The correlation program used for the evaluation of sampler

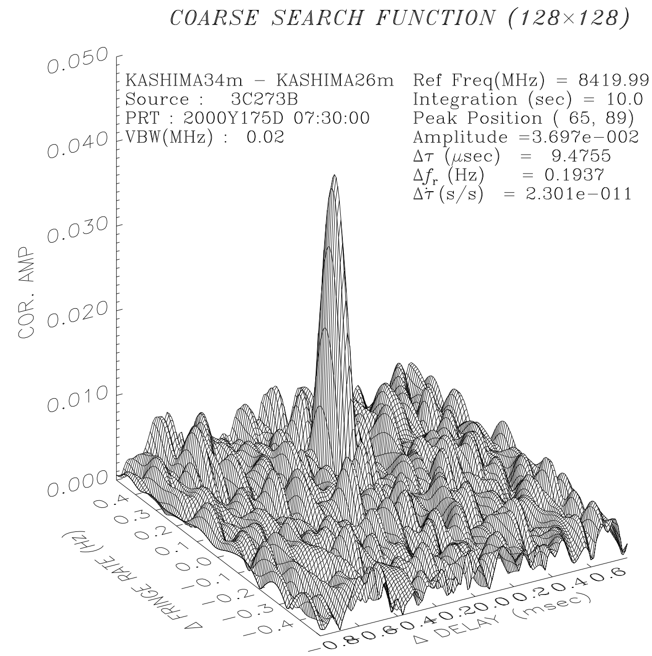

Figure 4: Detected fringes for 40 kHz sampling.

board is written by using the programming language PV-WAVE. Further speedup of the correlation processing is being attempted by improving an algorithm of processing. The latest version of correlation program can process 2 MHz sampling data in real-time when it runs on the Windows 98 with the Pentium III 1GHz processor. We are confident that further speedup will become possible soon.

4. CONCLUSIONS

The PCI sampler board under the development for the IP-VLBI has been evaluated by using signals from radio sources. Real-time characteristics have been evaluated using the LAN at Kashima Space Research Center. As a result, we confirmed the sufficient performance of “coherent sampling” up to 8 MHz sampling. We could not evaluate the performance of 16 MHz sampling due to problems still remaining in the software which interfaces the board and PCI-bus. However we have a prospect to solve the problems. Regarding the “real-time correlation processing” by using PC, we can process 2 MHz sampling data in real-time at present time. An improvement on algorithm to make correlation processing faster is in progress.

Lastly, we would like to thank Y. Fukuzaki of Geographical Survey Institute for approving the use of 26m antenna at Kashima for this technical development.

5. REFERENCES

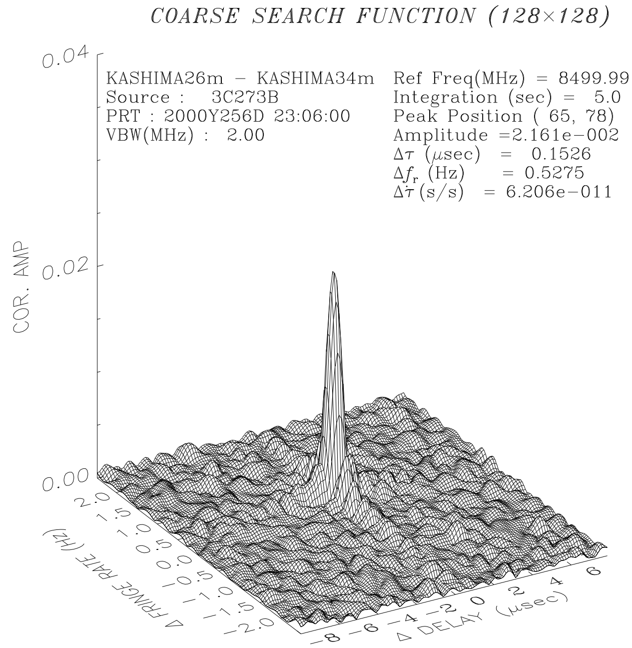

Figure 5: Detected fringes for 4 MHz sampling.

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DIURNAL AND SUBDIURNAL TERMS OF NUTATION: A SIMPLE THEORETICAL MODEL FOR A NONRIGID EARTH

A. BRZEZIŃSKI
Space Research Centre, Polish Academy of Sciences
Bartycka 18A, 00-716 Warsaw, Poland
e-mail: alek@cbk.waw.pl

ABSTRACT. This paper presents a simple theoretical description of the high frequency nutation. First we derive the equation describing the lunisolar excitation of polar motion. The underlying model of the Earth consists of the rotationally symmetrical elastic mantle and the liquid core, with no coupling between these two. Then, we give a systematic review of various components of the equatorial lunisolar torque and of the corresponding perturbation in Earth rotation. Our purpose is to find explicit analytical expressions involving both the parameters of geopotential and the tidal parameters, which gives us insight into the physical mechanism generating this minor, but not negligible, component of the lunisolar perturbation in Earth rotation and makes clear its geometry.

1. INTRODUCTION

Recent developments of the rigid Earth nutation theory, such as REN2000 (Souchay *et al.*, 1999), contain high frequency terms caused by the interaction of the lunisolar tidal potential with those geopotential components which express departures from the rotational symmetry. These are quasi diurnal nutations corresponding to the $(3,1)$ and $(4,1)$ components of the spherical harmonic expansion of geopotential, quasi semidiurnal nutations corresponding to the $(2,2)$ and $(3,2)$ components, and quasi terdiurnal nutations corresponding to the $(3,3)$ component, where the label (l,j) means degree l and order j. This part of nutation has a peak-to-peak size exceeding 100 μas therefore is significant at the current microarcsecond level of truncation. First study of the so-called diurnal prograde libration in polar motion, which is nothing but the terrestrial representation of the semidiurnal nutation, was done by Chao *et al.* (1991) with later corrections by Chao *et al.* (1996). The subject has been further developed by Bretagnon (1998), Bizouard *et al.* (2000).

This paper is a continuation of the earlier study (Brzeziński, 2000). First part contain derivation of the equation describing lunisolar excitation of polar motion (Section 2). The underlying model of the Earth consists of the rotationally symmetrical elastic mantle and the liquid core, with no coupling between these two. We will correct a mistake in the response function derived by Brzeziński (2000), eq.(17), and consider one additional case of a constant torque. Then, we will give in Section 3 a systematic review of various components of the equatorial lunisolar torque and of the corresponding perturbation in Earth rotation. Our purpose here is to find explicit analytical expressions involving both the parameters of geopotential and the tidal parameters. Such an approach gives us insight into the physical mechanism generating this minor component of the lunisolar perturbation in Earth rotation and makes clear its geometry.

As it was shown by Bizouard *et al.* (2000) and Brzeziński (2000), at the microarcsecond level of truncation all diurnal, semidiurnal and terdiurnal components of nutation are prograde (counterclockwise) therefore when expressed in the terrestrial frame they become the low frequency (prograde and retrograde), diurnal prograde and semidiurnal prograde variations, respectively, in polar motion. In this paper we will apply the terrestrial representation which is consistent with the treatment of the oceanic and atmospheric effects at similar frequencies (Brzeziński, 2000) and is in agreement with the recently adopted definition of the Celestial Intermediate Pole (CIP) – see Resolutions of the XXIV IAU General Assembly in (IAU, 2000). Detailed description of the conventional representation of the high frequency nutation and its transformation to the terrestrial frame can be found in (Brzeziński, 2000).

2. LUNISOLAR EXCITATION OF POLAR MOTION

2.1. *Perturbation scheme of Munk and MacDonald (1960)*

Let us denote

$$\vec{\omega} = \Omega \begin{pmatrix} m_1 \\ m_2 \\ 1{+}m_3 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} A + c_{11} & c_{12} & c_{13} \\ & B + c_{22} & c_{23} \\ & & C + c_{33} \end{pmatrix}, \tag{1}$$

the instantaneous rotation vector and the Earth's inertia tensor, expressed in the Terrestrial Reference System (TRS), where Ω is the mean value of the angular frequency of diurnal sidereal rotation. The corresponding angular momentum vector $\vec{H}$ is split up into the rotational and deformational components

$$\vec{H} = \mathbf{C}\vec{\omega} + \vec{h}, \qquad \text{with} \qquad \vec{h} = \iiint_V \vec{r} \times (\dot{\vec{r}} - \vec{\omega} \times \vec{r})\, \rho\, dV, \tag{2}$$

where $\vec{h}$ is the so-called relative angular momentum vector, V denotes the volume of the Earth and ρ its density.

Perturbations in Earth rotation caused by the external torque $\vec{L} = (L_1, L_2, L_3)^T$ are described by the conservation law of the angular momentum, which in the rotating Earth-fixed frame takes the following form

$$\frac{\partial \vec{H}}{\partial t} + \vec{\omega} \times \vec{H} = \vec{L}. \tag{3}$$

Linearization with respect to the small dimensionless quantities m_l, c_{lj}/C, $h_l/C\Omega$, for $l, j = 1, 2, 3$, and assuming rotational symmetry of the unperturbed inertia tensor, that is $A = B$, led Munk and MacDonald (1960) to the following linear equation of polar motion

$$A\Omega\dot{m} - i(C - A)\Omega^2 m + \Omega\dot{c} + i\Omega^2 c + \dot{h} + i\Omega h = L, \tag{4}$$

in which $i = \sqrt{-1}$ denotes imaginary unit, $m = m_1 + im_2$ expresses deviation of the instantaneous rotation axis from the terrestrial z-axis, that is polar motion, $c = c_{13} + ic_{23}$ is the complex combination of those off-diagonal components of the Earth's inertia tensor which are related to polar motion, and $h = h_1 + ih_2$, $L = L_1 + iL_2$ are equatorial components of the relative angular momentum and of the lunisolar torque, respectively.

Let us make now the following remarks.

1. We are not consistent when assuming the rotational symmetry of the Earth's inertia tensor, while considering the torques caused by the triaxiality of the Earth. Note, however, that the main factor expressing the rotational response of the Earth to the equatorial torque is the dynamical flattening $(C-A)/A$ which is about 150 times larger than the equatorial one $(B-A)/A$.

2. The usual assumption is that the terrestrial reference system TRS is the system of the Tisserand axes. That means that the motion of the Earth's particles does not contain the net rotation with respect to the TRS, which implies $\vec{h}=0$. Strictly speaking, this assumption concerns only the solid parts of the Earth excluding geophysical fluids, the atmosphere, the ocean and the core, but their influence on Earth rotation will be disregarded here.

3. We will adopt a simple assumption that for the perturbations considered here the core is not coupled to the mantle, that is it does not participate in the rotational variations. Equation (6) can be modified by replacing the moments of inertia C, A by their counterparts C_m, A_m referring to the mantle alone. But consequently, it is also necessary to replace the equatorial torque L on the Earth by the torque exerted on the mantle L_m.

2.2. *Elastic rotational deformations*

Centrifugal forces induced by the rotational variations cause elastic deformation which in turn contribute to the incremental components of the inertia tensor, eq.(1). These forces can be expressed by the second degree harmonic potential which, after using the well-known McCullag's relationships and expressing the elastic yielding of the Earth by the Love numbers, gives

$$c=\frac{k_2}{k_s}(C-A)m, \tag{5}$$

where $k_2=0.305$ is the second degree elastic Love number and $k_s=3G(C-A)/\Omega^2R_o^5=0.94$ is the secular Love number, see (Munk and MacDonald, 1960) or (Lambeck, 1988) for derivation. Here G denotes the gravitational constant and R_o is the mean radius of the Earth.

Equation (5) is not valid for the secular variations, in particular for the linear motion of the pole which will be considered here as a separate case. For such variation the Love number k_2 should be replaced by the so-called fluid love number k_f which numerically equals k_s (Munk and MacDonald, 1960, Chap.5, Sec.4). Hence, eq.(5) reduces to

$$c=(C-A)m. \tag{6}$$

If we take into account that $c/(C-A)$ describes terrestrial orientation of the principal axis of the perturbed inertia tensor, that is the instantaneous axis of the Earth's figure, eq.(6) can be interpreted in such a way that the rotation axis follows exactly the secular path of the figure axis.

2.3. *Elastic tidal deformations*

Let the torque L corresponds to the component u_{lj} of degree l and order j of the spherical harmonic expansion of the tidal potential, eq.(18). In addition to the elastic rotational deformation considered so far, there is also elastic deformation of the Earth in a direct response to the tidal force (Munk and MacDonald, Chap.5). The corresponding incremental gravitational potential at the displaced surface of the Earth is

$$\Delta U=k_l u_{lj}, \tag{7}$$

where k_l is the Love number of degree l ($k_2 = 0.305$, $k_3 = 094$, $k_4 = 0.042$; see (Lambeck, 1988) Table 3.3a). From the McCullag's theorem the incremental component c of inertia tensor which is related to polar motion, is proportional to ΔU_{21} – see eq.(16), hence within the linear theory considered here and expressed by eq.(4) with $h = 0$, only the u_{21} term of the tidal potential causes deformation which perturb polar motion. But this effect appears entirely as a contribution to the classical long periodic nutation, therefore *is not relevant* to the problem considered here.

Nevertheless, it is interesting to know which components of the lunisolar tidal potential are responsible for the diurnal and subdiurnal nutations. This problem will be addressed later, when deriving the expressions for the lunisolar equatorial torques.

2.4. *Solution of the equation of polar motion*

If we assume that the terrestrial reference frame is the Tisserand frame ($h = 0$) and that perturbations of the inertia tensor are entirely due to the elastic rotational variations expressed by eq.(5), eq.(4) reduces to

$$i\frac{\dot{m}}{\sigma_o} + m = \frac{i\,L}{(1 - k_2/k_s)(C - A)\Omega^2}, \tag{8}$$

with the resonant frequency $\sigma_o = (1 - k_2/k_s)/(1 + ek_2/k_s)\, e\Omega$ and the dynamical flattening of the Earth $e = (C - A)/A \approx 1/304.5$. By putting $k_2 = 0$ in eq.(8) we obtain the equation for a rigid Earth, with the frequency of resonance reduced to $\sigma_r = e\Omega$, that is the Euler frequency.

By assuming harmonic form of variables, $L = L_\sigma e^{i\sigma t}$, $m = m_\sigma e^{i\sigma t}$, where σ is the Earth-referred angular frequency, we can derive the frequency domain counterpart of eq.(8)

$$m_\sigma = \frac{iL_\sigma}{A\Omega\left(1 + \frac{k_2}{k_s}e\right)(\sigma_o - \sigma)}. \tag{9}$$

For the conventional intermediate pole p (currently the CIP, formerly the Celestial Ephemeris Pole CEP) the amplitudes should be further modified (Brzeziński, 1992)

$$p_\sigma = \frac{\Omega}{\sigma + \Omega}\, m_\sigma = \frac{iL_\sigma}{A\left(1 + \frac{k_2}{k_s}e\right)(\sigma + \Omega)(\sigma_o - \sigma)}. \tag{10}$$

Comparison with the corresponding solution for the rigid Earth yields the following ratio

$$\frac{p_\sigma}{p_\sigma^r} = \frac{m_\sigma}{m_\sigma^r} = q(\sigma)\,\frac{L_\sigma}{L_\sigma^r}\,, \tag{11}$$

in which the nonrigid Earth response function $q(\sigma)$ is given by

$$q(\sigma) = \frac{\sigma - \sigma_r}{\sigma - \sigma_c}\,\frac{A}{A_m}\,\left(1 + \frac{k_2}{k_s}\,e_m\right)^{-1}. \tag{12}$$

Here we replaced σ_o by the observed Chandler frequency σ_c and introduced the equatorial moment of inertia of the mantle A_m and the dynamical flattening of the mantle e_m in order to account for the assumption that the mantle is not coupled to the core. This is the corrected version of eq.(17) of Brzeziński (2000) who included erroneously the factor $1 - k_2/k_s(\sigma + \Omega)/\Omega$.

We will consider now separately the case of the constant torque. We insert eq.(6) expressing the elastic rotational deformation into the equation of polar motion, eq.(4), which gives

$$C\Omega\dot{m} = L, \tag{13}$$

and, after taking into account that for the secular motion $m = p$, the solution for the conventional pole is a linear motion

$$p = \frac{L}{C\Omega}(t - t_o) + p(t_o). \tag{14}$$

3. EQUATORIAL LUNISOLAR TORQUE

3.1. *General formulation*

The torque exerted on the Earth by the external point mass M_d (the Moon or the Sun) at the geocentric position $\vec{r}_d = (r_d, \theta_d, \lambda_d)$, where r_d, θ_d, λ_d denote the distance, co-latitude and longitude, can be expressed as follows

$$\vec{L} = -M_d\, \vec{r}_d \times \nabla U(\vec{r}_d) \tag{15}$$

where U is the external gravitational potential of the Earth given by the well-known spherical harmonic expansion

$$U(\vec{r}) = \frac{GM}{r}\sum_{l=0}^{\infty}\sum_{j=0}^{l}\left(\frac{R_o}{r}\right)^l P_{lm}(\cos\theta)[C_{lj}\cos j\lambda + S_{lj}\sin j\lambda], \tag{16}$$

in which M denotes the mass of the Earth, P_{lj} is the associated Legendre function of degree l and order j, and the (un-normalized) Stokes coefficients C_{lj}, S_{lj} are evaluated with reference to the mean radius of the Earth R_o.

Eq.(15) can be expressed in the rectangular terrestrial coordinates, then converted to the spherical coordinates which yields the following expression for the equatorial torque

$$L = L_1 + iL_2 = -i\, M_d \left(\frac{\partial U}{\partial \theta_d} + i\,\frac{\cos\theta_d}{\sin\theta_d}\frac{\partial U}{\partial \lambda_d}\right) e^{i\lambda_d}, \tag{17}$$

in which $e^{i\lambda_d} = \cos\lambda_d + i\sin\lambda_d$. To the accuracy of notation, these are equations (4.6) and (4.7) of McClure (1973). By inserting the spherical harmonic expansion of U, eq.(16), we can evaluate the torque in terms of the geocentric coordinates r_d, θ_d, λ_d of the torque-generating body. When taking into account the time dependence of these coordinates one can express L as an explicit function of time involving the coefficient of the tide generating potential

$$u(\vec{r}) = \frac{GM_d}{c_d}\sum_{l=2}^{\infty}\sum_{j=0}^{l}\left(\frac{r}{c_d}\right)^l P_{lj}(\cos\theta)\sum_m A_{ljm}\cos\left[\omega_{ljm}t + \beta_{ljm} + j\lambda + (l-j)\frac{\pi}{2}\right], \tag{18}$$

by using the following relationship

$$\left(\frac{1}{r_d}\right)^{l+1} W_{lj}P_{lj}(\cos\theta_d)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} j\lambda_d = \left(\frac{1}{c_d}\right)^{l+1}\sum_m A_{ljm}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\left[-\omega_{ljm}t - \beta_{ljm} - (l-j)\frac{\pi}{2}\right], \tag{19}$$

where $W_{lj} = 2(l-j)!/(l+j)!$ for $j = 1, \ldots, l$ and $W_{l0} = 1$, c_d denotes the mean distance of the disturbing body and the potential is evaluated at the point with geocentric coordinates $\vec{r} = (r, \theta, \lambda)$; see e.g. (*ibid.*), eq.(4.18) with derivation.

3.2. *The (2,2) term of geopotential*

Let us consider now in details the (2,2) term of the Earth's gravity potential, eq.(16):

$$U = \frac{GM}{r}\left(\frac{R_o}{r}\right)^2 P_{22}(\cos\theta)[C_{22}\cos 2\lambda + S_{22}\sin 2\lambda], \tag{20}$$

with $P_{22}(\cos\theta) = 3\sin^2\theta$. Inserting into eq.(17) yields

$$L = -i\,2\,\frac{GMM_dR_o^2}{r_d^3}\,P_{21}(\cos\theta_d)\,D_{22}\,e^{-i(\lambda_d - 2\lambda_{22})}, \tag{21}$$

where $P_{21}(\cos\theta) = 3\sin\theta\cos\theta$ and we denoted $C_{22} = D_{22}\cos 2\lambda_{22}$, $S_{22} = D_{22}\sin 2\lambda_{22}$. It can be shown (see e.g. (Chao *et al.*, 1991) with references) that the parameters D_{22} and λ_{22} are directly related to the equatorial moments of inertia, namely $D_{22} = (B{-}A)/4MR_o^2$ is proportional to the equatorial dynamical flattening while λ_{22} is the angle between the x-axis of the TRS and of the principal axis of inertia corresponding to A. For the GEM-3 values $C_{22} = 1.574410 \times 10^{-6}$, $S_{22} = -0.903757 \times 10^{-6}$ we derive $D_{22} = 1.815363 \times 10^{-6}$, $2\lambda_{22} = -29.8571^o$.

By taking into account that the angle λ_d varies diurnally in the retrograde direction while r_d and θ_d subject only small long periodic fluctuation, one can deduce that eq.(21) describes prograde quasi-diurnal variation in the terrestrial frame. After using eq.(19) in eq.(21) we arrive to the following expression

$$L = 6\,\frac{GMM_dR_o^2}{c_d^3}\,D_{22}\sum_m A_{21m}e^{i\,(\omega_{21m}t + \beta_{21m} + 2\lambda_{22})}, \tag{22}$$

which involves coefficients of the tesseral part u_{21} of the tidal potential, eq.(18), the same which is responsible for precession and long periodic nutation. Comparison with the expression for the diurnal retrograde torque (McClure, 1973, eq.(4.21)) shows that there is a change of sign of the argument $\omega_{21m}t + \beta_{21m}$ with additional shift of phase by $2\lambda_{22}$, while the amplitudes are systematically decreased by the factor of $2(C-A)/(B-A)$, that is about 300 times. The influence on polar motion is further diminished because diurnal prograde frequencies are far from the rotational resonances while the diurnal retrograde band contains the free core nutation (FCN) resonance.

The corresponding solution for polar motion of the rigid Earth can be derived from eq.(10) with $k_2 = 0$ and $\sigma_o = e\Omega$

$$p = i\,\frac{6GMM_dR_o^2}{c_d^3}\,D_{22}\sum_m \frac{A_{21m}}{(\omega_{21m} + \Omega)(e\Omega - \omega_{21m})}\,e^{i\left(\omega_{21m}t + \beta_{21m} + 2\lambda_{22} + \frac{\pi}{2}\right)}. \tag{23}$$

This is solution for the CIP which in the case of a rigid Earth is the same as the pole of the axis of figure. For the nonrigid Earth the amplitudes should be additionally multiplied by the frequency dependent response coefficient $q(\omega_{21m})$ given by eq.(12). Note, however that also the factor MD_{22} needs modification in order to account for the decoupling of the liquid core, that is the factor L_σ/L_σ^r in eq.(11). This problem will be addressed by Brzeziński *et al.* (2001, in preparation).

The maximum amplitude of polar motion can be estimated directly from equations (21) and (10) with $\sigma = \Omega$ and $\sigma_c = 0$. It is reached when the torque-generating body is far away from the equatorial plane, that is at $\theta = 90 \pm 23.5^o$ for the Sun and $\theta = 90 \pm 28.5^o$ for the Moon, and takes the following values

$$p = \begin{cases} 1.1225 \times 10^{-10} \ \text{rad} \\ 0.4491 \times 10^{-10} \ \text{rad} \end{cases} = \begin{cases} 23 \ \ \mu\text{as} \quad \text{for the Moon} \\ \ 9 \ \ \mu\text{as} \quad \text{for the Sun.} \end{cases}$$

3.3. *The (3,1) term of geopotential*

The (3,1) term of the Earth's gravity potential, eq.(16), is

$$U = \frac{GM}{r} \left(\frac{R_o}{r}\right)^3 P_{31}(\cos\theta)[C_{31}\cos\lambda + S_{31}\sin\lambda], \tag{24}$$

with $P_{31}(\cos\theta) = \sin\theta\,(15/2\cos^2\theta - 3/2)$. Inserting into eq.(17) yields

$$L = i\,3\,\frac{GMM_dR_o^3}{r_d^4}\,D_{31}\Big[2P_{30}(\cos\theta_d)\,e^{i\lambda_{31}} + \frac{1}{6}P_{32}(\cos\theta_d)\,e^{i(2\lambda_d - \lambda_{31})}\Big], \tag{25}$$

where $P_{30}(\cos\theta) = 5/2\cos^3\theta - 3/2\cos\theta$, $P_{32}(\cos\theta) = 15\sin^2\theta\cos\theta$, and as before we denoted $C_{31} = D_{31}\cos\lambda_{31}$ and $S_{31} = D_{31}\sin\lambda_{31}$. For the GEM-3 values $C_{31} = 2.190182 \times 10^{-6}$, $S_{31} = 0.269185 \times 10^{-6}$ we derive $D_{31} = 2.206662 \times 10^{-6}$, $\lambda_{31} = 7.0068^o$.

This equatorial torque comprises long periodic variation and retrograde quasi semidiurnal oscillation. The $(3,1)$ geopotential coefficients C_{31}, S_{31} are of the same order as the $(2,2)$ coefficients, therefore from the comparison of eq.(25) with eq.(21) it can be seen that the $(3,1)$ torque is smaller than the $(2,2)$ torque by the factor of r_d/R_o which is about 60 for the Moon and as much as 23500 for the Sun. Obviously, there will be a similar decrease in amplitudes of the corresponding lunisolar variations in polar motion with exception of the long periodic term, where the resonant enhancement due to the proximity of the Chandler wobble can compensate the decrease of the torque. Consequently, we will neglect the last term of eq.(25) for which the corresponding retrograde semidiurnal polar motion has an amplitude of about 0.2 μas, and consider in a more detailed way only the first term exciting the long periodic polar motion. Substituting eq.(19) yields the expression involving the coefficients of the 3-rd order zonal tidal potential

$$L = i\,6\,\frac{GMM_dR_o^3}{c_d^4}\,D_{31}e^{i\lambda_{31}}\sum_m A_{30m}\sin\left(\omega_{30m}t + \beta_{30m}\right), \tag{26}$$

and the corresponding polar motion is

$$p = -\,6\,\frac{GMM_dR_o^3}{c_d^4}\,D_{31}e^{i\lambda_{31}}\sum_m \frac{A_{30m}}{(\omega_{30m} + \Omega)(e\Omega - \omega_{30m})}\ \sin\left(\omega_{30m}t + \beta_{30m}\right). \tag{27}$$

Again, the amplitudes in the last equation should be further modified by applying the response coefficient (12) accounting for the Earth's non-rigidity. Particularly important parameter is the frequency of resonance $e\Omega$ which is within the frequency band of the zonal tide, therefore should be replaced by the observed value of the Chandler frequency $\sigma_c = \Omega/434$.

¿From the last equation it can be seen that the pole moves along the line inclined to the Greenwich meridian by the angle $\lambda_{31} = 7.0068^o$ which is argument of the complex combination of the geopotential coefficients $C_{31} + iS_{31}$. This angle is in perfect agreement with the phase estimated from the rigid Earth amplitudes of the diurnal nutation by Brzeziński (2000), but he obtained a slight ellipticity of the polar motion waves. This discrepancy is probably due to the fact that when modeling the response of the Earth in

Sec.2, we assumed rotational symmetry of the Earth's inertia tensor while in the previous work we used the rigid Earth solution of (Souchay *et al.*, 1999) assuming the triaxial Earth.

Equation (25) shows that the maximum torque is when the torque-generating body is at the maximum angle from the equatorial plane, that is at $\theta = 90 \pm 23.5^o$ for the Sun and $\theta = 90 \pm 28.5^o$ for the Moon. For the frequency $\sigma = \Omega/27.4$ expressing the lunar revolution around the Earth, we estimate from equations (25) and (10) the amplitude of polar motion

$p = 9.84 \times 10^{-13}$ rad $= 20$ μas.

3.4. *The (4,1) term of geopotential*

The equatorial torques corresponding to the (3,2) and (3,3) terms of geopotential can be studied in the same way as those considered so far. These torques, though interesting from the point of view of theory, nevertheless will be disregarded here because the corresponding polar motion, prograde diurnal and semidiurnal, respectively, has amplitudes of the order of 0.1 μas. Let us consider now the (4,1) term of the Earth's gravity potential, eq.(16), that is

$$U = \frac{GM}{r}\left(\frac{R_o}{r}\right)^4 P_{41}(\cos\theta)[C_{41}\cos\lambda + S_{41}\sin\lambda], \tag{28}$$

with $P_{41}(\cos\theta) = \sin\theta\,(35/2\cos^3\theta - 15/2\cos\theta)$. Inserting into eq.(17) yields

$$L = -i\,\frac{GMM_dR_o^4}{r_d^5}\Big[10P_{40}(\cos\theta_d)\,(C_{41} + iS_{41}) + \frac{1}{2}P_{42}(\cos\theta_d)\,(C_{41} - iS_{41})e^{i2\lambda_d}\Big], \tag{29}$$

where $P_{40}(\cos\theta) = 1/8(35\cos^4\theta - 30\cos^2\theta - 3)$, $P_{42}(\cos\theta) = 5/2\sin^2\theta(21\cos^2\theta - 3)$.

For obvious reasons the last term of eq.(29) is far negligible. Also the terms depending on θ_d can be shown to have not important contribution to polar motion. An interesting feature of this torque is a quasi constant term which can be written as follows

$$L = -i\,\frac{15}{4}\,\frac{GMM_dR_o^4}{r_d^5}\,D_{41}e^{i\lambda_{41}}, \tag{30}$$

where we denoted $C_{41} = D_{41}\cos\lambda_{41}$ and $S_{41} = D_{41}\sin\lambda_{41}$. For the GEM-3 values $C_{41} = -0.508638 \times 10^{-6}$, $S_{41} = -0.449141 \times 10^{-6}$ we derive $D_{41} = 0.678557 \times 10^{-6}$, $\lambda_{41} = -138.55^o$. By replacing the geocentric distance to the Moon r_d by its mean value c_d, we can evaluate the value $|L| = 3.9007 \times 10^{15}$ kg m^2/s^2. Finally, from eq.(14) we derive the corresponding linear drift in polar motion

$$\dot{p} = 8.66\,\mu\text{as/yr} \cdot e^{i131.45^o}. \tag{31}$$

This drift is much smaller and has almost opposite direction than the observed secular motion of the pole of about 3 mas/yr towards the 90^oW longitude.

4. CONCLUSIONS

The main results of Section 3 can be summarized in the following table.

Stokes coefficients	Tidal potential	Nutation	Polar motion	PM amplitude	PM argument
C_{22}, S_{22}	u_{21}	prograde semidiurnal	prograde diurnal	23μas (Moon) 9μas (Sun)	phase shift $\lambda = 60^o E$
C_{31}, S_{31}	u_{30}	prograde diurnal	long periodic	20μas (Moon)	meridian $\lambda = 97^o E$
C_{41}, S_{41}	u_{40}	prograde diurnal	linear drift	$\dot{p} =$ 8.7 μas/yr	direction $\lambda = 131^o E$

We should remain here that the polar motion amplitude estimates were rough values involving various simplifications. Highly accurate values for the rigid Earth can be found in Table 2 of Brzeziński (2000). The only exception is the term with the terrestrial argument zero, whose amplitude should be replaced there by the first order polynomial; see Section 3.4. Note, however, that according to the results of Section 2, the column labelled "elastic" should be divided by the factor of $1 - 0.324(\sigma + \Omega)/\Omega$ which is approximately 0.676 for the long periodic waves and 0.351 for the prograde quasi diurnal waves.

Further development of the present study (Brzeziński *et al.* 2001, in preparation) concerns the factor $Ł_\sigma / L^r_\sigma$ of eq.(11), which depends on the core-mantle boundary shape. In Table 2 of Brzeziński (2000) this factor was just set to 1, but its true value can be significantly different.

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ON THE USE OF THE METHOD OF COMBINED SMOOTHING TO COMBINE EOP DATA FROM DIFFERENT TECHNIQUES

J. VONDRÁK[1], A.ČEPEK[2]

[1]Astronomical Institute and Center for Earth Dynamics Research
Boční II, 141 31 Prague 4, Czech Republic
e-mail vondrak@ig.cas.cz

[2]Czech Technical University and Center for Earth Dynamics Research
Thákurova 7, 166 29 Prague 6, Czech Republic
e-mail: cepek@fsv.cvut.cz

ABSTRACT. The main ideas of the recently developed method of combined smoothing and its transfer functions are described, and a method of calculating selected covariances of the adjusted values is outlined. The method is tested with simulated data to demonstrate its properties, and the optimal choice of coefficients of smoothing is proposed. Then the example of combined smoothing, using real data (six years of weekly VLBI-based UT1 observations and daily GPS-based l.o.d. observations), is given. It is demonstrated that the new estimator takes over the long-term stability of less frequent UT1 series and combines it with the short-time high accuracy of more dense l.o.d. series. The result is two sufficiently smooth and dense series of UT1 and l.o.d. (defined in all points with observations of any kind), the latter being strictly the time derivative of the former.

1. INTRODUCTION

When combining the Earth orientation Parameters (EOP) obtained by different techniques, we are quite often facing a problem of different stability of the results at different frequencies. This holds especially in cases when the difference between Universal and Atomic Time scale UT1–TAI is observed by VLBI, and its time derivative, length-of day changes (l.o.d.) is observed by satellite methods. The values of UT1 are observed with very high long-term stability and rather low time resolution (typically one week), while l.o.d. values are observed with a very high short-term accuracy and high time resolution (typically 1–3 days), but its long-term stability is low, due to unmodeled long-periodic changes of the orientation of satellite orbits (Vondrák & Gambis 2000). Therefore a new method of combining simultaneously observed values of a function and its time derivative has recently been worked out. It is a generalization of an older method of smoothing (Vondrák 1969, 1977), and it is shortly described by Vondrák & Gambis (2000) and in detail by Vondrák & Čepek (2000).

2. THE METHOD OF COMBINED SMOOTHING

Generally speaking, two series of observations are available, at unequally spaced epochs and with different uncertainties: one with measured values of a function (whose analytic expression is unknown), and the second with measured values of time derivatives of the same function; the epochs of both series need not be identical. They are as follows (prime denoting the observed value, bar the first derivative):

- n function values y_i', measured at instants x_i with weights p_i;
- $\bar{n}$ time derivative values $\bar{y}_i'$, measured at instants $\bar{x}_j$ with weights $\bar{p}_j$.

Since the arguments x_i, $\bar{x}_j$ can partially overlap, the total number of all arguments with at least one observation is $N \le n + \bar{n}$. We are looking for two new series y_i, $\bar{y}_i$, defined at all N points with any observation, whose values lie on two relatively smooth curves satisfying three conditions:

- the first one must fit as close as possible to the first data series;
- the second one must fit as close as possible to the second data series;
- both curves must be tied by constraints assuring that the latter is the time derivative of the former.

To satisfy the conditions above, we define the following values that should be minimized:

1. $S = \frac{1}{x_N - x_1} \int_{x_1}^{x_N} \varphi'''^2(x)\mathrm{d}x$ – 'smoothness' of the first series, in which φ''' is estimated from cubic Lagrange polynomial fitted to four consecutive points of the smoothed function values;
2. $F = \frac{1}{n} \sum_{i=1}^{N} p_i (y_i' - y_i)^2$ –'fidelity' of the first curve to the observed function values in a least-squares sense;
3. $\bar{F} = \frac{1}{\bar{n}} \sum_{i=1}^{N} \bar{p}_i (\bar{y}_i' - \bar{y}_i)^2$ – 'fidelity' of the second curve to the observed values of derivatives in a least-squares sense;
4. $\bar{y}_j = \sum_{i=1}^{N} a_i y_i$ – N constraints assuring that the second curve is the time derivative of the first one. Coefficients a_i are given as rather complicated expressions, derived from the same Lagrange polynomial as the smoothness S.

The first condition applied alone (i.e., $S = 0$) would lead to any quadratic function of time (and corresponding linear function of time for the derivative), the second condition ($F = 0$) would lead to a first curve running through all points $< x_i; y_i >$, and the third one ($\bar{F} = 0$) to a second curve running through all points $< \bar{x}_j; \bar{y}_j >$. It is clear that we should look for a weighted compromise among these three conditions, of the form

$$Q = S + \varepsilon F + \bar{\varepsilon}\bar{F} = \text{min.} \quad \Rightarrow \frac{\partial Q}{\partial y_i} = \frac{\partial Q}{\partial \bar{y}_i} = 0 \ \text{ for } i = 1,\ 2, \ldots N\,, \tag{1}$$

where ε, $\bar{\varepsilon}$ are non-negative constants, coefficients of smoothing. Their proper choice assures that the result is an appropriate balance between the smoothness of the curves and their fidelity to the observations. Equation (1) would normally lead to $2N$ equations ($\partial Q/\partial y_i = 0$, $\partial Q/\partial \bar{y}_j = 0$); in order to diminish the number of these equations to one half, we use the constraints above to express $\bar{y}_j$ in terms of y_i in $\bar{F}$. We can then use only

$\partial Q/\partial y_i = 0$, leading to the system of N linear equations for y_i, and the values $\bar{y}_j$ are then calculated directly from the constraints.

The analytical expressions of transfer functions of the filter (assuming that the weights $p_i = \bar{p}_j = 1$) depend on coefficients of smoothing ε, $\bar{\varepsilon}$ and frequency f:

$$T = \frac{1}{1 + \varepsilon^{-1}(2\pi f)^6} \qquad \text{for the function values and} \tag{2}$$

$$\bar{T} = \frac{1}{1 + \bar{\varepsilon}^{-1}(2\pi f)^4} \qquad \text{for the time derivatives.} \tag{3}$$

3. CALCULATION OF COVARIANCES

In order to estimate the uncertainties of the points on the smoothed curve and/or their functions, we need to know the full matrix of covariances. Taking into consideration often a very large number of the equations, it is recommendable to make use of the sparsity of the matrix; here we outline a method of calculating a selected subset of covariance matrix.

For combined smoothing method we solve the system of linear equations, with (symmetric) positive definite band matrix $\mathbf{A}$, which can be expressed in matrix notation as

$$\mathbf{A}\mathbf{y} = \mathbf{B}\begin{pmatrix} \mathbf{y}' \\ \bar{\mathbf{y}}' \end{pmatrix}, \qquad \mathbf{B} = (\mathbf{B}_1, \mathbf{B}_2)\,. \tag{4}$$

Covariance matrix Σ_{yy} of smoothed values is

$$\Sigma_{yy} = \mathbf{A}^{-1}\Sigma_B\mathbf{A}^{-1}, \qquad \Sigma_B = \mathbf{B}\begin{pmatrix} \Sigma_{y'y'} & 0 \\ 0 & \Sigma_{\bar{y}'\bar{y}'} \end{pmatrix}\mathbf{B}^{\mathrm{T}}. \tag{5}$$

For uncorrelated observed data covariance submatrices $\Sigma_{y'y'}$ and $\Sigma_{\bar{y}'\bar{y}'}$ are both diagonal and band matrix Σ_B is positive definite, because matrix $\mathbf{B}$ cannot have linearly depended rows

$$\mathbf{B}_1 = \frac{\varepsilon}{n}\mathrm{diag}\{\mathrm{p}_1, \ldots, \mathrm{p}_\mathrm{n}\}.$$

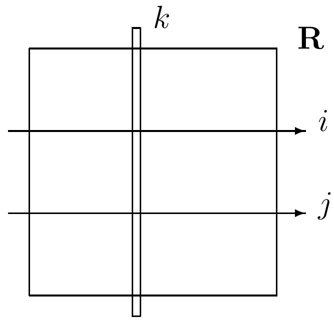
Bandwidth of Σ_B is 4, its inverse is a dense matrix and we cannot simply compute selected covariances of smoothed values as in the case of smoothing without first derivatives (see, e.g., Čepek & Vondrák 2000).

If we denote Cholesky decomposition of Σ_B as $\mathbf{L}$, then $\Sigma_B = \mathbf{L}\mathbf{L}^{\mathrm{T}}$ and we can express the covariance matrix Σ_{yy} of smoothed values as

$$\Sigma_{yy} = \left(\mathbf{A}^{-1}\mathbf{L}\right)\left(\mathbf{L}^{\mathrm{T}}\mathbf{A}^{-1}\right) = \mathbf{R}\mathbf{R}^{\mathrm{T}}. \tag{6}$$

This way the covariances of smoothed values are equal to dot products of corresponding row vectors of $\mathbf{R}$. We can easily compute a selected subset $\mathcal{Z}$ of elements of covariance matrix Σ_{yy} by simply changing summation order as follows:

1. compute Cholesky decomposition of $\mathbf{A}$;
2. allocate memory for $\mathcal{Z}$ elements and set them all to zero;
3. for $k = 1, \ldots, n$ compute k-th column of $\mathbf{R}$ as the solution of linear system $\mathbf{A}r_{*k} = l_{*k}$ (using Cholesky decomposition from the first step) and add the product $r_{ik}r_{jk}$ to z_{jk} for all elements in $\mathcal{Z}$.



Note: column vectors r_{*k} could be separated into several groups and processed in parallel.

4. TESTING THE METHOD WITH SIMULATED DATA

In order to study the behavior of the new method with expected result known beforehand, a number of tests with simulated data were made. One example of testing the method with the signal and noise similar to real observations of UT1 and l.o.d. is given here. The signal consists of annual, semi-annual and fortnightly terms, in which $t = (x - 50448)/365.2422$:

$$\begin{aligned} y &= -0.020 \sin 2\pi t + 0.012 \cos 2\pi t + 0.006 \sin 4\pi t - 0.007 \cos 4\pi t + 0.0008 \sin 52\pi t \\ \bar{y} &= \frac{-\pi}{365.2422}(0.040 \cos 2\pi t + 0.024 \sin 2\pi t - 0.024 \cos 4\pi t - 0.028 \sin 4\pi t - 0.0416 \cos 52\pi t), \end{aligned}$$

(in seconds and seconds per day respectively) and we added a pseudo-Gaussian noise ($\sigma = 20\mu$s, $\bar{\sigma} = 20\mu$s/day) to both series. In addition to this, a step function of -20μs/day before $x = 50400$ and $+20\mu$s/day after was added only to the second series, in order to test the reaction of the estimator to the situation caused by a change of orbit modeling in satellite-based l.o.d. The data were generated on the interval $< 50200; 50400 >$, with one day spacing, and then subject to combined smoothing with many different combinations of ε, $\bar{\varepsilon}$.

The results led to the set of rules for choosing the coefficients of smoothing that can be summarized as follows:

1. Best approximation of the original signal is obtained if using $\varepsilon = (2\pi/P)^6$, $\bar{\varepsilon} = (2\pi/P)^4$, in which P is the period lying roughly between one third and one half of the shortest period known to be contained in the signal.

2. Using substantially smaller values than above leads to smoother curves, in which the short-periodic signal is suppressed.

3. Keeping the same ε and using substantially smaller value of $\bar{\varepsilon}$ than above does not do much harm – it only gives smaller weights to observed derivatives (in extreme case of $\bar{\varepsilon} = 0$ the method reduces to 'original smoothing', in which only function values are taken into account).

4. Keeping the same ε and using substantially larger value of $\bar{\varepsilon}$ than above should be avoided – it often produces spurious unrealistic peaks in the resulting curves.

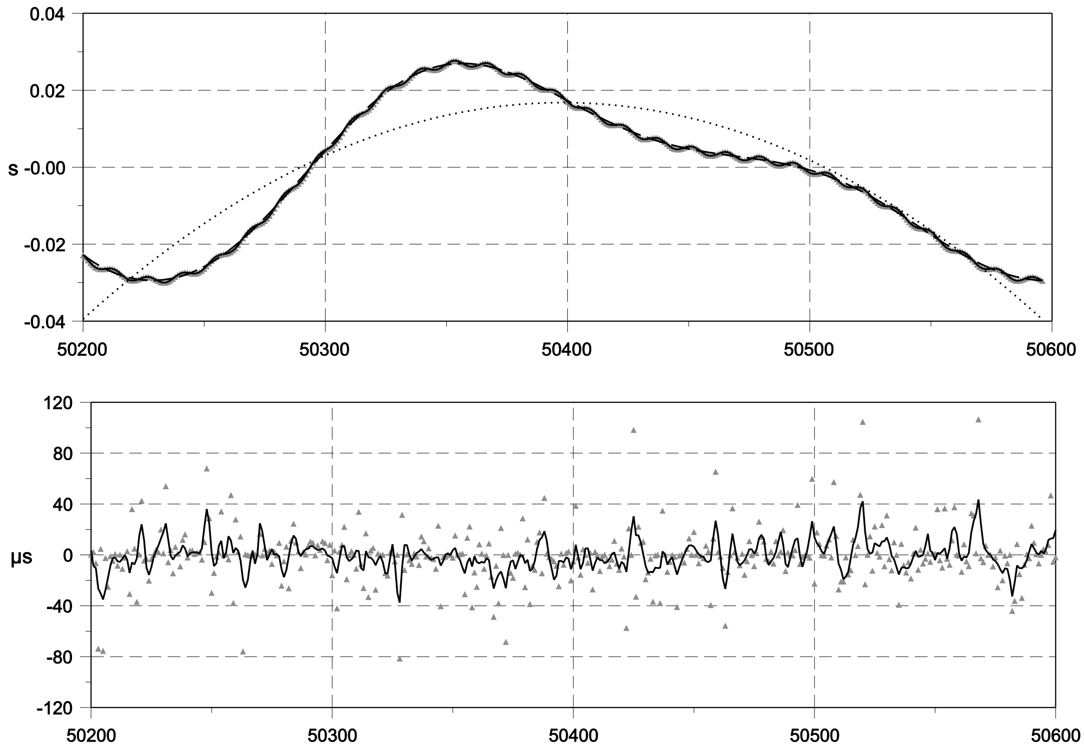


Figure 1: Combined smoothing of simulated data – function values. Grey triangles represent the simulated observations, dotted, dashed and full lines the strong, medium and weak smoothing (top plot). The simulated values and the weakest smoothing minus the original signal are depicted in bottom plot.

5. Using both values substantially larger than above generally leads to more 'ragged' curves that tend to follow the noise rather than signal.

To demonstrate the properties of the method, three examples of smoothing simulated data described above are given in Figures 1 and 2, in which three different sets of coefficients of smoothing were applied:

1. $\varepsilon = \bar{\varepsilon} = 0$. It is the strongest smoothing, yielding the quadratic function for y and linear function for $\bar{y}$.

2. $\varepsilon = 3.5 \times 10^{-7}$ day^{-6}, $\bar{\varepsilon} = 4.9 \times 10^{-5}$ day^{-4}. The values are calculated on the assumption that the shortest period contained in the signal is 150–180 days, therefore $P = 75$ days was used. All periods shorter than 30 days are completely suppressed.

3. $\varepsilon = 3.9$ day^{-6}, $\bar{\varepsilon} = 2.5$ day^{-4}. These values were calculated from $P = 5$ days, i.e. on the assumption that the shortest period in the signal is 10–14 days. From the transfer functions of the filter it can be deduced that it passes completely all periods of 14 days and longer, suppresses the period of 5 days to one half and suppresses completely all periods shorter than 2 days. This smoothing yields the best reproduction of the original signal.

Figure 1 shows the function, Figure 2 its time derivatives; in both figures triangles represent the simulated observations and the dotted, dashed and full line depict the three smoothed curves described above. In order to show more details that are not well visible

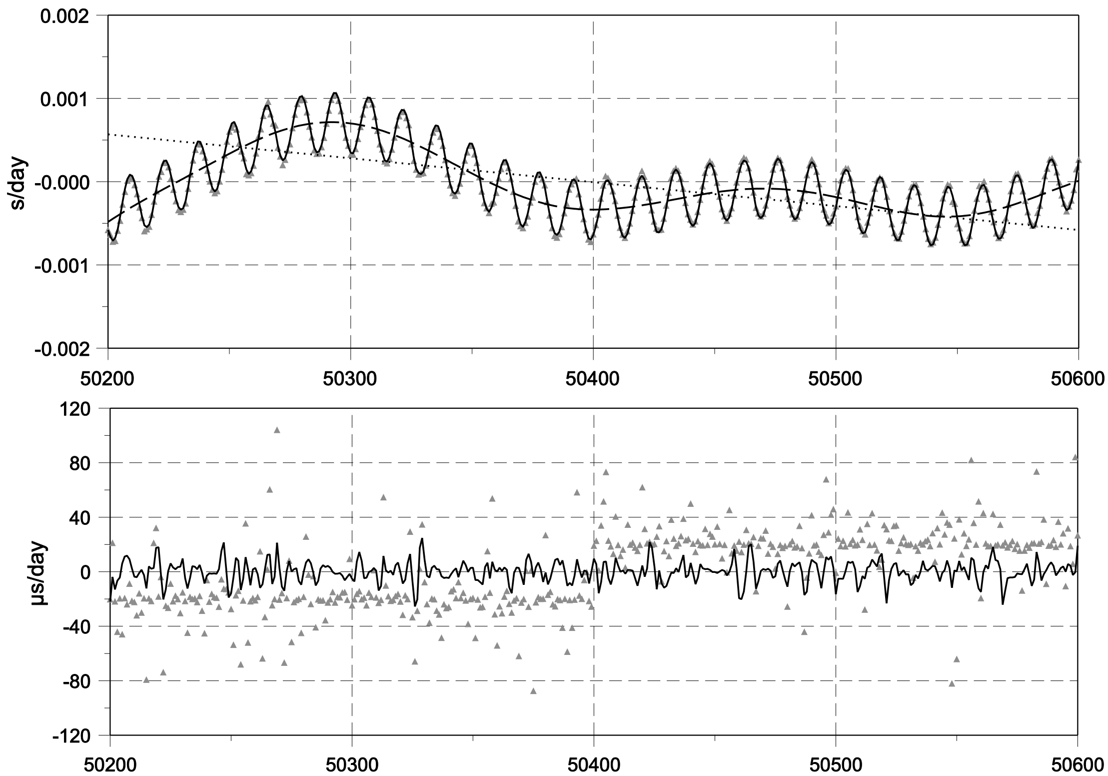


Figure 2: Combined smoothing of simulated data – time derivatives. Grey triangles represent the simulated observations, dotted, dashed and full lines the strong, medium and weak smoothing (top plot). The simulated time derivatives and the weakest smoothing minus the original signal are depicted in bottom plot.

in upper plots of both figures, the same values with the original signal subtracted are shown in lower plots in a much enlarged scales. Only the best combination (full line) is shown in this case. It can be seen that the combined smoothed curve i s much closer to original signal than the observations, and also that the artificial step introduced in the simulated time derivative is completely suppressed in the combination (Figure 2, lower plot). It means that the long-periodic trend is taken over from the function values, and only short-periodic signal of the derivatives contributes to the combined solution. The dispersion of the combined values from the signal (full line) is equal to only one half of the noise of simulated data (triangles), which is a great improvement.

5. COMBINATION OF REAL OBSERVATIONS

Six years of simultaneous observation of UT1 by VLBI and l.o.d. by GPS are combined, using the solutions made at U.S. Naval Observatory, Washington D.C. (USNO) and University of Berne (CODE). Both values are tied by well-known relation

$$\mathrm{l.o.d.} = -\mathrm{d}(\mathrm{UT1} - \mathrm{TAI})/\mathrm{d}x,$$

if the time argument x is expressed in days. As in the case of simulated data, we assume that the shortest period existing in the real signal is 10–14 days, and we use exactly the same coefficients of smoothing that were found appropriate in Section 4. Weights of individual observations were calculated from their formal uncertainties, given by both analysis centers. The average values of these uncertainties over the whole period are

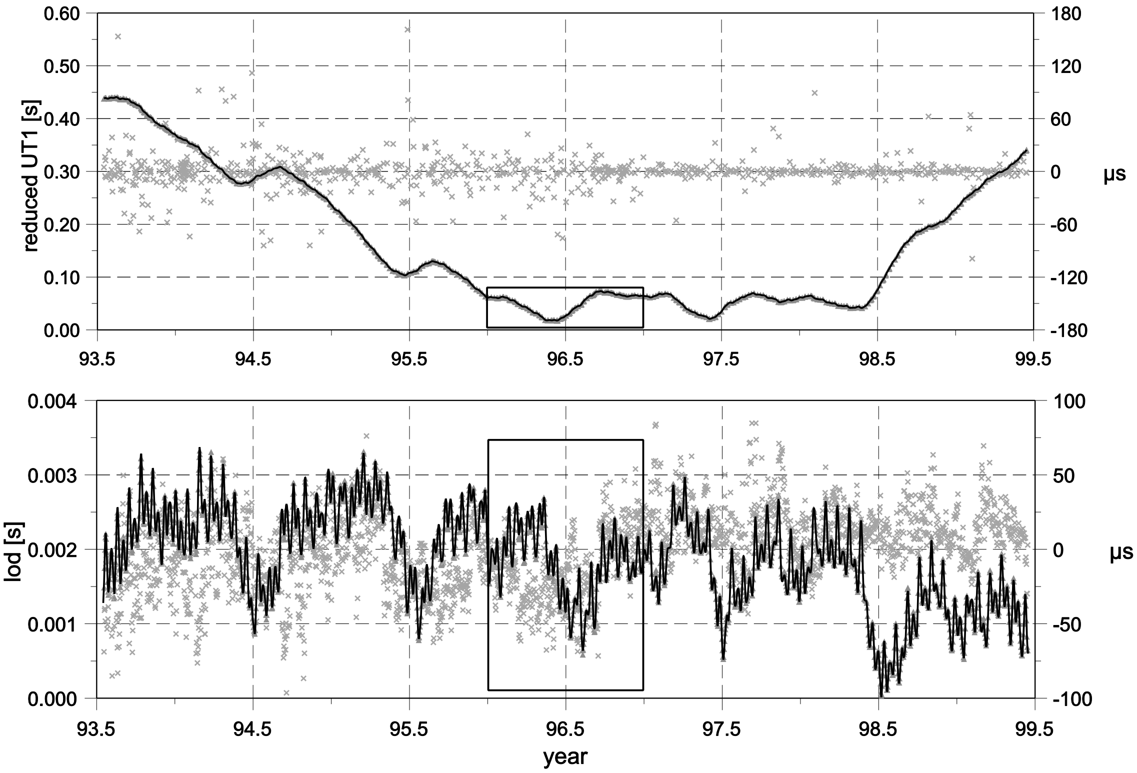


Figure 3: Combined smoothing of UT1 (VLBI) and length-of-day (GPS). Grey triangles represent the observations, full lines the smoothed curves and grey crosses the residuals in enlarged scale on the right (reduced UT1–TAI in top plot, length-of-day in bottom). Rectangular frames in both plots show the areas whose close-ups are depicted in Fig. 4.

11.6μs for UT1(USNO) and 1.7μs for l.o.d.(CODE). The results of the whole period are displayed graphically in Figure 3. Triangles correspond to observed values, full lines to combined smoothed curves (scale on the left), and crosses to residuals (highly enlarged scale on the right), upper plot depicts UT1–TAI from which a constant and linear trend have been removed, lower plot l.o.d. In order to see more details, close-up of the central parts of both plots (framed in boxes) is shown in Figure 4.

The a posteriori standard errors, calculated from the residuals, are nevertheless quite different from the formal uncertainties reported by analysis centers – they equal to 6.3μs for UT1(USNO) and 24.9μs for l.o.d.(CODE), respectively. The large value for l.o.d. can be, at least partially, explained by evident systematic long-periodic deviations that can be seen in the residuals in both lower plots of Figures 3 and 4. Remarkably evident is a sudden jump in the residuals around 1996.7, of about 40μs. This is due to the change of the model, introduced by CODE.

6. CONCLUSIONS

The tests of the new method of combined smoothing helped to find an optimal choice of coefficients of smoothing, provided that at least an approximate a priori knowledge on the observed process is given. It is recommended to use only the values of ε, $\bar{\varepsilon}$ leading to approximately the same transfer functions, passing the shortest periods expected in the signal. The method proves to combine the advantages of both series: long-term stability of measured function values and short-term accuracy of measured time derivatives. Thus

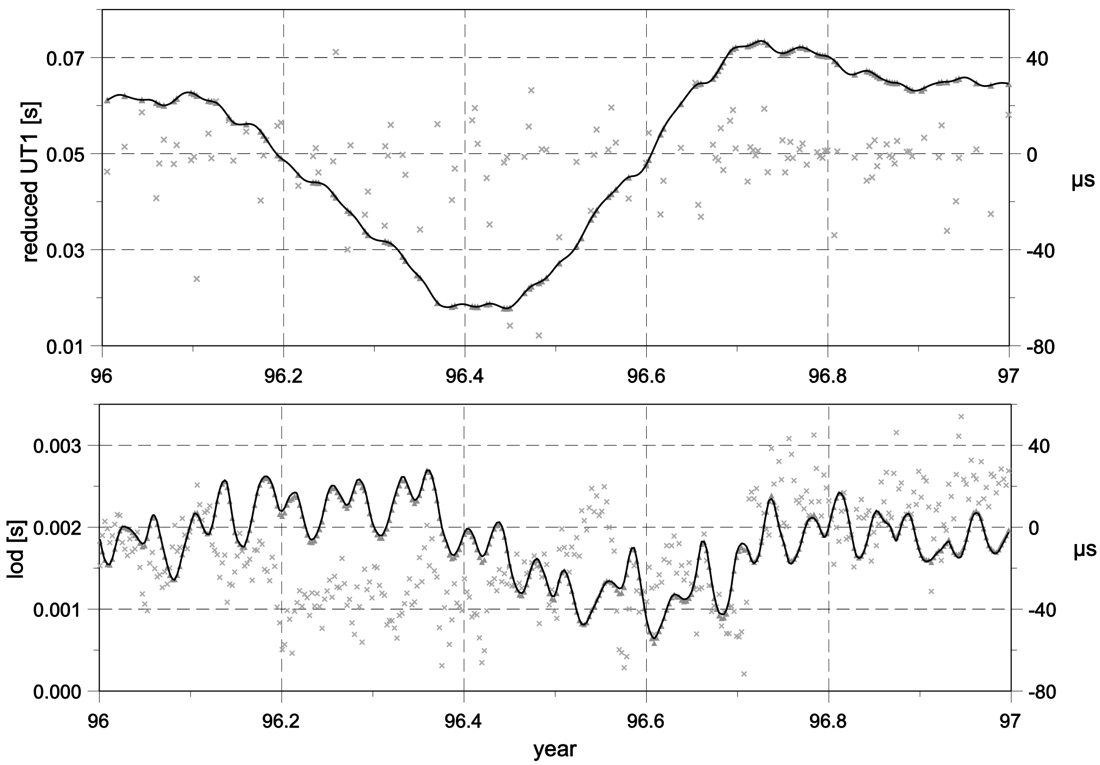


Figure 4: Close-up of the framed parts of Fig.3. Grey triangles represent the observations, full lines the smoothed curves and grey crosses the residuals in enlarged scale on the right (reduced UT1–TAI in top plot, length-of-day in bottom).

an optimal combination of UT1 measured by VLBI with l.o.d. measured by GPS is achieved, compatible with both data series. It also proved to be a good tool to reveal systematic deviations between the two data series. The usage of the method is however not limited only to the application demonstrated in this paper – it can be used, e.g., also to combine observations of polar motion with polar motion rate, celestial pole offsets observed by VLBI with their rates measured by GPS (Weber 2001) or for combining worldwide comparison of atomic clocks with the comparison of their frequencies.

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SHORT PERIODIC NUTATIONS : COMPARISON BETWEEN SERIES AND INFLUENCE ON POLAR MOTION

C. BIZOUARD[1], M. FOLGUEIRA[2], J. SOUCHAY[1]

[1] DANOF/UMR8630-CNRS
Observatoire de Paris, 61 avenue de l'Observatoire
e-mail: bizouard@danof.obspm.fr

[2] Instituto de Astronomía y Geodesia (UCM-CSIC),
Facultad de Ciencias Matemáticas
Universidad Complutense de Madrid, 28040 Madrid. Spain
e-mail: martafl@iagmat1.mat.ucm.es

ABSTRACT. The three up date series of diurnal and subdiurnal lunisolar nutations for a rigid Earth (REN 2000, SMART 1997, RDAN 1997) are compared, firstly. The largest offsets in time domain, over one century, are 2-4 microarcsecond. Secondly, the effects due to the non rigidity of the Earth are estimated. Finally the influence of these terms in the polar motion is investigated.

1. INTRODUCTION

Nutation is caused by the luni-solar torque on the Earth. As a first approximation the Earth is an axial and homogeneous ellipsoid, so that the diurnal rotation does not affect the luni-torque. The variation of this torque is associated with the fact that the sun and the moon are slowly moving up and down with respect to the equatorial bulge. Therefore nutation terms have period greater than a few days at least.

But any departure from an axial and homogeneous ellipsoid produce slight diurnal or subdiurnal variation of the torque, and hence nutation. Actually the distribution of masses inside the Earth is not perfectly axisymetric. The distribution of masses depends slightly on the longitude. The main perturbation, detected by satellite observation through the geopotential, is triaxality of the inertia axes: besides the axial inertia axis (inertia C), there are two distinct inertia axes in the Earth's equatorial plane, of inertia A and B (with $B > A$). The axis A lays in the direction -14.92° with respect to the Greenwich meridian. It corresponds to an extra-bulge at 90° from the axis of inertia A (northern part of the indian ocean and antipode). Because of the rotation of the Earth, this extra-bulge passes two times a day in front of the moon and the sun. In turn a semidiurnal torque and nutation can be predicted.

Semidiurnal nutation have been computed by Kinoshita and Souchay (1990) and Chao

(1991). They present variations up to 20 microarcseconds in time domain. This is the level of accuracy of the Earth orientation parameters determined by modern geodetic techniques. Hence astrometric determination of the Earth's orientation as well as the interpretation of the Earth's orientation parameters require the knownledge of the short period nutation terms.

Three series for short period nutations, established by three independant teams in the frame of a rigid Earth model, are available. We aim at providing with a comparison between these series, as well as a complete description of the phenomenom for astrometric purposes.

2.COMPARISON OF THE AVAILABLE SERIES FOR THE SHORT PERIOD NUTATIONS

Presently, there are three accurate available rigid Earth nutation series computed by different analytical methods:

- SMART97 *(Solution du Mouvement de l'Axe de Rotation de la Terre)* : the rigid Earth nutation series of Bretagnon et al. (1997, 1998) based on the Eulerian equations for the motion of the Earth in the space,
- RDAN97 *(Roosbeek Analytical Nutation)*: the rigid Earth nutation series of Roosbeek (1999), computed using the torque approach,
- REN 2000 *(Rigid Earth Nutation)*: the rigid Earth nutation series of Souchay and Kinoshita (1996, 1997) and Souchay et al. (1999), based on the Hamiltonian equations of a rotating body.

All the effects having an influence at the 0.1 microarsecond (μas) level such as Earth's triaxiality (C_{22} and S_{22} effects), J_3 and J_4 effects, non-zonal harmonics of third and fourth degree influences, planetary indirect and direct effects are taken into account in these three theories. The non-zonal harmonics induce the short-period nutations and can be divided into three categories (Souchay et al., 1999):

- The quasi-diurnal terms, related to C_{31}, S_{31}, C_{41} and S_{41} harmonics
- The semi-diurnal terms, due to C_{22}, S_{22}, C_{32} and S_{32} harmonics
- The third-diurnal terms, related to C_{33} and S_{33} harmonics

The nutation terms are classically expressed as a nutation in longitude $\Delta\psi$ and a nutation in obliquity $\Delta\varepsilon$:

$$\Delta\psi = \psi_s \sin\theta(t) + \psi_c \cos\theta(t) \tag{1}$$
$$\Delta\varepsilon = \varepsilon_s \sin\theta(t) + \varepsilon_c \cos\theta(t) \tag{2}$$

where θ is the astronomical argument of the nutation given by a linear combination of the Delaunay arguments and the angle Φ, which represents the Greenwich sidereal time minus the effect of the moving ecliptic :

$$\theta(t) = i_1 l_M + i_2 l_S + i_3 F + i_4 D + i_5 \Omega + i_6 \Phi$$

where l_M is the mean anomaly of the Moon, l_S the mean anomaly of the Sun, F the difference between the mean longitude of the Moon and the mean longitude of the node of the moon, D the difference between the mean longitudes of the Moon and of the Sun, Ω the mean longitude of the node of the Moon. The argument $\theta(t)$ can be expressed as the sum of a secular term λt, where λ is the frequency of the nutation concerned and a quasi-constant term.

In Table 1 we present the main quasi-diurnal and subdiurnal terms of REN 2000 series according to the above representation (the complete series contain 252 terms in longitude and 190 in obliquity). In the same table, we report the term-to-term differences with SMART97 and RDAN 2000 models. The largest offsets are below 0.8 μas.

We proceeded also the time comparison of those series over one thousand years. The Table 2 provides us with the maximum absolute values of the differences between each of the series.

3. CIRCULAR COMPONENTS AND NON-RIGID EFFETCTS

From a geophysical point of view, the best information is contained in the circular components of the nutation terms. The prograde and retrograde circular terms, expressed in the terrestrial frame have totally different frequency. In turn they constitute different phenomenona with respect to the Earth. Thus we can expect, that they are perturbated in different way by the geophysical properties of the Earth.

Misceallenous ways for expressing the circular components of the nutations exist. A given nutation term, in the mean equatorial frame, can be expressed by the complex coordinate

$$P = \Delta\psi \sin\varepsilon_0 + i\,\Delta\varepsilon \tag{3}$$

According to equations (1) and (2) and by using Euler decomposition of cosinus and sinus terms, we get the following expression in circular components :

$$P = i(a^+ e^{i\theta(t)} + a^- e^{-i\theta(t)}) \tag{4}$$

$$\begin{aligned} a^+ &= a_r^+ + i\,a_i^+ = \frac{1}{2}(-\Delta\psi_s \sin\varepsilon_0 + \Delta\varepsilon_c) + i\frac{1}{2}(-\Delta\psi_c \sin\varepsilon_0 - \Delta\varepsilon_s) \quad (5)\\ a^- &= a_r^- + i\,a_i^- = \frac{1}{2}(\Delta\psi_s \sin\varepsilon_0 + \Delta\varepsilon_c) + i\frac{1}{2}(-\Delta\psi_c \sin\varepsilon_0 + \Delta\varepsilon_s) \quad (6)\end{aligned}$$

If the frequency λ of the nutation term is positive, then the term a^+ provide us with the prograde component, whereas a^- provide us with the retrograde component. In the contrary, if the frequency λ of the nutation term is negative, then the term a^+ provide us with the retrograde component, whereas a^- provide us with the prograde component.

It turns out that short period nutations are prograde, the retrograde components being less than 1 μas in absolute amplitude. In Table 3 we give the prograde components associated with the model REN 2000 and greater than 1 μas. We give also the period with which these terms appear in a terrestrial reference frame. The quasi-diurnal terms get mapped into long period polar motion, whereas the semi-diurnal ones into prograde diurnal polar motion $p = x - iy$ according to the equation :

$$p = -Pe^{-i\Phi} \tag{7}$$

Until now the computation of the diurnal and subdiurnal nutation was restricted to a rigid Earth model. Some resonance effects with the Chandler frequency can be expected

Table 1: Main quasi-diurnal and semi-diurnal nutations of the figure axis : REN-2000 values and differences with SMART97 and RDAN97 models. The unit is μas.

	Argument						Period	Longitude ($\Delta\psi$)		Obliquity ($\Delta\varepsilon$)	
	Φ	l_M	l_S	F	D	Ω		sin	cos	sin	cos
	1	0	0	-1	0	0	1.03521	-5.30	-.65	-.27	2.17
RDAN97-REN2000								-.03	.01	.02	-.02
SMART97-REN2000								-.03	-.00	.00	-.01
	1	0	0	-1	0	-1	1.03505	-35.40	-4.35	-1.59	12.91
RDAN97-REN2000								.58	.07	-.04	.39
SMART97-REN2000								.58	.08	-.05	.46
	1	1	0	-1	0	-1	.99758	-19.94	-2.45	-.97	7.91
RDAN97-REN2000								.03	.02	.00	-.02
SMART97-REN2000								.09	-.04	-.02	-.03
	1	-1	0	1	0	1	.99696	24.14	2.97	1.16	-9.47
RDAN97-REN2000								-.21	-.03	.01	-.05
SMART97-REN2000								-.22	.03	.03	-.05
	1	-1	0	1	0	0	.99682	4.03	.50	.20	-1.60
RDAN97-REN2000								-.01	-.01	.00	.00
SMART97-REN2000								-.01	.01	.00	.00
	1	1	0	1	-2	1	.99216	-7.06	-.87	-.35	2.81
RDAN97-REN2000								-.04	-.01	-.00	.01
SMART97-REN2000								-.02	-.01	-.00	.01
	1	0	0	1	0	1	.96215	-38.23	-4.70	-1.86	15.11
RDAN97-REN2000								.09	.01	-.07	.02
SMART97-REN2000								.10	.00	-.01	.02
	1	0	0	1	0	0	.96201	-6.04	-.74	-.29	2.35
RDAN97-REN2000								.03	.12	.05	-.01
SMART97-REN2000								-.00	-.00	-.00	.00
	2	-1	0	-2	0	-2	.52743	-5.10	2.93	1.18	2.06
RDAN97-REN2000								-.09	.05	-.00	.00
SMART97-REN2000								-.05	.03	-.01	-.01
	2	0	0	-2	0	-1	.51756	-4.71	2.70	1.08	1.88
RDAN97-REN2000								-.11	.07	.02	.04
SMART97-REN2000								-.07	.04	.01	.02
	2	0	0	-2	0	-2	.51753	-25.53	14.65	5.68	9.89
RDAN97-REN2000								-.00	-.00	.15	.26
SMART97-REN2000								.17	-.10	.11	.20
	2	0	0	-2	2	-2	.50000	-10.44	6.00	2.37	4.12
RDAN97-REN2000								-.23	.13	.07	.12
SMART97-REN2000								-.19	.10	.06	.10
	2	0	0	0	0	0	.49863	31.25	-17.94	-7.12	-12.40
RDAN97-REN2000								.81	-.47	-.21	-.35
SMART97-REN2000								.60	-.34	-.16	-.27
	2	0	0	0	0	-1	.49860	4.28	-2.46	-.95	-1.66
RDAN97-REN2000								.06	-.04	-.04	-.06
SMART97-REN2000								.04	-.02	-.03	-.06

Table 2: Comparison in time domain between SMART97, RDAN97 and REN2000 short period nutations. The unit is μas.

	$\Delta\psi \sin\varepsilon$	$\Delta\varepsilon$
SMART97 – REN2000	1.74	3.52
RDAN97 – REN2000	2.40	6.80
RDAN97 – SMART97	1.40	6.80

Table 3: REN2000 prograde nutations above 2 μas and corresponding values for a non-rigid Earth according to Mathews, Herring and Buffet (MHB 2000). We give the in-phase a_r^+ (ip) and out-of-phase a_i^+ (op) terms defined hereabove. The unit is μas.

Argument						Period	Period	Rigid Earth		Non rigid Earth	
Φ	l_M	l_S	F	D	Ω	in space	in Earth	ip	op	ip	op
1	-1	0	-1	0	-1	1.07545	-13.71824	1.2	0.2	1.4	0.2
1	0	0	-1	0	0	1.03521	-27.20986	2.1	0.3	2.5	0.3
1	0	0	-1	0	-1	1.03505	-27.32087	13.5	1.7	15.6	2.0
1	1	0	-1	0	0	0.99772	-2203.84398	1.2	0.1	1.8	0.3
1	1	0	-1	0	-1	0.99758	-3193.94631	7.9	1.0	12.2	2.1
1	-1	0	1	0	1	0.99696	3222.61635	-9.5	-1.2	-15.8	-3.0
1	-1	0	1	0	0	0.99682	2216.40169	-1.6	-0.2	-2.7	-0.5
1	1	0	1	-2	1	0.99216	193.68628	2.8	0.3	2.1	0.2
1	0	0	1	0	1	0.96215	27.32239	15.2	1.9	16.6	2.0
1	0	0	1	0	0	0.96201	27.20994	2.4	0.3	2.6	0.3
1	1	0	1	0	1	0.92969	13.71960	1.1	0.1	1.3	0.2
2	-1	0	-2	0	-2	0.52743	1.11951	2.0	-1.2	2.3	-1.3
2	0	0	-2	0	-1	0.51756	1.07596	1.9	-1.1	2.1	-1.2
2	0	0	-2	0	-2	0.51753	1.07583	10.0	-5.8	11.3	-6.5
2	0	0	-2	2	-2	0.50000	1.00275	4.1	-2.4	4.7	-2.7
2	0	0	0	0	0	0.49863	0.99725	-12.4	7.1	-14.0	8.0
2	0	0	0	0	-1	0.49860	0.99713	-1.7	1.0	-1.9	1.1

for the diurnal terms, since they appears as a long period polar motion in the terrestrial frame (around 1 month for the biggest term).

Generally the tidally induced nutation for a non rigid Earth are deduced from those of a rigid Earth thanks to a transfer function involving geophysical properties. The transfer functions are constantly refined by taking account new processes as well improved values of geophysical parameters. We have used the recent transfer function as set up by Mathews, Herring and Buffet (MHB 2000) such as :

- any prograde diurnal nutation of frequency σ' in the celestial frame - $\sigma = \sigma' - \omega$ in the terrestrial frame with $\omega = 1$ cycle per sidereal day - has to be multiplied by:

$$T(\sigma) \approx \frac{A}{A_m} \frac{\sigma - \sigma_e}{\sigma - \sigma_{CW}} \tag{8}$$

  where A is the equatorial moment of inertia of the Earth, A_m the equatorial moment of inertia of the mantle ($A/A_m = 1.1284$) , $\sigma_e = \omega/304$ the Euler frequency, and $\sigma_{CW} = 0.0023074 + i0.0001244$ cycle per sidereal day the Chandler frequency

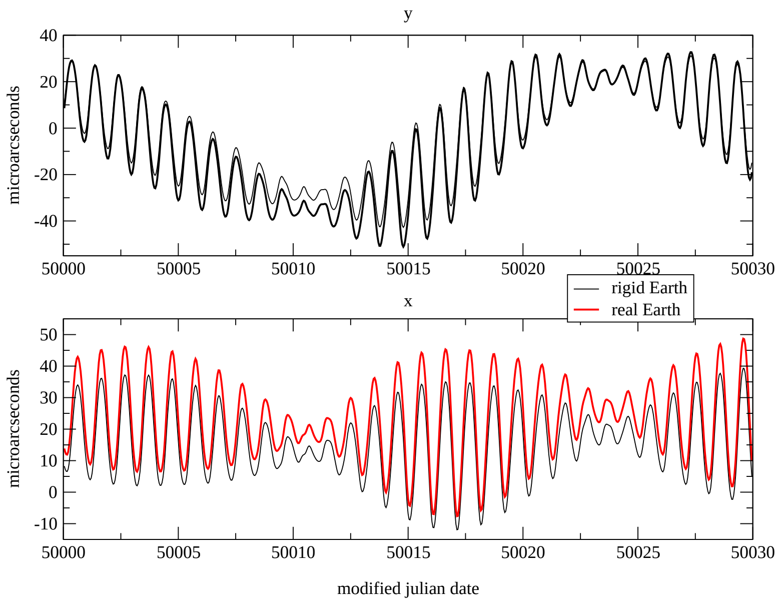
- any prograde semidiurnal nutation has to be multiplied by the factor $A/A_m = 1.1284$.

The prominent values (larger than 1 μas) are reported in Table 3 for the circular components. It has to be noted an increasing of the amplitudes of the nutation terms for a rigid Earth model at the level of a few μas.

5. INFLUENCE ON POLAR MOTION

According to the recomendations adopted by the IAU General Assembly in August 2000, the diurnal and subdiurnal nutations have to be expressed as polar motion of the CIP (Celestial Intermediate Pole). The transformation is ensurred by equation (7). The

Figure 1: Contribution of short periodic nutations to polar motion. Values for rigid Earth model and real Earth model in μas.



diurnal nutations become long periodic polar motion, the semidiurnal ones prograde diurnal polar motion. By applying relation (7) to the short periodic nutation waves, the equivalent polar motion has been computed in time domain over a 30 days span. The result is displayed in Fig.1.

This gravitationally-induced polar motion was investigated in GPS polar motion series. The diurnal contribution (equivalently the semi-diurnal nutations) was removed from the hourly CODE GPS series stretching from 1995 to 1998. The long periodic contribution was removed from the daily CODE GPS series (1993-2000). VLBI series were also considered. In all cases the rms of the series have been increased, except on some restricted time spans. We concluded there is no observational evidence of the short periodic nutations.

6. CONCLUSION

The new level of truncation (0.1 μas) of the rigid Earth?s nutation series requires to take into account the effects due to the triaxiality and the non-zonal harmonics of the geopotential of third and fourth degree, which originate the nearly diurnal and subdiurnal nutations. The agreement between the short-period nutations of the three available rigid Earth?s nutation series is, at the level of 1 μas, satisfactory. The diurnal nutations are increased by the non rigidity of the Earth up to 5 μas, the semidiurnal ones are less affected. The contribution of diurnal and semidiurnal nutation on polar motion has not been detected in GPS and VLBI observed polar motion series.

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THE CHANGES OF PARAMETERS OF CHANDLER NUTATION AND ANNUAL OSCILLATION BY USING FEW KINDS OF POLAR MOTION DATA

G. DAMLJANOVIĆ (1) and M.S. DE BIASI (2)

(1) Astronomical Observatory
Volgina 7, 11160 Belgrade, Yugoslavia
E-mail: gdamljanovic@aob.bg.ac.yu

(2) Facultad de Ciencias Astronómicas y Geofísicas (UNLP)
Paseo del Bosque s/n, 1900 La Plata, Argentina,
Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina
E-mail: debiasi@fcaglp.fcaglp.unlp.edu.ar

ABSTRACT. We used about 80 years (for the period 1899.806 - 1978.972) of the International Latitude Service - ILS data (Yumi and Yokoyama 1980 reduction), about 30 years (1962.01 - 1992.99) of the Bureau International de l'Heure - BIH data (IERS, 1993) and about 92 years (1899.7 - 1992.0) of the newest optical astrometry time series solution called OA00 received from Vondrák (Vondrák, 2000) to determine and investigate the variation of parameters (the period, the amplitude and the phase) of Chandler nutation and annual oscillation of polar motion. The Direct Fourier Transforms method of spectral analysis and the Least Squares Method were applied to the data. Some interested results are presented here. Very indicative is the quasi - periodic changes of the amplitude of Chandler nutation. The period of these changes is about 40 years.

1. INTRODUCTION

The goal of this paper is to give us more informations of the decadal variations of polar motion data. Some time ago, we calculated the parameters of quasi - periodic instability of the Chandler nutation amplitude (Damljanović et al., 1997) by using the 37 years of Belgrade visual zenith -telescope latitude data (in the list of BIH series denoted by BLZ). After that, we did it for the 13 years of Punta Indio (in the list of BIH series denoted by PIP) photographic zenith tube latitude data, and had a good agreement with the results of both data.

In this paper, we analysed different series of polar motion data and wanted to examine the time stability of the amplitudes and the phases (or the periods) of the Chandler nutation and the annual wobble. First at all, we used about 80 years of ILS data (Yumi and Yokoyama, 1980) for the period 1899.806 - 1978.972 with 1/12 years of time spans and the number of the input data Nils=951, then the BIH ones (IERS, 1993) for the period 1962.01 - 1992.99 (about 30 years) with 5 days time spans (Nbih=2264), and the newest

(August 2000) optical astrometry solution called OA00 and referred to the HIPPARCOS catalogue (Vondrák, 2000) for the period 1899.7 - 1992.0 (about 92 years) with Nv=6693. To obtain the equidistant data for Vondrák's series, the OA00 data were interpolated by the cubic spline method, and after the interpolation we had Nvi=6739 with 5 days time spans. For our investigations it was important to have the input data extended far away in the past and the homogenized ones.

In the papers of Rykhlova (1996) for the interval of 119 years and Nastula et al. (1993) for the interval of 140 years the quasi - periodic instability of the Chandler nutation amplitude was detected (the period lay in the range 30 - 40 years). In the paper of Vondrák and Ron (1995), based on the data reduced to the HIPPARCOS reference frame, we can see the similar result. It is particularly interesting because the HIPPARCOS stars coordinates and proper motions are free of known systematic errors and local distortions of classical astrometry ones. Also, the results of other authors (Markowitz, 1960; McCarthy, 1972; Vicente and Currie, 1976; Wilson and Vicente, 1980; Guinot, 1972, 1982; Vondrák, 1985, etc.) give some informations about the decadal variations of polar motion. But, until now, there is not a complete physical explanation of the phenomenon.

2. CALCULATION

1. During the first step of our calculation we determined (by using the Least Squares Method - LSM) and removed from each mentioned polar motion series the linear drift (for both polar coordinates, x and y). The results are:

- ILS data, $0\sec 00065/yr \pm 0\sec 00019/yr$ for x, $0\sec 00347/yr \pm 0\sec 00019/yr$ for y,
- BIH data, $0\sec 00204/yr \pm 0\sec 00032/yr$ for x, $0\sec 00476/yr \pm 0\sec 00031/yr$ for y,
- OA00 after our interpolation, $0\sec 00117/yr \pm 0\sec 00007/yr$ for x, $0\sec 00205/yr \pm 0\sec 00007/yr$ for y.

2. After that, the Direct Fourier Transforms method (DFT) was applied to each series to determine the amplitudes, the periods and the phases. The ILS and OA00 were divided into three subintervals because of long series (ILS and OA00) and decadal variations of x and y. The subintervals for ILS data are: 1899.806 - 1924.972 (ILSI, 303 points), 1925.056 - 1945.972 (ILSII, 252 points) and 1946.056 - 1978.972 (ILSIII, 396 points). The subintervals for OA00 data are: 14925 MJD - 24115 MJD (OA00I, 1839 points), 24120 MJD - 31760 MJD (OA00II, 1529 points) and 31765 MJD - 48615 MJD (OA00III, 3371 points). The results of DFT for Chandler, annual and semiannual wobbles are presented in Table 1.
3. We used the results of step 2. to remove sa and a wobbles from the data, and after that calculated the amplitude and the phase of ch wobble by using LSM (with P from step 2.) and 1.25 years input subintervals of residues. The phase epoch is 1900.0 . The results are presented in Fig. 1. (changes of amplitude of Chandler polar wobble) and Fig. 2. (changes of phase of Chandler polar wobble). The ILS curves are presented with “achx”, “achy”, “fchx” and “fchy”, the BIH curves with “achxb”, “achyb”, “fchxb” and “fchyb”, and OA00 curves with “achxv”, “achyv”, “fchxv” and “fchyv”.

Table 1. The periods (P), the amplitudes (A), and the phases (F, the epoch is $1900.^{yr}0$ for ILS and BIH data, but 14925 MJD for OA00 ones) of the Chandler (ch), annual (a) and semiannual (sa) wobbles

	P, ch	A, ch	F, ch	P, a	A, a	F, a	P, sa	A, sa	F, sa
ILSI, x	$1.^{yr}20$	0 sec 151	352°	$1.^{yr}00$	0 sec 089	238°	$0.^{yr}50$	0 sec 004	69°
ILSI, y	1.20	0.152	264	1.00	0.078	156	0.49	0.005	216
ILSII, x	1.16	0.077	96	1.00	0.092	288	0.50	0.006	55
ILSII, y	1.16	0.073	3	1.00	0.076	205	0.50	0.007	288
ILSIII, x	1.19	0.180	126	1.00	0.100	243	0.50	0.005	114
ILSIII, y	1.19	0.185	37	1.00	0.078	159	0.50	0.007	291
BIH, x	1.18	0.157	237	1.00	0.097	299	0.50	0.006	307
BIH, y	1.18	0.156	145	1.00	0.089	209	0.51	0.004	182
OA00I, x	1.19	0.158	109	1.00	0.101	318	0.51	0.007	259
OA00I, y	1.19	0.161	0	1.00	0.081	228	0.50	0.008	221
OA00II, x	1.14	0.071	335	1.00	0.090	12	0.49	0.006	306
OA00II, y	1.15	0.063	208	1.00	0.068	217	0.50	0.011	334
OA00III, x	1.18	0.176	314	1.00	0.096	54	0.50	0.006	115
OA00III, y	1.18	0.179	225	1.00	0.080	337	0.50	0.004	8

4. During this step we removed ch wobble from the data (we used the results of step 3.), and calculated the amplitudes and the phases of a and sa wobbles by using LSM (with P from step 2.) and 1.0 years subintervals of residues. The phase epoch is 1900.0 . The results are presented in Fig. 3. (changes of amplitude of annual polar wobble) and Fig. 4. (changes of phase of annual polar wobble). The ILS curves are presented with "agx", "agy", "fgx" and "fgy", and the OA00 curves with "agxv", "agyv", "fgxv" and "fgyv".
5. Finally, we put our attention to the results presented in Fig. 1. to determine the amplitude, the phase and the period of the quasi - periodic instability of Chandler nutation amplitude by using LSM. It means, to determine the sinusoidal approximation for Chandler nutation amplitude. The criterion for LSM was the minimum of standard deviation of residuals (the approximation minus the results of Fig. 1. data) with varying the period of sinusoid (the lag was 0.5 years). We have got the formula $p(i) = q + r * COS(s(i) - t)$, and the results (q and r are in arcseconds, t in degrees, and P in years):

- for BIH, x, q=0.165, r=0.03, t=138, P=38.0,
- for BIH, y, q=0.161, r=0.03, t=135, P=38.0,
- for ILS, x, q=0.143, r=0.07, t=95, P=42.5,
- for ILS, y, q=0.145, r=0.07, t=86, P=44.0,
- for OA00, x, q=0.151, r=0.06, t=108, P=40.0,
- for OA00, y, q=0.148, r=0.07, t=107, P=40.5 .

The results can be useful for the physical explanation of the phenomenon, for the predication of polar motion, etc.

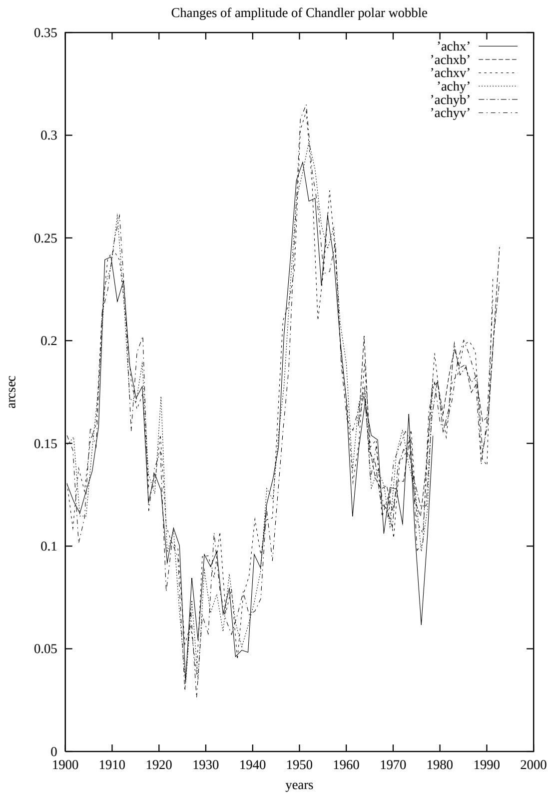

Figure 1

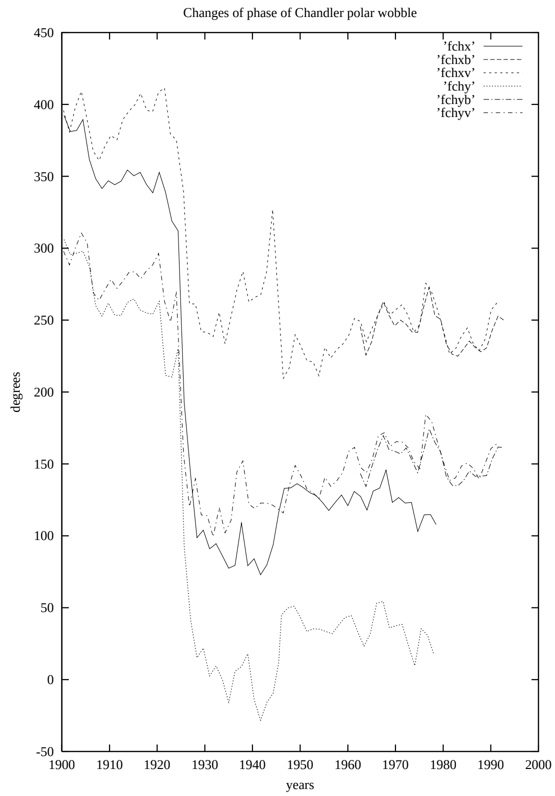

Figure 2

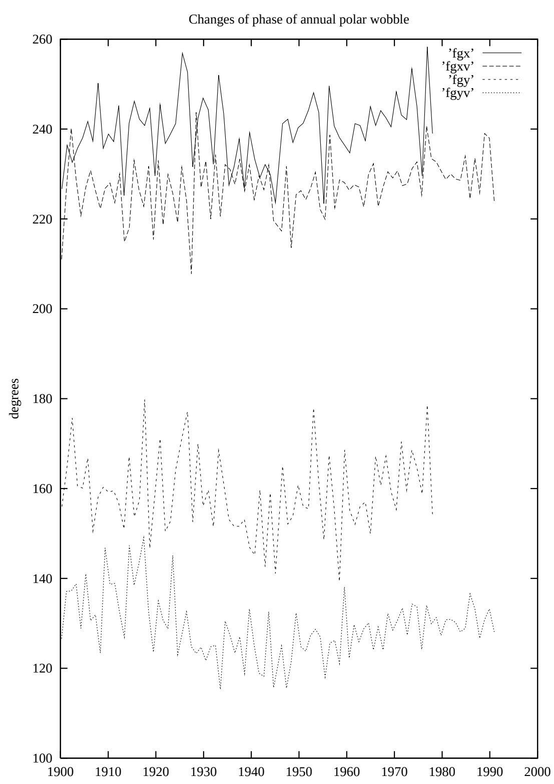

Figure 3

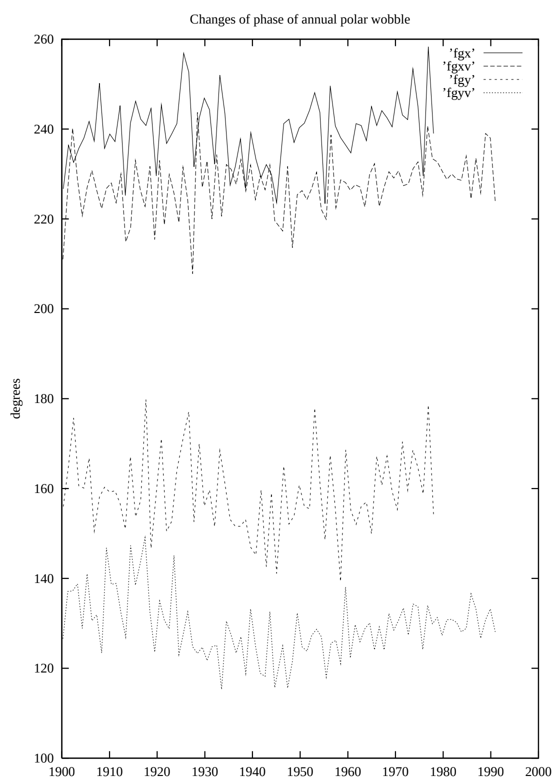

Figure 4

3. CONCLUSION

In few kinds of homogenized series of polar motion data the decade variation of Chandler nutation amplitude is detected. The period of variation is (in years): 42.5 (ILS, x), 44.0 (ILS, y), 38.0 (BIH, x and y), 40.0 (OA00, x), 40.5 (OA00, y). The amplitude of variation is (in arcseconds): 0.07 (ILS, x and y), 0.03 (BIH, x and y), 0.06 (OA00, x), 0.07 (OA00, y). The values of r for OA00 are similar to the values of r for ILS.

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APPLICATIONS OF ARCH AND GARCH TIME SERIES ANALYSIS METHODS IN STUDY OF EARTH ROTATION

J. Hefty[1], M. Komorníkova[2], T. Bognár[2]
[1]Department of Theoretical Geodesy
[2]Department of Mathematics and Descriptive Geometry
Slovak University of Technology,
Radlinského 11, 81368 Bratislava, Slovak Republic
e-mail: hefty@cvt.stuba.sk

ABSTRACT. Non-linear methods of Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) modelling are applied for analysis of short-term (periods less than 100 days) fluctuations of ERP. It is shown that 1-day sampled time series of x, y and $UT1R$ from 1993.0 to 1999.3 can be modelled as linear autoregressive process and non-linear time dependent variance. The latter is well modelled as GARCH(1,1) process for x and y and ARCH(2) process for $UT1R$.

1. INTRODUCTION

The Earth rotation variations can be generally divided into the regular and irregular part, even though this separation is valid only in the first approximation. The regular variations such as Chandler wobble, seasonal changes, tidal terms, secular trends, etc. can be described by well defined deterministic functions (polynomials and/or periodic functions). The irregular part can be described predominantly by statistical time series modelling. As shown by (Frede and Mazzega, 1999) the non-linear methods have to be applied as the dynamics of the forcing terms so the Earth response are non-linear in principle. In this paper we will try to solve the stochastic modelling of the short-term part of Earth rotation variations by non-linear analysis methods using the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. These models enable to describe the time dependent variance of stochastic time series - an effect that is observed when analysing the Earth rotation data.

This paper is concentrated on stochastic modelling of short-term variations in Earth rotation parameters. Only the fluctuations with effects shorter than 100 days will be analysed. For the residuals of x, y and $UT1R - TAI$ series sampled at 1-day interval we will find their representation in form of the linear Autoregressive (AR) model and the non-linear representation of the residual variance applying the ARCH and GARCH modelling.

2. HOMOSKEDASTIC AND HETEROSKEDASTIC TIME SERIES MODELS

The fundamental assumption of time series modelling is that the value of the series at time t, Y_t depends only on its previous values (deterministic part) and on random disturbance (stochastic part). The linear Autoregressive AR(p) model for time series $\{Y_t\}$ with p AR coefficients $\{\phi_1, \phi_2, \ldots \phi_p\}$ then we can write

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + D_t \tag{1}$$

The stochastic (disturbance) part in linear modelling can be described as

$$D_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots + \theta_q Y_{t-q} \tag{2}$$

where $\{Z_t\}$ is the zero-mean white noise process, i.e. $E(Z_t) = \mu = 0$ and the variance $var(Z_t) = \sigma^2$ is constant. The time independence of variance of $\{Z_t\}$ is called homoskedasticity. The time series $\{D_t\}$ is then called as Moving Average (MA) process (Box and Jenkins, 1970). Combination of (1) and (2) results into the mixed Autoregressive Moving Average process ARMA(p, q)

$$Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} - \ldots - \phi_p Y_{t-p} + D_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots + \theta_q Y_{t-q} \tag{3}$$

For the heteroskedastic models is characteristic that the variance of disturbing part $\{D_t\}$ is not constant - periods of relative tranquility are followed by periods of large volatility (Enders, 1992). The conditional variance of $\{Y_t\}$ is generally defined as

$$var(Y_{t+1}|X_t) = \sigma^2 X_t^2 \tag{4}$$

where X_t is independent variable that can be observed at period t. Let $\{\tilde{D}_t\}$ denote the estimated residuals from ARMA(p, q) model. Then the ARCH(r) process is defined as

$$\tilde{D}_t = \nu_t \sqrt{(h_t)} \tag{5}$$

where $\{\nu_t\}$ is random sequence with zero mean and unit variance and $\{h_t\}$ is autoregressive process for squares of estimated residuals $\{\tilde{D}_t^2\}$. The h_t coefficients are defined as

$$h_t = \alpha_0 + \alpha_1 \tilde{D}_{t-1}^2 + \alpha_2 \tilde{D}_{t-2}^2 + \ldots + \alpha_r \tilde{D}_{t-r}^2 \tag{6}$$

Finally, GARCH(s, r) model is defined if $\{h_t\}$ is following ARMA(r, s) model

$$h_t = \alpha_0 + \alpha_1 \tilde{D}_{t-1}^2 + \alpha_2 \tilde{D}_{t-2}^2 + \ldots + \alpha_r \tilde{D}_{t-r}^2 + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \ldots + \beta_s h_{t-s} \tag{7}$$

The combined AR-ARCH model is defined when $\{Y_t\}$ is following AR(p) model and the disturbances $\{D_t\}$ are following ARCH(r) model. Similar definition is valid for combined AR-GARCH time series when the disturbances are following GARCH(s, r) model.

Estimation procedures for solving of AR or ARMA process are well described in literature, e.g. (Box and Jenkins, 1970), (Enders, 1992). The algorithms for time series treatment are available e.g. in Time Series Package of Mathematica 3.0 software (Wolfram, 1995). The Akaike's and Bayesian information criteria are used for optimum model identification. The parameter estimation for AR model is based on solving $\{\phi_k\}$ coefficients from Yule-Walker equations

$$\tilde{\gamma}(k) = \phi_1 \tilde{\gamma}(k-1) + \phi_2 \tilde{\gamma}(k-2) + \ldots + \phi_p \tilde{\gamma}(k-p), k = 1, 2, \ldots, p \tag{8}$$

where $\tilde{\gamma}(k)$ are values of sample covariance function of time series $\{Y_t\}$, $\gamma(k) = Cov(Y_{t+k}, Y_t)$. The Hannan-Rissanen procedure is applied for estimating ARMA parameters.

The input time series for estimation of ARCH and GARCH models are the residuals $\{\tilde{D}_t\}$ from optimum AR(p) model for $\{Y_t\}$. After testing if variance of $\{\tilde{D}_t\}$ is not constant in time, the AR or ARMA modelling is applied for $\tilde{D}_t^2$. If $\tilde{D}_t^2$ follows AR(r) model then $\{\tilde{D}_t\}$ is ARCH(r) model. If $\tilde{D}_t^2$ follows ARMA(r, s) model then $\{\tilde{D}_t\}$ is GARCH(s, r) model.

3. APPLICATION OF AR-GARCH MODELS TO ANALYSIS OF EARTH ROTATION PARAMETERS

For stochastic modelling of short-term variations in ERP we used as input the combined x, y and $UT1 - UTC$ series EOP(IERS)C03 at 1-day interval from 1993.00 to 1999.33 (IERS, 1999). The series represent the unsmoothed raw observed data and is continuous for polar motion. For $UT1-UTC$ the missing values were interpolated. Mean uncertainties of polar motion coordinates are 0.29 mas and for $UT1-UTC$ they are 0.017 ms. Uncertainties of all three series are significantly decreased from 1993 to 1998. We converted the $UT1-UTC$ series to $UT1R-TAI$ series by removing the complete IERS Conventions tidal model and by referencing to TAI.

As in this analysis we are interested in the short-term variations only, the Vondrak filtering (Vondrák, 1969) has been used for removing oscillations with periods larger than 100 days. The analysed series of short-term ERP oscillations are plotted in Figure 1. Note the significant difference between the scatter of polar motion x and y short-term oscillations. The amplitude spectra of the series analysed are shown in Figure 2 (black lines).

Figure 1: Short-term oscillations of ERP, the variations with periods larger than 100 days are completely removed

Figure 2: Amplitude spectra of short-term ERP oscillations (black line) and of residuals from AR modelling (gray line)

The AR modelling of the series shown in Figure 1 results to optimum models AR(3) for x and y, and AR(2) for $UT1R - TAI$. The residuals from AR modelling are plotted in Figure 3, their clearly pronounced time dependent variance has been confirmed also by statistical testing. Spectra of the residual series are plotted in Figure 2 with grey line. It is evident that the AR linear modelling is sufficient to explain dominant part of the ERP short-term oscillations.

The residuals plotted in Figure 3 have been analyzed in order to find their ARCH/GARCH representation. For polar motion series we found as optimum the GARCH(1,1) model and for $UT1R - TAI$ the ARCH(2) model. The time dependent variance of the series in Figure 3 is demonstrated in Figure 4. For comparison also the observation uncertainties of EOP(IERS)C03 series are plotted. The comparison of observed and modelled variance time dependency shows very good consistency, the ERP short-term oscillations are in general above the observational noise. Only short periods of inconsistency between observed variance and ARCH/GARCH model are visible in the interval from 1993.3 to 1999.0, namely around 1994.3 for polar motion and around 1996.7 for $UT1R$.

Figure 3: Residuals of AR modelling

4. CONCLUSIONS

The analysis of short-term ERP oscillations (fluctuations less than 100 days) can be well modelled by AR-ARCH and AR-GARCH models. The linearity of short-term oscillations is observed up to level of 0.4 mas for polar motion and up to 0.03 ms for $UT1R$. The non-linear behavior of ERP is demonstrated as time dependent variance varying from 0.05 to 0.35 mas^2 for polar motion and from 0.0005 to 0.0025 ms^2 for $UT1R$. All these variations are above the observation uncertainties. The ARCH and GARCH models describe properly these variance fluctuations, only short periods of model inconsistency are observed.

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Figure 4: Observed $\sigma^2(t)$ from 100-day intervals ARCH/GARCH models and observation uncertainty

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IRREGULAR VARIATIONS OF THE EARTH ROTATION IN 1897-1989

A.KORSUN[1], G.KURBASOVA[2]
(1) Main Astronomical Observatory of Ukrainian National Academy of Sciences
03680, Golosiiv, Kyiv-127, Ukraine
e-mail: akorsun@mao.kiev.ua

(2) Crimean Astrophysical Observatory
p/o Nauchnyi, Crimea, 98677, Ukraine
e-mail: gsk@crao.crimea.ua

ABSTRACT. The "turning points" corresponded to changes in the length of day by several milliseconds and " breaks" in the Earth polar motion in 1897-1989 are discussed. In our analysis we used statistical methods. The irregular variations of the Earth rotation were compared with some geophysical phenomenons. If correlation does exit, it could very well be between the noise of the data of Earth rotation and geophysical phenomenons.

The turning points corresponded to the changes in the lenght of day by several milliseconds were discussed W. Mank and G.McDonald in 1960. The causes of such irregular variations have been a subject of considerable controversy. Now the question of the turning points in data of the Earth rotation remains open.
We made statistical analysis of irregular variations of some series data for the determination their characters and the correlations.

In our analysis we used following the annual mean of data for 1897-1989: ΔZ - the variations of the radius vector of polar motion, δP - variations of the length of day, W- the relative sunspot numbers, E- integrated seismic energy.
The desire to establish cause-and-effect relationships between these phenomena is based on the idea that solar activity can exert a modulating effect on various geodynamical and geophysical parameters. First of all we processed the initial data to remove trends and periodical components.

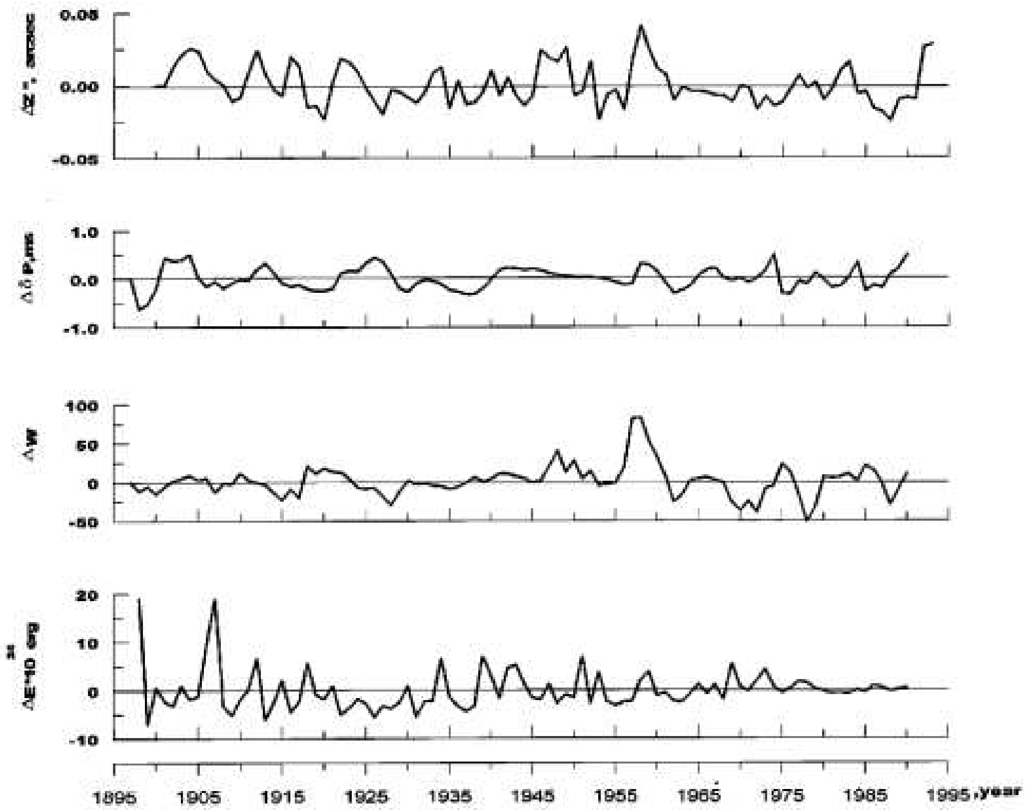

Figure 1. The irregular variations of the series : ΔZ, $\Delta\delta P$, ΔW, ΔE.

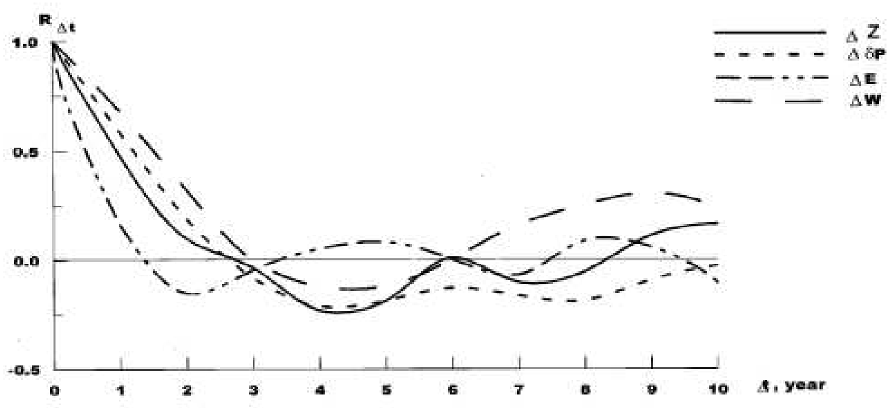

Figure 2. The dependence of the coefficient of the correlations $R_{\Delta t}$ on the time displacement in the data of the irregular variations.

In fig. 3 the coefficients of the mutual correlation of the different series are given.

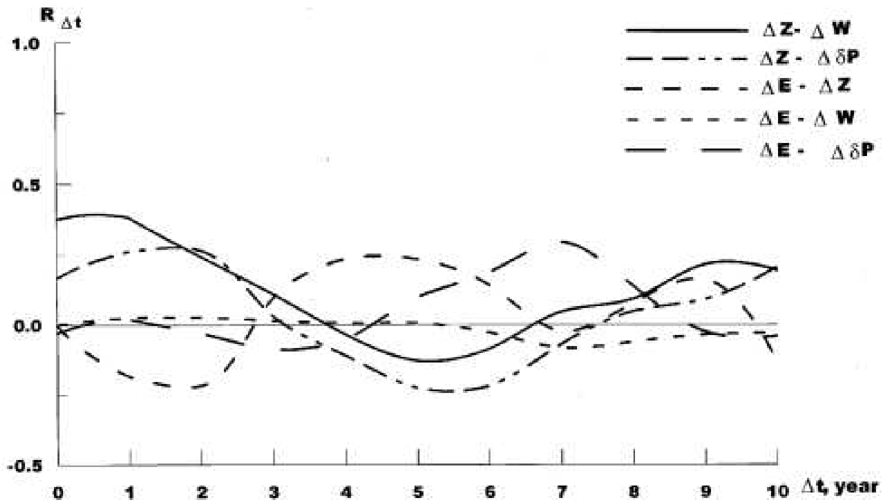

Figure 3.The dependence of the coefficient of the mutual correlationf $R_{\Delta t}$ between different series of the data on time Δt (standardized correlogramms).

Analysis of the mutual correlations does not indicate on the strongly dependence the irregular variations between different series of the data. The determinations even the faint connections (fig.3) demant the investigations of the existence of the reserved (latent) periodical components. For this purpose we used a two-channel autoregression analysis by a Nuttal-Strand algorithm (1976,1977).
The results of this analysis are given in fig.4.

A two-channel autoregression analysis indicates that low-period components are small amplitudes and that the correlations between the series of the data are marginally significance.

CONCLUSIONS

The irregular variations of the Earth rotation are caused by many phenomenons both the Earth's inner and the external origin.
We discussed only some of them.
We did not discover correlation between irregular variations of the analysed data.If correlation does exist it could very well be between the noise of analysed series data.

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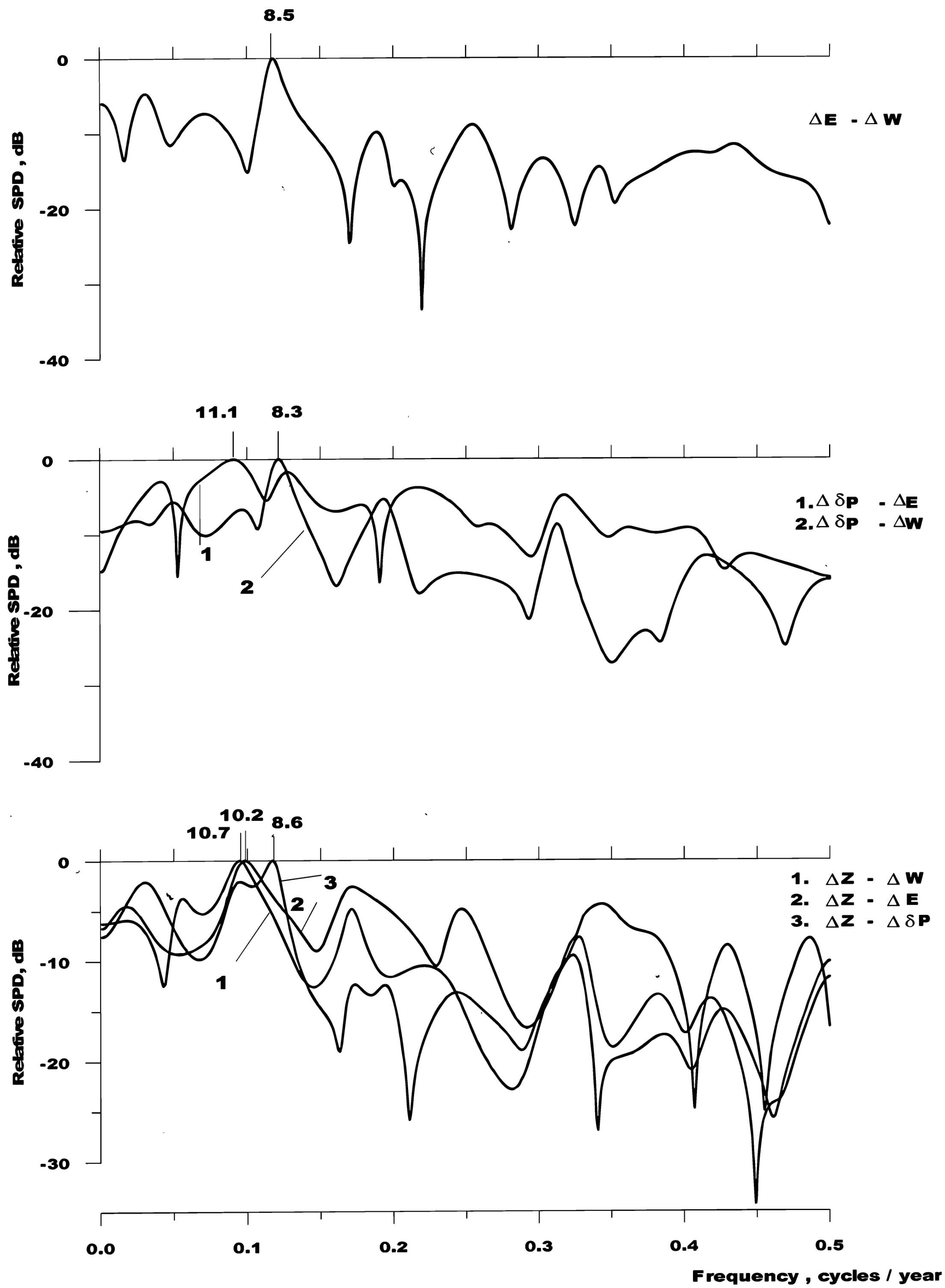

Figure 4. Two-channel autoregression analysis of different series of the data.

EARTH IRREGULAR ROTATION EFFECTS IN ARTIFICIAL SATELLITE ORBITS

V. MIOC and M. STAVINSCHI
Astronomical Institute of the Romanian Academy
Str. Cuţitul de Argint 5, RO-75212 Bucharest, Romania
e-mail: vmioc@roastro.astro.ro, magda@roastro.astro.ro

ABSTRACT. The motion of artificial satellites in low eccentric orbits perturbed by the Lense-Thirring effect is being studied. The Earth's rotational velocity is considered as being periodically changing. The corresponding relativistic first order perturbations for the orbital parameters of a satellite over one nodal period are determined. Resonant solutions are pointed out. The analytic results are valid not only for artificial satellites, but for all satellites orbiting other planets of the solar system.

1. INTRODUCTION

The mathematical model we tackle here is that of a test particle orbiting a rotating central body. As it is known since the second decade of the 20th century, considering the problem within a general relativistic framework, the rotation of the central mass generates a gravitomagnetic field that entails an *inertial frame dragging* effect undergone by the particle (Lense and Thirring 1918; Thirring 1918). A description of this relativistic phenomenon can be found *e.g.* in the textbooks of Misner et al. (1973), or Soffel (1989): the spacetime in the neighbourhood of the rotating central mass is influenced by this one as if it were immersed in a viscous fluid that transfers a part of its rotational energy to the surrounding medium by means of drag forces.

Under the action of the inertial frame dragging, the test particle will experience the so-called Lense-Thirring acceleration. In general, its effects on the particle motion are studied perturbatively; this means that the Lense-Thirring acceleration is considered to be a perturbing acceleration that alters the Keplerian motion in the Newtonian field of the central body.

This relativistic effect in the orbital motion was analytically studied by several authors, who dealt with the changes of the Keplerian orbital elements or of the nodal period (*e.g.* Soffel 1989; Pal et al. 1993, 1994; Stavinschi et al. 1996; Mioc and Stavinschi 1997). They found periodic variations for the majority of the parameters, and secular changes only for the longitude of ascending node (orbit precession) and the argument of pericentre (apsidal motion).

All the above quoted authors considered that the field-generating body rotates uniformly. A first generalization from this standpoint was performed by Mioc and Stavinschi

(1995), who tackled the case of a periodic variation of the rotational velocity of the central body, but only for initially circular orbits. They concretized their study to artificial satellites orbiting the Earth (of gravitational parameter μ and mean equatorial radius R_E, rotating with angular velocity ω_E) at geocentric distance r.

In this paper we resume and generalize this research for initial orbits of small (but nonzero) eccentricity. (This is the case for many artificial satellites, as for instance the GPS ones, but also for almost all planetary satellites in our solar system.) We shall characterize the relative motion of the particle (satellite) via the Keplerian orbital elements $\{y \in Y; u\}$,all time-dependent, where

$$Y := \{p, \Omega, i, q = e\cos\omega, k = e\sin\omega\},$$

and p = semilatus rectum, Ω = longitude of ascending node, i = inclination, e = eccentricity, ω = argument of pericentre, u = argument of latitude. Yet, we shall keep the notation R_E and ω_E, though our study covers much more astronomical situations than the artificial Earth satellite case.

The changes of the parameters $y \in Y$ over one nodal period of the particle will be analytically established under the following hypotheses:

(i) the rotational velocity ω_E of the central body varies periodically;

(ii) the parameters $y \in Y$ experience small variations over one revolution of the particle (this is likely, because the Lense-Thirring acceleration is a relativistic effect);

(iii) the initial orbit of the particle is low eccentric (namely in all expansions in powers of e the terms in e^n, $n \geq 2$, will be neglected).

As a last remark, taking also into account hypothesis (ii), our results will be of first order with respect to the small parameter characterizing the Lense-Thirring effect (see Section 3 below).

2. BASIC EQUATIONS

We have chosen the nodal period as basic time interval. Accordingly, let us describe the motion via the Newton-Euler equations written with respect to the argument of latitude (*cf.* Mioc and Stavinschi 1995, 1997):

$$\begin{aligned}
dp/du &= 2(Z/\mu)r^3T, \\
d\Omega/du &= (Z/\mu)r^3[B/(pD)]N, \\
di/du &= (Z/\mu)r^3(A/p)N, \\
dq/du &= (Z/\mu)\{r^2BR + r^2[r(q+A)/p + A]T + r^3[kBC/(pD)]N\}, \\
dk/du &= (Z/\mu)\{-r^2AR + r^2[r(k+B)/p + B]T - r^3[qBC/(pD)]N\}, \\
dt/du &= (Z/\sqrt{\mu p})r^2.
\end{aligned} \tag{1}$$

Here R, T, N are the radial, transverse, and binormal components of the perturbing acceleration, respectively, and we abridged

$$\begin{aligned}
(A, B) &: = (\cos, \sin)(u), \\
(C, D) &: = (\cos, \sin)(i);
\end{aligned}$$

$$Z := [1 - r^2C(d\Omega/dt)/\sqrt{\mu p}]^{-1}.$$

By virtue of the hypothesis (ii), the orbital elements $y \in Y$ may be considerd to be constant and equal to their initial values $y_0 = y(u_0) = y(u(t_0))$ in the right-hand side of equations (1), therefore these ones can be separately integrated. Consequently, after one nodal period, the parameters $y \in Y$ can be written as $y = y_0 + \Delta y$, with the first order changes (in a small parameter ε) changes Δy provided by

$$\Delta y = \int_0^{2\pi} (dy/du)_{y=y_0} du, \tag{2}$$

the respective integrands being yielded by equations (1). The small parameter ε (not specified yet) features the perturbing factor.

These changes, the main goal of the present paper, will be estimated by successive approximations, with $Z \approx 1$ (in the sequel we shall omit the factor Z in the equations of motion). Having in view the hypothesis (ii), the process will be limited to the first order approximation.

3. EQUATIONS OF MOTION IN THE GRAVITOMAGNETIC FIELD

The components of the Lense-Thirring acceleration have the following expressions (*e.g.* Soffel 1989):

$$\begin{aligned} R &= \alpha\mu h \omega_E C/r^4, \\ T &= -\alpha\mu h\omega_E Ce\sin v/(pr^3), \\ N &= \alpha\mu h\omega_E (D/r^4)[2B + (r/p)eA\sin v], \end{aligned} \tag{3}$$

where v = true anomaly, and $h := \sqrt{\mu p}$. We also abridged $\alpha := 2(\gamma+1)R_E^2/(5c^2)$, in which $\gamma \approx 1$ is the space curvature parameter, whereas c stands for the speed of light.

The periodic variation of ω_E entails a periodic character of the perturbation undergone by the satellite motion. The intricate character of this variation makes us resort to a Fourier series expansion in the expressions (3) (see also Mioc and Stavinschi 1995). Also, using the fact that $u = \omega + v$, as well as the expressions of q and k, formulae (3) become

$$\begin{aligned} R &= \mu hCS/r^4, \\ T &= -\mu hCS(Bq - Ak)/(pr^3), \\ N &= \mu hD(S/r^4)[2B + (r/p)(ABq - A^2k)], \end{aligned} \tag{4}$$

where the Fourier series expansion appears under the form

$$S := \sum_{n=0}^{\infty} [a_n \cos(n\nu u) + b_n \sin(n\nu u)]. \tag{5}$$

All coefficients a_n, b_n are small quantities; let us denote by ε (small parameter, too) their maximum order of magnitude.

Substituting (4) into (1), using the orbit equation written in polar coordinates $r = p/(1 + e\cos v)$ under the form

$$r = p/(1 + Aq + Bk), \tag{6}$$

and introducing the abbreviating notation $L := h/p^2 = \sqrt{\mu}/p^{3/2}$, the equations of motion acquire the form

$$dp/du = -2pLCS(Bq - Ak),$$

$$
\begin{aligned}
d\Omega/du &= LS[2B^2+3(A-A^3)q+(3B^3-B)k], \\
di/du &= LDS[2AB+3(B-B^3)q+(2A-3A^3)k], \\
dq/du &= LCS[B+2(1+B^2)k-Bq^2+3B^3k^2+(5A-3A^3)qk], \\
dk/du &= -LCS[A+2(1+B^2)q+(4A-3A^3)q^2-Ak^2+(B+3B^3)qk], \\
dt/du &= 1/[L(1+Aq+Bk)^2].
\end{aligned} \tag{7}
$$

Equations (7) represent the departure point for the determination of the orbital element changes.

4. VARIATIONS OF THE ORBITAL ELEMENTS. NONRESONANT CASE

According to the hypothesis (ii), $y=y_0$, $y \in Y$, in the right-hand side of equations (7). Also, by virtue of hypothesis (iii), we truncate these right-hand sides to first order in eccentricity (that is, equivalently, in q and k).

Replacing the resulting expressions in (**??**), the respective integrands will contain only explicit functions of u (through A, B, and S) and quantities considered constant over one revolution. The first order (in ε on the one hand, in q and k on the other hand) changes of the orbital elements $y \in Y$ become

$$
\begin{aligned}
\Delta p &= -2L_0p_0C_0(I_{01}q_0-I_{10}k_0), \\
\Delta\Omega &= L_0[2I_{02}+3(I_{10}-I_{30})q_0+(3I_{03}-I_{01})k_0], \\
\Delta i &= L_0D_0[2I_{11}+3(I_{01}-I_{03})q_0+(2I_{10}-3I_{30})k_0], \\
\Delta q &= L_0C_0[I_{01}+2(1+I_{02})k_0], \\
\Delta k &= -L_0C_0[I_{10}+2(1+I_{02})q_0],
\end{aligned} \tag{8}
$$

where, taking into account (5), we have abridged

$$
I_{ij} := \sum_{n=0}^{\infty}(a_n\widetilde{C}_{n,ij}+b_n\widetilde{S}_{n,ij}),
$$

and

$$
\begin{aligned}
\widetilde{C}_{n,ij} &:= \int_0^{2\pi}\cos(n\nu u)A^iB^j du, \\
\widetilde{S}_{n,ij} &:= \int_0^{2\pi}\sin(n\nu u)A^iB^j du.
\end{aligned}
$$

Performing the integrations, and denoting for sake of brevity

$$
\begin{aligned}
f_n &:= a_n\sin(2\pi n\nu)+b_n[1-\cos(2\pi n\nu)], \\
g_n &:= a_n[1-\cos(2\pi n\nu)]-b_n\sin(2\pi n\nu),
\end{aligned} \tag{9}
$$

we obtain

$$
\Delta p = -2L_0p_0C_0\sum_{n=0}^{\infty}\frac{g_nq_0+n\nu f_nk_0}{1-n^2\nu^2},
$$

$$
\begin{aligned}
\Delta\Omega &= L_0 \sum_{n=0}^{\infty} \left[\frac{4f_n}{n\nu(4-n^2\nu^2)} - \frac{6n\nu f_n q_0 - (9+n^2\nu^2) g_n k_0}{(1-n^2\nu^2)(9-n^2\nu^2)} \right], \\
\Delta i &= L_0 D_0 \sum_{n=0}^{\infty} \left[\frac{2g_n}{4-n^2\nu^2} + \frac{(3-n^2\nu^2)(3g_n q_0 + n\nu f_n k_0)}{(1-n^2\nu^2)(9-n^2\nu^2)} \right], \\
\Delta q &= L_0 C_0 \sum_{n=0}^{\infty} \left[\frac{g_n}{1-n^2\nu^2} + \frac{4f_n}{n\nu(4-n^2\nu^2)} k_0 \right], \\
\Delta k &= L_0 C_0 \sum_{n=0}^{\infty} \left[\frac{n\nu f_n}{1-n^2\nu^2} - \frac{4f_n}{n\nu(4-n^2\nu^2)} q_0 \right].
\end{aligned}
\tag{10}
$$

These expressions are the variations of the five independent parameters $y \in Y$ of a satellite in quasicircular orbit, over one nodal period, due to the considered variable rotation of the Earth.

5. VARIATIONS OF THE ORBITAL ELEMENTS. RESONANT CASE

Let us examine more closely the orbital element changes (10). As long as $n\nu \notin \{0,1,2,3\}$, the periodicity is obvious. But, in concrete astronomical situations, natural values of n (N_0, N_1, N_2, N_3, say) could likely exist such that $N_0\nu = 0$ ($N_0 = 0$), $N_1\nu = 1$, $N_2\nu = 2$, $N_3\nu = 3$, creating resonances. Separating the resonant terms ($n = N_j$, $j = \overline{0,3}$) in (5), substituting the resulting expressions in (7) and integrating (observing the hypotheses (ii) and (iii), of course), we get

$$
\Delta p = -2L_0 p_0 C_0 \left[\pi(b_{N_1} q_0 - a_{N_1} k_0) + \sum_{n=1, n\neq N_1}^{\infty} (g_n q_0 + n\nu f_n k_0)/(1-n^2\nu^2) \right],
$$

$$
\begin{gathered}
\Delta\Omega = L_0 \Big\{ \pi(2a_0 - a_{N_2}) + \textstyle\sum_{n=1,n\neq N_2}^{\infty} 4f_n/[n\nu(4-n^2\nu^2)] + \\
\Big[2\pi(-a_{N_1} + a_{N_3}) - \textstyle\sum_{n=1,n\neq N_1,N_3}^{\infty} 6n\nu f_n/[(1-n^2\nu^2)(9-n^2\nu^2)] \Big] q_0 + \\
\Big[(\pi/4)(5b_{N_1} - 3b_{N_3}) + \textstyle\sum_{n=1,n\neq N_1,N_3}^{\infty} (9+n^2\nu^2) g_n/[(1-n^2\nu^2)(9-n^2\nu^2)] \Big] k_0 \Big\},
\end{gathered}
$$

$$
\begin{gathered}
\Delta i = L_0 D_0 \Big\{ \pi b_{N_2} + \textstyle\sum_{n=1,n\neq N_2}^{\infty} 2g_n/(4-n^2\nu^2) + \\
\Big[(3\pi/4)(b_{N_1} + b_{N_3}) + \textstyle\sum_{n=1,n\neq N_1,N_3}^{\infty} 3(3-n^2\nu^2) g_n/[(1-n^2\nu^2)(9-n^2\nu^2)] \Big] q_0 + \\
\Big[(-\pi/4)(a_{N_1} + 3a_{N_3}) + \textstyle\sum_{n=1,n\neq N_1,N_3}^{\infty} n\nu(3-n^2\nu^2) f_n/[(1-n^2\nu^2)(9-n^2\nu^2)] \Big] k_0 \Big\},
\end{gathered}
\tag{11}
$$

$$
\begin{gathered}
\Delta q = L_0 C_0 \Big\{ \pi b_{N_1} + \textstyle\sum_{n=1,n\neq N_1}^{\infty} g_n/(1-n^2\nu^2) + \\
\Big[2 + \pi(2a_0 - a_{N_2}) + \textstyle\sum_{n=1,n\neq N_2}^{\infty} 4f_n/[n\nu(4-n^2\nu^2)] \Big] k_0 \Big\},
\end{gathered}
$$

$$
\begin{gathered}
\Delta k = L_0 C_0 \Big\{ \pi a_{N_1} + \textstyle\sum_{n=1,n\neq N_1}^{\infty} n\nu f_n/(1-n^2\nu^2) - \\
\Big[2 + \pi(2a_0 - a_{N_2}) + \textstyle\sum_{n=1,n\neq N_2}^{\infty} 4f_n/[n\nu(4-n^2\nu^2)] \Big] q_0 \Big\}.
\end{gathered}
$$

These expressions of the orbital element changes are equivalent to (10), but they put into evidence the resonant terms.

6. CONCLUDING REMARKS

Examining our results (11), which are the most likely, we can formulate some concluding remarks (bearing in mind that these results are valid to first order in the small parameter ε, on the one hand, and in the eccentricity e, on the other hand). After one nodal period, the corresponding osculating orbit will be featured by:

- changes in the semilatus rectum: zero for initially polar orbits, more and more pronounced as the initial inclination is closer to zero (being however far enough from the equatorial case to have the node well defined);
- changes in the longitude of ascending node (precession of the orbit);
- changes in the inclination: maximum for initially polar orbits, and monotonically decreasing as the initial orbit plane is closer to the equatorial plane;
- orbit deformation: for satellites in initially polar orbits the eccentricity variation is zero; closer to the equatorial plane is the initial orbit, greater the eccentricity variation will be;
- apsidal motion: the pericentre shift observes the same conditions as the orbit deformation above.

All these variations (over one nodal period) contain both secular and periodic terms. Of course, for the iterative algorithms (which consider the orbital parameters after one nodal period as initial conditions for the next iteration), only the secular terms are important. Further numerical studies will deal with the stability time-scate for such resonant trajectories.

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IRREGULAR VARIATIONS IN EARTH ROTATION: THE SINGULAR SPECTRUM ANALYSIS AND WAVELETS

V.V. VITYAZEV
Sobolev Astronomical Institute, St.-Petersburg University
198904, St.-Petersburg, Petrodvorets, Bibliotechnaya pl.2, Russia
vityazev@venvi.usr.pu.ru

ABSTRACT

The paper deals with estimation of the parameters of the irregular signals in the EOP. Three techniques are applied to the analysis of the Chandlerian and annual components of the Polar motion in the time series reduced to the system of the HIPPARCOS catalogue. They are: the FOURIER TRANSFORM, the WAVELET TRANSFORM and the Singular Spectrum Analysis (the "CATERPILLAR" routine). Of all the three, it is only the "CATERPILLAR" that permits estimating of the INSTANTANEOUS values of the periods. The time-dependent phases and the time-dependent periods of the Chandlerian and annual wobbles are estimated with the time resolution 1 year.

1. INTRODUCTION

The longer the EOP time series become the more evidence we have that their components are not regular. It is a common practice to describe the irregularities of, say, the Chandlerian wobble with the help of the harmonic functions with amplitude and period (phase) depending on time. If the Fourier transform is applied to all the data available the resulting spectrum displays all the frequencies that are specific to the wobble within the whole time span without indication when the change of the frequency had occured. To confirm this the Fig.1 presents the Fourier spectra of the Chandlerin component in the x- and y-coordinate of the polar motion within the time span 1900-1992 in the system of the HIPPARCOS catalogue (Vondrak at al., 1997). To evaluate the "instantaneous" values of these parameters one must use as short pieces of the time series as possible, and in the case of the Fourier transform this yields very poor frequential resolution. The same fault shares the wavelet analysis (Vityazev, 1997), so to get the reasonable frequential resolution one must take the effective time span over which the wavelets operate as long as 10-12 yr. (Fig.1). The main purpose of this paper is to show that there exists a method which is able to solve the problem of the instantaneous determination of the irregular signal parameters.

2. THE CATERPILLAR METHOD

This method (D.Danilov, A.Zhiglyavsky, 1998; http://vega.math.spbu.ru/caterpillar/ru) belongs to the SINGULAR SPECTRUM ANALYSIS technique. The short outline of the method is given below.

Given is a one-dimensional time series

$x_i = f(\Delta t(i-1)),\ i = 1, ..., N,$

from which the M-dimensional time series (the 'caterpillar' matrix) is formed according to

$$X = (x_{ij})_{i,j=1}^{k,M} = \begin{pmatrix} x_1 & x_2 & x_3 & \dots & x_M \\ x_2 & x_3 & x_4 & \dots & x_{M+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_k & x_2 & x_{k+1} & \dots & x_N \end{pmatrix},$$

where M – a free parameter (the length of the 'caterpillar') and $k = N - M + 1$. At the next step the correlation matrix is formed

$$R = \frac{1}{k} X^*(X^*)^T,$$

and the eigenvalues Λ and the eigenvectors P are calculated

$$R = P\Lambda P^T.$$

After that the principal components of the time series under consideration may be found

$$X^*P = Y = (y_1, y_2, ..., y_M),$$

and the relative input of a principal component y_i to the initial signal is estimated by the ratio λ_i/M.

The restoration or filtering of the inital time series is possible, if all or only needed principal components are taken into account in the following equation

$$X^* = YP^T.$$

3. THE PRINCIPAL COMPONENTS

For each regular signal there exists a pair of two equal eigenvalues and a pair of the sine- and cosine-type principal components. For the irregular signal we suppose that the principal components may be represented as follows

$$c(t) = a(t)\cos\Omega(t),$$

$$s(t) = b(t)\sin\Omega(t).$$

The squared values of them are

$$c^2(t) = \frac{a^2(t)}{2}(1 + \cos 2\Omega(t)),$$

$$s^2(t) = \frac{b^2(t)}{2}(1 + \sin 2\Omega(t)),$$

from which we get

$$a(t) = \sqrt{2 < c^2(t) >},$$
$$b(t) = \sqrt{2 < s^2(t) >},$$

where $< . >$ denotes the operator of low frequency filter.
Now we may evaluate

$$\Omega(t) = artan \frac{s(t)}{c(t)} \frac{a(t)}{b(t)}.$$

For regular signal with $\omega_0 = const$ and $\phi_0 = const$ we get well known representation

$$\Omega(t) = \omega_0 t + \phi_0.$$

In the case of irregular wobble we may use two approaches. The first one implies that $\phi_0 = const$ but the frequency is the time-dependent function. In this case we have

$$\Omega(t) = \int_0^t \omega(t) dt,$$

from which one finds

$$\omega(t) = \frac{d\Omega(t)}{dt}.$$

In the second approach we may consider that $\omega_0 = const$ but the phase is the time-dependent function. This yields the equation

$$\Omega(t) = \omega_0 t + \phi_0 + \Delta\phi(t).$$

Subtracting the linear trend from $\Omega(t)$ we obtain the phase variations

$$\Delta\phi(t) = \Omega(t) - \omega_0 t - \phi_0.$$

Both approaches are much the same. Really, suppose

$$\omega(t) = \omega_0 + \Delta\omega(t),$$

then

$$\Omega(t) = \omega_0 t + \phi(t),$$

where

$$\phi(t) = \phi_0 + \Delta\phi(t) = \int_0^t \Delta\omega(t) dt.$$

and

$$\Delta\omega(t) = \frac{d\phi(t)}{dt}.$$

4. RESULTS AND CONCLUSIONS

We applied the CATERPILLAR technique to study the x- and y-coordinates (Vondrak et al., 1997) of the polar motion reduced to the time interval 0.1 yr. The length of the "caterpillar" was taken $M = 60$. The method produced two principal components which correspond to the Chandlerian and to the annual wobbles. The time-dependent phases and the time-dependent periods of both the wobbles were derived (Figs.2-3). In the first approach the periods of the chandlerian and annual components (averaged over x- and y-components) turned out to be 428.699 ± 0.091 and 365.393 ± 0.015 days correspondingly. The intercomparison between the three techniques applied to the time series under consideration leads us to the following conclusions.

- The FOURIER TRANSFORM displays all features of the irregular signal but does not localize them in the time domain.
- The WAVELET TRANSFORM does localize the features of the signal in the time domain but yields poor resolution in the frequential domain. To achieve reasonable resolution one has to increase the time span over which the WV operates. This prevents us to see the variance of the Chandlerin wobble parameters with the time resolution less than 6-10 years.
- The "CATERPILLAR" routine permits determining of the "instantaneous" characteristics of the irregular wobbles. Theoretically, the time resolution may reach the temporal interval of the time grid on which the time series under consideration is given. In practice, due to presence of noise, the time resolution of the "CATERPILLAR" routine may be worse. We have found that dependence of the Chandler frequency on time can be traced with the time resolution about 1 year, which is better than the ability of the wavelet analysis. That is why the "caterpillar" plots of the time-dependent parameters display much more details than the corresponding wavelet plots.

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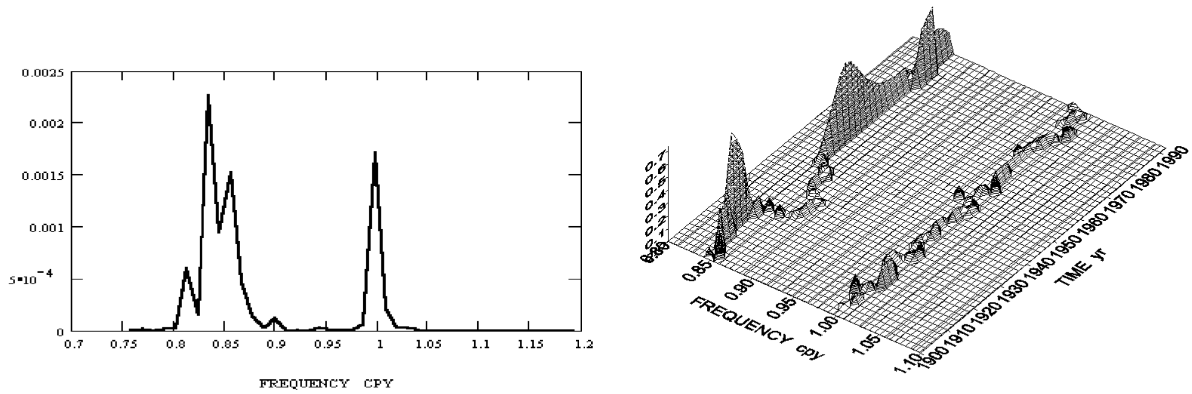

Figure 1: Fourier spectrum (left) and the wavelet spectrum (right) averaged over x- and y-components of the polar motion.

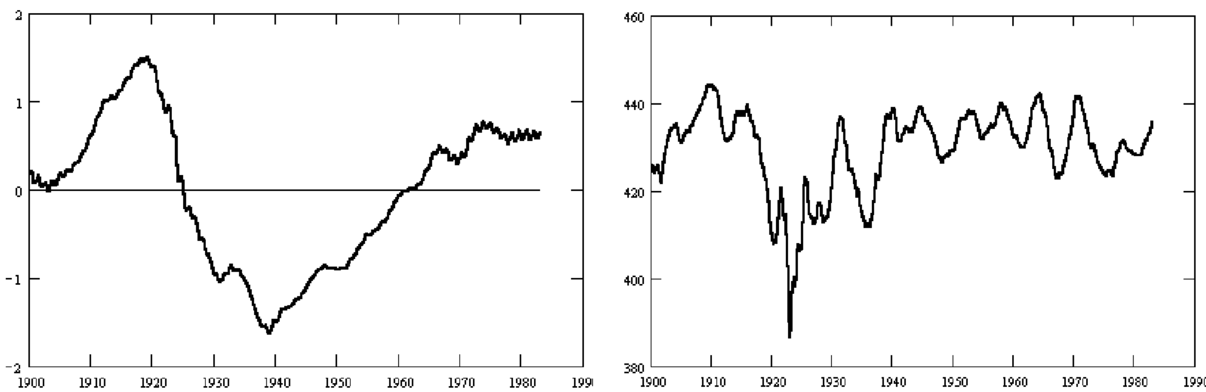

Figure 2: The time dependent phase (left) and the time dependent period (right) of the chandlerian component averaged over x- and y-components of the polar motion.

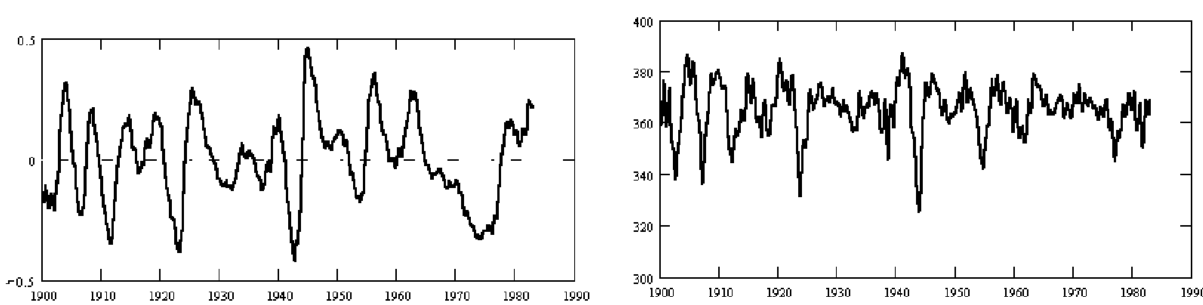

Figure 3: The time dependent phase (left) and the time dependent period (right) of the annual component averaged over x- and y-components of the polar motion.

COMPARISON AND COMBINATION OF LAGEOS SLR AND VLBI ESTIMATIONS FOR POLE MOTION AND UNIVERSAL TIME

P. YAYA
DANOF/UMR8630-CNRS, Observatoire de Paris
61 avenue de l'Observatoire 75014 Paris - FRANCE
e-mail: philippe.yaya@obspm.fr

ABSTRACT. Solutions for pole motion and universal time have been obtained with the CNES (Centre National d'Etudes Spatiales) orbitography software from two different set of data : SLR and VLBI observations. A new method of combination, which implies a mixing of the normal equations of the problem, have been tested and the different solutions - individual and combined - have been analyzed through their internal consistency and compared to the IERS reference series EOPC04 (McCarthy, 1996). The improvement or degradation of the combined solutions are discussed.

1. INTRODUCTION

Different modern techniques are able to provide the Earth Orientation Parameters (EOP). Each one has its own qualities and failings. For example GPS gives good results for the short period components of nutation and DORIS is much more useful for the realization of the terrestrial frame. In this preliminary work we have used data from two distinct techniques : Very Long Baseline Interferometry (VLBI) and Satellite Laser Ranging (SLR). The latter gives good results for the components of the pole (X and Y) at 3-day interval but is not really interesting for UT1. On the contrary, VLBI gives rather excellent results for the whole EOPs at 7-days interval. A combination of both techniques should then improve each individual result. Indeed, the goal of a combination is to take the best of each technique in order to improve the final solution. The combination have been taken with the CNES software called GINS, initially dedicated to the analyses of satellite data. In such a combination technique the observations are "mixed" together in the processing as compared to the classical combination in which the results of each technique are combined.

2. THE OBSERVATIONS

2.1 SLR data

The one month SLR data are taken from LAGEOS 1 & 2 observations. The source is the CDDIS and they range from 5 September 1999 to 5 October 1999. We have analyzed approximately 6000 observations. As for SLR stations, they present an unhomogeneous distribution on Earth surface : 25 stations over 28 are distributed in the North hemisphere. That should introduce bias in the processing of the data.

2.2 VLBI data

The source is the GSFC and cover the same period. The main difference with SLR data is the repartition of the observations : VLBI data are not homogeneous as they consist of four 24-hour sessions, with a one week gap between each of them. The number of observation is half time less numerous than SLR ones (3000). and the distribution of the 10 VLBI stations used in that work is better than for SLR.

3. PROCESSING WITH GINS SOFTWARE

GINS was developed at CNES (the French spatial agency, in Toulouse) about twenty years ago. It was initially dedicated to orbitography and still give satellites orbits, as well as EOP and other parameters which appear in the models, after a least-square processing of the data. New routines have been implemented in the software in 1999 in order to process VLBI data (Meyer *et al.*, 2000). Although there are no orbit integration for VLBI, the first results seemed to be encouraging in comparison with the JPL MODEST software.

Our software uses a classical least-square method to get the parameters, which are estimated by minimizing the differences (the residuals) between the observed and the calculated quantities. For SLR those quantities are the distances between a station and the satellite (in fact a time, changing in distance using a precise model of wave propagation). As for VLBI, the equivalent quantities are the time delays between two station observations of a same quasar.

The combination technique used in GINS software is different from other classical ones which combine the individual solution given by each technique. GINS combines more upstream : the normal equations are mixed before inversing the final matrix. We obtain then a unique solution. This method has already given good results for the processing of several satellite arcs for example. But it is still under experiment for the estimation of EOP (this paper is one step of this work).

4. RESULTS AND ANALYSIS

The processing of SLR observations with GINS has proven itself for years at GRGS. The orbits residuals obtained for the test period of this paper have a root mean square (RMS) of about 2cm for a 6-day interval, which is a common value for that kind of measurements. As for the VLBI, the usual value for the residuals (obtained with MODEST (JPL) or GLORIA (Gontier, 1992) is about 40 picoseconds (ps) for a 1-day estimation. Our processing leads to a delay residuals RMS between 160 and 220 picoseconds (ps), depending of the arc. We have then a RMS up to 5 time larger than the classical value. One of the main differences between GINS and the other software is the number of tropospheric parameters daily estimated. We only estimate one parameter as we should estimate at least 4. But the following results will prove that even with that kind of processing we can obtain good results.

To obtain figure 1 (showing the daily EOP differences) we have removed an offset (about 1 milliarcsecond (mas)) for the SLR solutions but none for VLBI, whose offset was

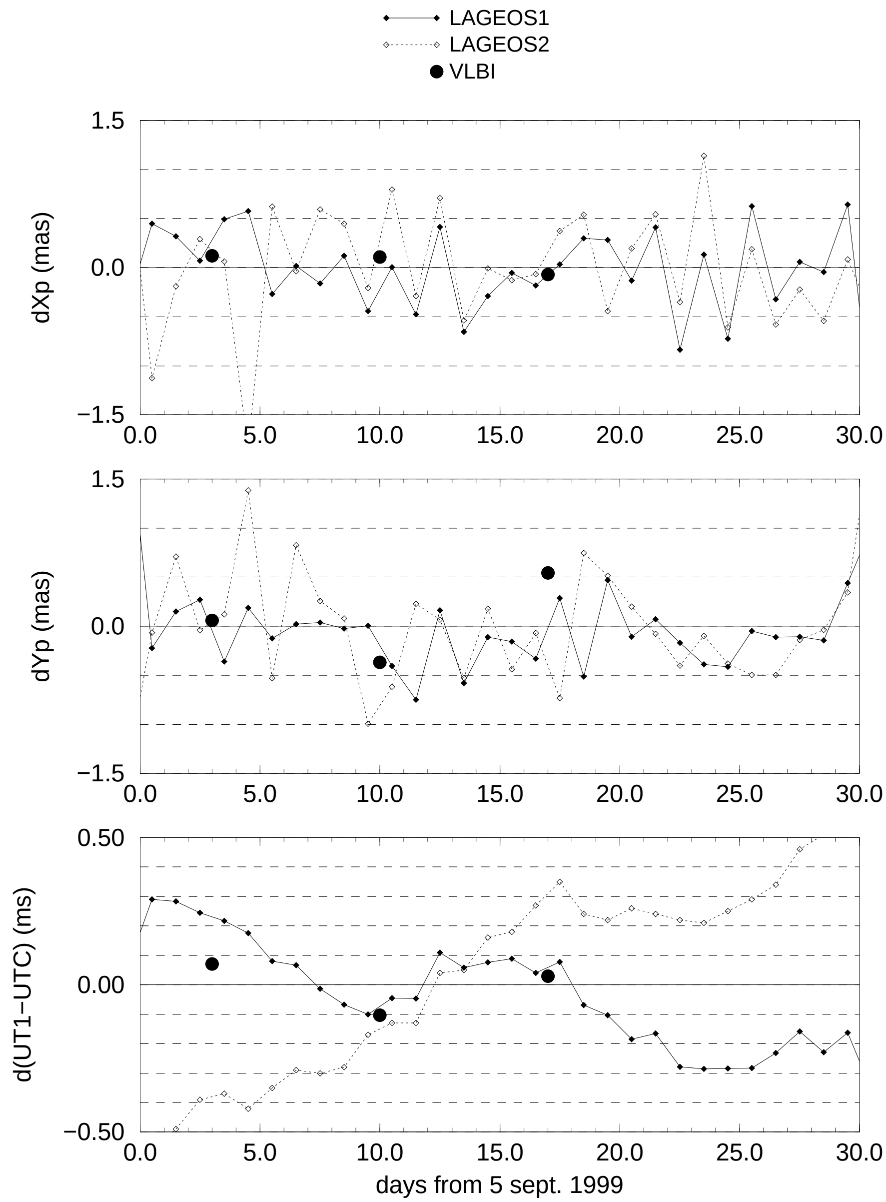

Figure 1: EOP differences between the IERS EOPC04 series and LAGEOS/VLBI solution series

Table 1: Comparison with EOP C04 (after removing offsets)

		LAGEOS1	LAGEOS1+2	LAGEOS1+2 + VLBI
	X(mas)	0.46	0.44	0.21
RMS	Y(mas)	0.49	0.66	0.49
	UT(mas)	3.87	0.11	0.06
formal	X(mas)	0.98	0.79	0.12
errors	Y(mas)	1.17	1.58	0.13
	UT(mas)	18.24	19.14	0.01

negligeable. Thus it confirms the very good stability of VLBI data and explains why the solutions based on them are used to build an absolute reference frame. The figure shows then the dispersion of the residuals, whose RMS are written down in table 1, as well as the RMS of the combined solutions.

Concerning the pole there is improvement for the X component as the RMS is divided by a factor two : it decreases from 0.46 mas for LAGEOS1 solution to 0.21 mas for LAGEOS+VLBI solution. The Y component does not present an equivalent improvement, probably due to LAGEOS2 observations, as the its residuals seems have great values at the beginning of the period (see figure 1). The explanation is a lack of data at this moment. But it is interesting to see that adding the VLBI serie decreases the RMS for both component.

As for the UT component the combination shows here its advantage : combining LAGEOS1 and LAGEOS2, the RMS decreases consequently from about 4 mas to 0.1 mas. This is due to the the fact that LAGEOS1 et LAGEOS2 have complementaries orbits, it is obvious on figure1.

5. CONCLUSION

This preliminary work on the combination of SLR and VLBI data gives encouraging results as the residuals of the combined serie with respect to the IERS serie are lower than the individual ones. The succes of the combination results from the complementarity of LAGEOS1 and LAGEOS2 orbits and the inertial stability of VLBI frame. However some improvements could be added, such as a next processing of the data weight in order to have lower residuals for LAGEOS2. Another interesting point would be the estimation of other tropospheric parameters in order to decrease the delay residuals of VLBI observations. Anyway this work leads to optimistic prospects concerning the combination of others techniques (LLR, GPS or DORIS)

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NONLINEAR EFFECTS DETECTION IN EOP SERIES

O. KUDLAY
Main Astronomical Observatory of National Academy of Sciences of Ukraine
03680, Golosiiv,Kyiv-127,Ukraine
e-mail: kudlay@mao.kiev.ua

ABSTRACT. Irregularity of the Earth Orientation Parameters (EOP) series is considered as a result of nonlinear effects influence. To prove the hypothesis we apply system analysis technique–Volterra-Wiener-Korenberg method to EOP series as to a "black box " and demonstrate that Universal time and the length of the day are really chaotic values on long time interval, and Atmospheric Angular Momentum – marginally chaotic one.

1. THE ANALYSIS

Irregularity of the polar motion and Universal time data is well known perhaps all the time they studied. Spectral analysis revealed that EOP data have complicated continuous spectra which traditionally are interpreted as a sum of the finite number of harmonics followed from linear theory of the Earth rotation plus observation noise. But continuous spectrum excitation is very hard problem for linear approach which was not resolved till nowadays.

Here we investigate the hypothesis that irregular EOP variations are the sum of deterministic chaotic component and observation noise which form together with proper and forced oscillations spectra observed in the Earth rotation. The origin of the chaotic component should be revealed during the process. To test the hypothesis we should apply method that can be able to work with short time series about N=1000 points – usual length of the EOP data. As such we took Volterra-Wiener method, recently modified by Korenberg [1] – technique of system analysis that is new for chaotic component detection not only for EOP data case.

Consider after [1] the usual description of dynamical system (in our case Liouville equations) as a "black box" with input x_n and output y_n at time n=1, ..., N with sampling time τ. Volterra series [4] is a Taylor-like polynomial expansion of y_n in terms of $x_n, x_{n-1}, \ldots, x_{n-k+1}$, where k is the memory of the system. For the autonomous dynamical systems, as Liouville system is, a close-loop version when output y_n feeds back as delayed input consider than. So, discrete Volterra-Wiener-Korenberg (VWK) series of degree d and memory k is constructed to calculate the predicted time series y_n^{calc}:

$$y_n^{calc} = \qquad a_0 + a_1 y_{n-1} + a_2 y_{n-2} + \ldots + a_k y_{n-k} +$$

$$a_{k+1}y_{n-1}^2 + \ldots + a_{k+2}y_{n-1}y_{n-2} + a_{M-1}y_{n-k}^d \tag{1}$$

where all distinct combinations of $(y_{n-1}, y_{n-2}, \ldots, y_{n-k})$ up to degree d is composed, and total terms number $M = (k+d)!(d!k!)$.

Coefficients a_m are recursively estimated through a Gramm-Shmidt procedure from linear and nonlinear autocorrelations of data series itself.

One-step-ahead prediction power of a model is measured by error:

$$\varepsilon^2(k,d) = \sum_{n-1}^{N}(y_n^{calc}(k,d) - y_n)^2 / \sum_{n-1}^{N}(y_n - \bar{y})^2$$

where $\bar{y} = 1/N \sum_{n-1}^{N} y_n$ and $\varepsilon^2(k,d)$ is - normalized variance of error residuals. The best model $\{k_{opt}, d_{opt}\}$ minimizes the following information criterion

$$C(r) = \log \varepsilon(r) + r/N \tag{2}$$

where r- number of polynomial terms of truncated Volterra expansion for a certain pair {k,d}.

So, for each data series the best linear model is obtained by searching k^{lin} which minimizes C(r) with d=1. To obtain the best nonlinear model, procedure is repeated with increasing k and $d > 1$. A standard F-test will serve to reject, with a certain level of confidence, the hypothesis that nonlinear model are no better than linear models as one-step-ahead predictors. Due to the paper size limitation for other mathematical details please see references in [1].

The primary objective we tend in the work is to test the EOP series that are longest in the time. So, next EOP series were used for analysis:

Table 1.

IERS EOPC01	Normal values of EOP at 0.05 year intervals (Vondrak solution [2] 1900-1961 from optical astrometry and modern LOD data).
Stephenson & Morrison 1623.5-1997.5 (S&M)	Mean values of the duration of the day [2]
TDT-UT, LOD (M&B)	McCarthy & Babcock [3] data 1657-1984.5
aam.ncep. reanalysis	NCEP reanalysis data for Atmospheric Angular Momentum
IPMS (Y&Y)	Yumi & Yokoyama polar motion data 1899.9-1979.0 from optical astrometry [5]

When it was possible the standard data length N=1024 was taken to compare results with more extended AAM series . Table 2 contains calculation results for above mentioned series (Fisher criterion values are F(0.01)=1.17, F(0.05) $\approx$ 1.11) .

Main results of the work - is the confident, statistically significant on 99% level of confidence identification universal time and duration of the day as nonlinear chaotic processes. Volterra–Wiener–Korenberg method robustly works even on super short series with high level of observational noise as S&M (N=355) and M&B (N=655).

Table 2 demonstrate that both series give comparable results with big confidence. It means that long-term axial rotation of the Earth is chaotic value.

Table 2.

series	linear	nonlinear	$\varepsilon^2_{lin}/\varepsilon^2_{nlin}$	F(0.01)(F(0.05))
M&S	-2.9002	-3.212296	1.869218	1.31 (1.20)
TDT-UT M&B	-2.6624	-2.86755	1.50727	1.28 (1.20)
LOD M&B	-1.79216	-2.03362	1.6208	1.28 (1.20)
LOD EOPC01	-1.19	-1.05	0.75578	1.20 (1.12)
AAM-Y wind	-0.61591	-0.67683	1.12957	1.20 (1.12)
AAM-Y pressure	-0.69666	-0.74463	1.10069	1.20 (1.12)
AAM-X pressure	-0.41094	-0.449858	1.10442	1.20 (1.12)
Vondrak polar motion	-1.16248	-1.09749	0.94409	1.20 (1.12)
Y&Y polar motion	-2.75871	-2.78457	1.00937	1.20 (1.12)

Well known that atmosphere is the main excitation source for LOD and UT variation. So, it would be natural to study Atmospheric Angular Momentum series. The same analysis as for S&M and M&B series but with N=1024 points really revealed resemble but not identical picture. Only y-component of AAM-wind value occurred statistically significant, and x-; y-AAM pressure terms - marginally significant. Any z-AAM component is not significant. So, the reason of long-term chaotic behaviour of the Earth axial rotation rather is its proper unstable motion and may be interaction with liquid core.

Polar motion series IPMS and EOPC01 demonstrate wholly linear predictability and it is a little strange thing. On the other hand there exist interesting phenomenon: AAM and polar motion dependence on y-coordinate.

2. CONCLUSIONS

Volterra–Wiener–Korenberg method – powerful and robust technique, capable to detect chaotic nonlinear phenomena in noisy, super short series. It's application to EOP and AAM parameters revealed:

1. **Axial rotation of the Earth (universal time and length of the day data) on the long time intervals is the chaotic phenomenon.** More accurate but shorter in time new techniques observations didn't reveal the effect.

2. AAM-wind y-component confidently and AAM-pressure x- and y-components marginally are chaotic values also. Z-component is regular one.

3. Polar motion is regular value on the interval of optic observations.

From fruitful discussion with Dr. V.Frede during Journees 2000 I learned that she obtained similar results [6],[7] applying last modification of traditional Lyapunov exponents and some other techniques to VLBI and SLR data , so proved that in the high frequency domain as EOP and AAM data are chaotic . Some differences in our results must be explored (first of all my data length limitation) but the fact that two authors independently applying different mathematical methods obtaine similar results for opposite spectral domaines assures that chaotic component in Earth Orientation Parameters series really exists .

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PREDICTION OF EARTH ORIENTATION PARAMETERS BY ARTIFICIAL NEURAL NETWORKS

H. SCHUH
Inst. für Geodäsie und Geophysik
Technische Universität Wien
Gusshausstr. 27-29, 1040 Wien, Austria
hschuh@luna.tuwien.ac.at

M. ULRICH
Lehrstuhl für Photogr. u. Fernerkundung
Technische Universität München
Arcisstraße 21, 80290 München, Germany
ulrich@photo.verm.tu-muenchen.de

ABSTRACT. The Earth orientation parameters (EOP) were effects which can be described by functional models, e.g. influences of the solid Earth tides and the ocean tides or seasonal atmospheric variations of the EOP were removed. Only the differences between the modeled and the observed EOP, i.e. the quasi-periodic and irregular variations, were used for training and prediction. The results of the prediction were analyzed and compared with those obtained by other methods. The accuracy of the prediction is equal or even better than by other prediction methods.

1. PREDICTING EARTH ORIENTATION BY NEURAL NETWORKS

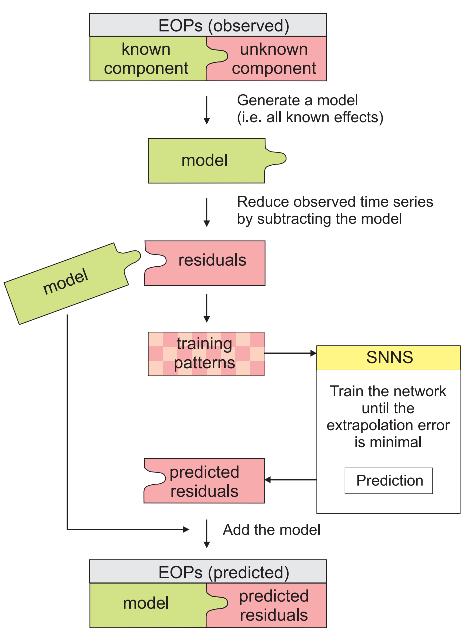

Figure 1: The neural network approach for EOP prediction

For the prediction of the EOP in terms of polar motion and length of day (LOD), UT1-UTC respectively, a procedure was applied as described in the flow chart of figure 1. The observed EOP can be split up into a first part, which is known rather well because a functional model exists and a second, unknown, stochastic part. The known component called *a-priori model* consists of periodic effects like influences of the solid Earth tides and the ocean tides on the EOP and seasonal variations. The a-priori model of polar motion additionally contains the Chandler Wobble (CW). After reduction of the observed time series by the a-priori model, training patterns were formed out of the residuals. These patterns were used for training the neural network until the extrapolation error became a minimum. The subsequently predicted residuals were then added to the a-priori model to obtain the predicted values of the EOP. The Stuttgart Neural Network Simulator (SNNS) (Zell, 1994) which is a very powerful software tool developed at the University of Stuttgart was applied to construct and to validate different types of neural networks in order to find the optimal topology of the net, the most economical learning algorithm and the best procedure to feed the net with data patterns.

EOP series from various sources, e.g. the C04 series from the International Earth Rotation Service (IERS) and the re-analyses optical astrometry series based on the Hipparcos frame served for training the neural network for both short-term and long-term prediction. The study is described in detail by Ulrich (2000) and Schuh et al. (2001) where the results of short-term and long-term prediction are shown and further references are given.

2. PREDICTION ERRORS AND COMPARISON WITH OTHER METHODS

A comparison of the results with other prediction methods is given in figures 2 and 3. The accuracy of the prediction of polar motion and of the short-term prediction of UT1-UTC is equal to the best prediction methods found in the literature. Mean-term prediction beyond 100 days is even substantially better than the results of other methods.

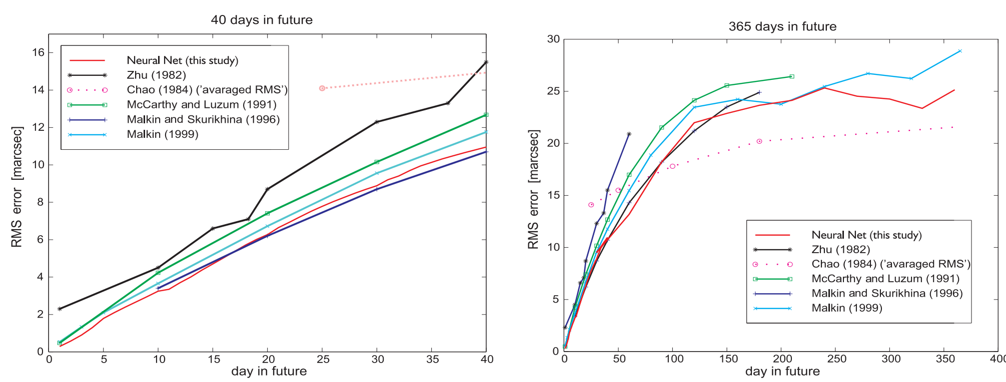

Figure 2: RMS errors of prediction of polar motion; the RMS errors given by Chao (1984) are averaged values, i.e. the value of the nth prediction day represents the mean value of the prediction days 1 to n.

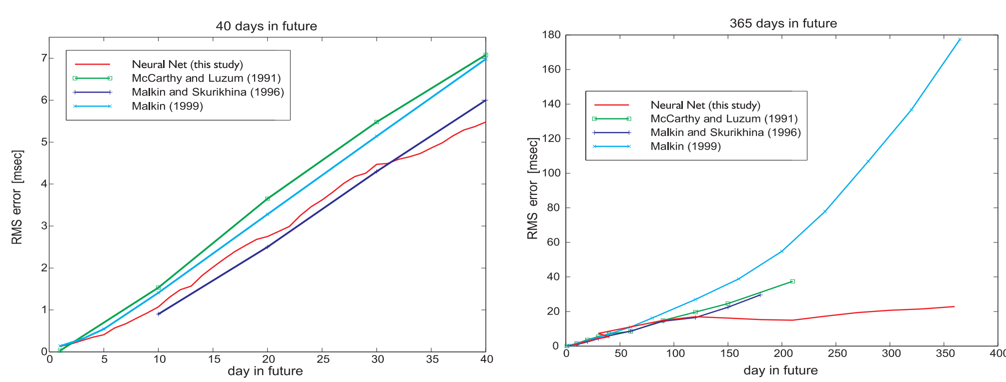

Figure 3: RMS errors of predicion of UT1-UTC

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POSTFACE

JOURNÉES 2001 SYSTÈMES DE RÉFÉRENCE SPATIO-TEMPORELS

"Influence of geophysics, time and space reference frames on Earth rotation studies"

Brussels, 24-26 September 2001

Scientific Organizing Committee

V. Dehant, Belgium (Chair); A. Brzeziński, Poland; N. Capitaine, France; M. Soffel, Germany; J. Vondrák, Czech R.; Ya. Yatskiv, Ukraine

Local Organizing Committee

A. Berger (UCL), C. Bruyninx, P. Defraigne, V. Dehant, O. de Viron, J. Jean, P. Pâquet, F. Roosbeek, B. Sépulchre (G. Lemaître Prize), T. Van Hoolst, R. Warnant

The Journées 2001 "Systèmes de référence spatio-temporels" (JSR) with the sub-title "Influence of geophysics, time and space reference frames on Earth rotation studies" will be organised in Brussels jointly by the Royal Observatory of Belgium, the Institut d'Astronomie et de Géophysique G. Lemaître from the Catholic University of Louvain (UCL) and the Observatoire de Paris. They will be held on 24, 25 and 26 September 2001. These Journées will be the thirteenth conferences in this series of which the purpose is to discuss the problems, from the concepts and realizations of space and time reference systems, to the scientific interpretations of precise observations referred to these systems.

The problems to be discussed in 2001 are related to various geophysical effects on Earth rotation such as geophysical fluids, tides, oceanic and atmospheric perturbations, as well as to the effects of time and space reference frames, such as time transfer, realisation of time scales and terrestrial and celestial frames. This also includes the problem of sensitivity of the different techniques of observation of Earth rotation. Earth rotation studies are also extended to other planets. The programme of the Journées 2001 will include the five following sessions, half-a-day each; for each session one or two members of the LOC are in charge of the local organisation; their names are given at the end of the proposed sessions :

Session I : Influence of geophysical and other effects on Earth's orientation; sensitivity of the observing systems (O. de Viron, P. Defraigne).

Session II : Ephemeris and dynamical reference systems (F. Roosbeek)

Session III : Time and time transfer (P. Defraigne, C. Bruyninx)

Session IV : Local, regional and global terrestrial frames, station positions and their interpretation; influence of the geophysical fluids, tidal, ocean and atmospheric effects (C. Bruyninx, R. Warnant)

Session V : Geodesy and rotation of the other planets (T. Van Hoolst, V. Dehant)

At this occasion the G. Lemaître Prize, of the University of Louvain, will be awarded to a geophysicist; a special session will be organised in Louvain on September 25 (lecture by the winner and JSR evening dinner).

Contact : P. Pâquet, Observatoire Royal de Belgique, avenue Circulaire 3, 1180 Bruxelles, Belgium, Fax: 32-2-374.98.22 at the attention of P. Pâquet, e-mail: Paul.Pâquet@oma.be

Or see : http://www.astro.oma.be/JSR2001/index.html