Enhanced term of order G^3 in the time transfer function: discussion for solar system experiments

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Journées Systèmes de Référence 2013,
16-18 September 2013, Paris

Introduction

Knowing the light travel time $t_B - t_A$ between $\mathbf{x}_A$ and $\mathbf{x}_B$ as a "reception time transfer function"

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See also data from INPOP13a (Verma *et al* 2013).

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- light rays are quasi-Minkowskian null geodesics:

$$\Longrightarrow \quad \mathcal{T}_r(\mathbf{x}_A, t_B, \mathbf{x}_B) = \frac{|\mathbf{x}_B - \mathbf{x}_A|}{c} + \sum_{n=1}^{\infty} \mathcal{T}_r^{(n)}(\mathbf{x}_A, t_B, \mathbf{x}_B)$$

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- Integration of geodesic eqs. (Richter & Matzner 1983, Brumberg 1987, Klioner & Zschocke 2010)
- World function (John 1975, Le Poncin *et al* 2004) or iterative solution of an eikonal eq. (Teyssandier & Le Poncin 2008).

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The aim of this talk:
to show that the enhanced term in $\mathcal{T}^{(3)}$ must be taken into account for modeling the determination of γ at the level 10^{-8} in solar system experiments.

Expression of $\mathcal{T}$ up to G^3

Metric ouside a central body of mass M ($m = GM/c^2$):

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and

$$\mathcal{B}(r)^{-1} = 1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \frac{1}{16}\gamma_4\frac{m^4}{r^4} + \sum_{n=5}^{\infty}(\gamma_n - 1)\frac{m^n}{r^n}.$$

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In GR:

$$\beta = \beta_3 = \beta_4 = \beta_5 = \cdots = 1, \quad \gamma = \epsilon = \gamma_3 = \gamma_4 = \gamma_5 = \cdots = 1.$$

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For $n = 1, 2, 3$:

$$\mathcal{T}^{(1)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{(1+\gamma)m}{c} \ln\left(\frac{r_A + r_B + |\mathbf{x}_B - \mathbf{x}_A|}{r_A + r_B - |\mathbf{x}_B - \mathbf{x}_A|}\right), \qquad \text{(Shapiro 1964)}$$

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$$\mathcal{T}^{(2)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{m^2}{r_A r_B}\frac{|\mathbf{x}_B - \mathbf{x}_A|}{c}\left[\kappa \frac{\arccos \mathbf{n}_A.\mathbf{n}_B}{|\mathbf{n}_A \times \mathbf{n}_B|} - \frac{(1+\gamma)^2}{1 + \mathbf{n}_A.\mathbf{n}_B}\right], \qquad \text{(Le Poncin } \textit{et al } \text{2004)}$$

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$$\mathcal{T}^{(3)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{m^3}{r_A r_B}\left(\frac{1}{r_A} + \frac{1}{r_B}\right)\frac{|\mathbf{x}_B - \mathbf{x}_A|}{c(1+\mathbf{n}_A.\mathbf{n}_B)}\left[\kappa_3 - (1+\gamma)\kappa\frac{\arccos \mathbf{n}_A.\mathbf{n}_B}{|\mathbf{n}_A \times \mathbf{n}_B|} + \frac{(1+\gamma)^3}{1+\mathbf{n}_A.\mathbf{n}_B}\right], \qquad \text{(Linet \& T. 2013)}$$

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where

$$\mathbf{n}_A = \frac{\mathbf{x}_A}{r_A}, \qquad \mathbf{n}_B = \frac{\mathbf{x}_B}{r_B}$$

and

$$\kappa = 2(1+\gamma) - \beta + \frac{3}{4}\varepsilon, \quad \kappa_3 = 2\kappa - 2\beta(1+\gamma) + \frac{1}{4}(3\beta_3 + \gamma_3)$$

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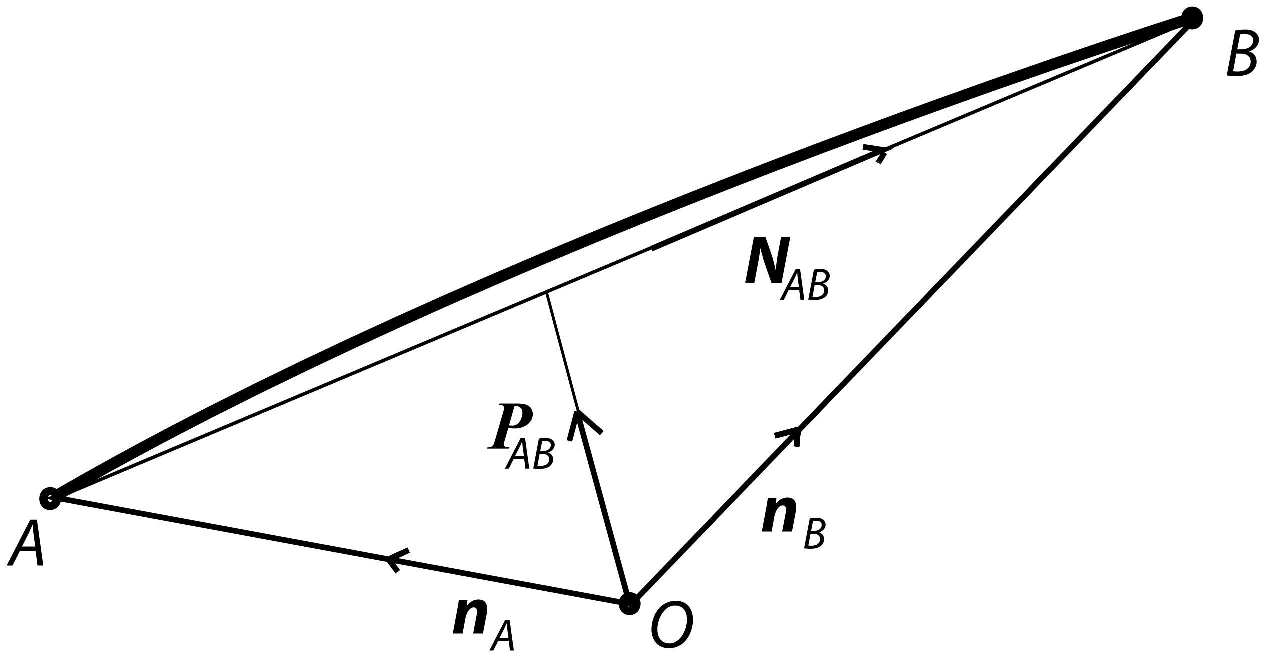

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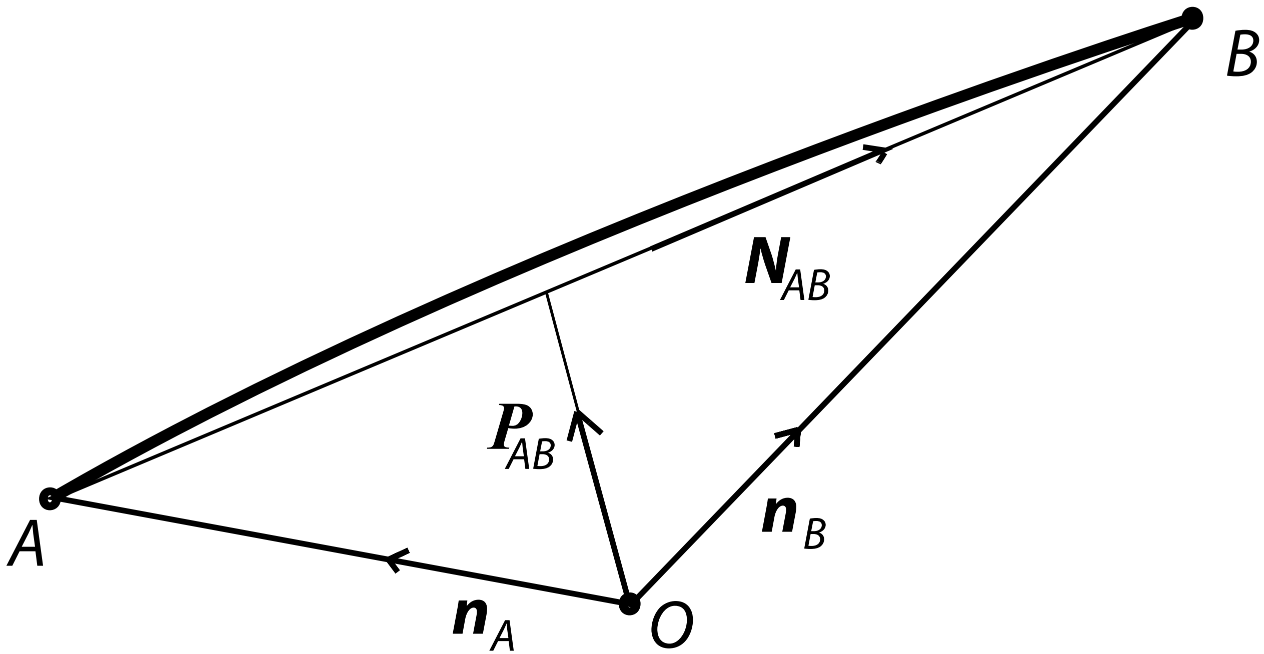

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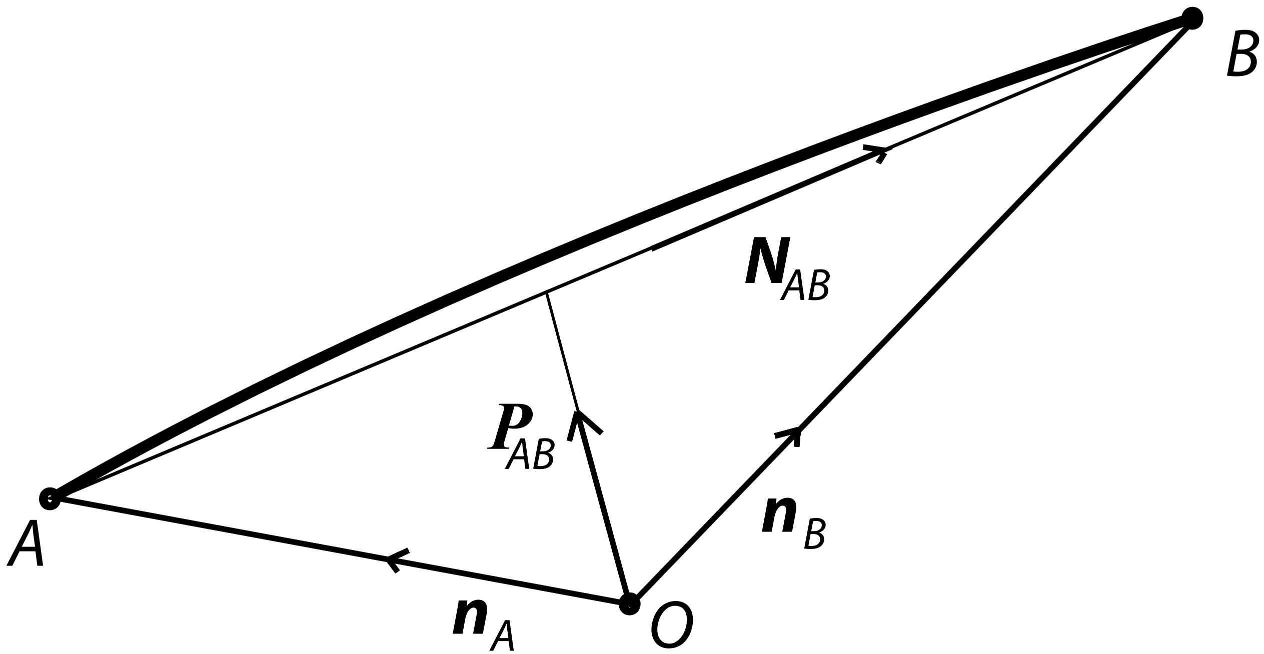

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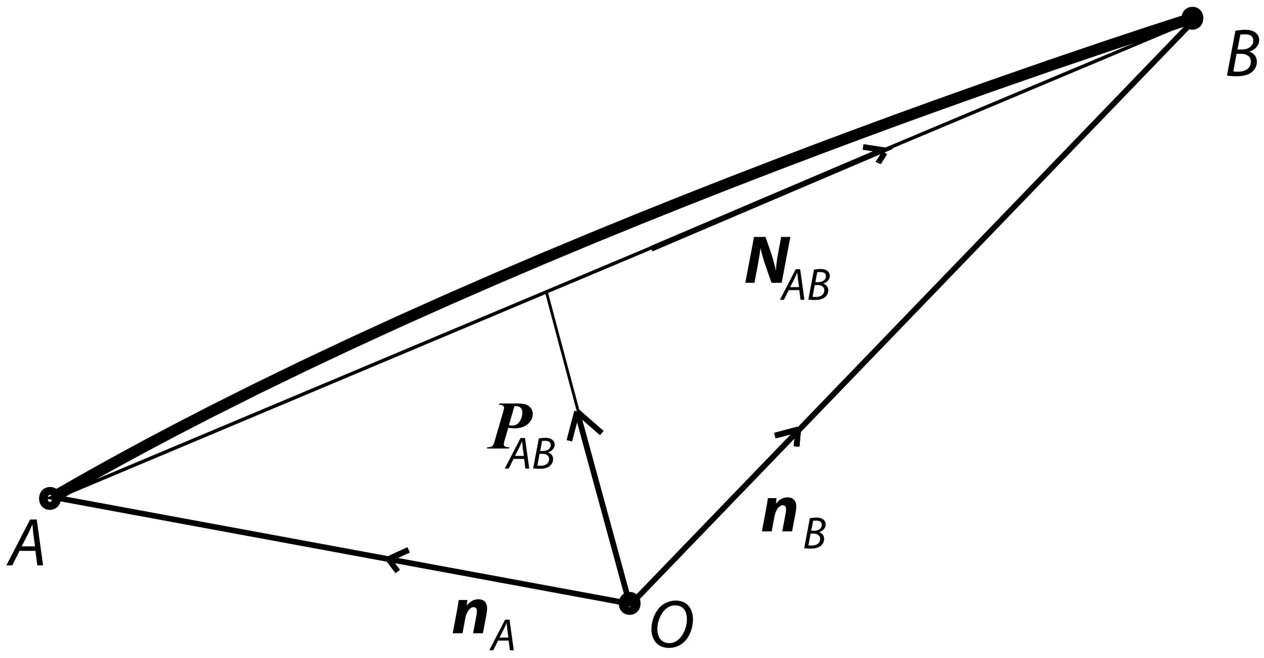

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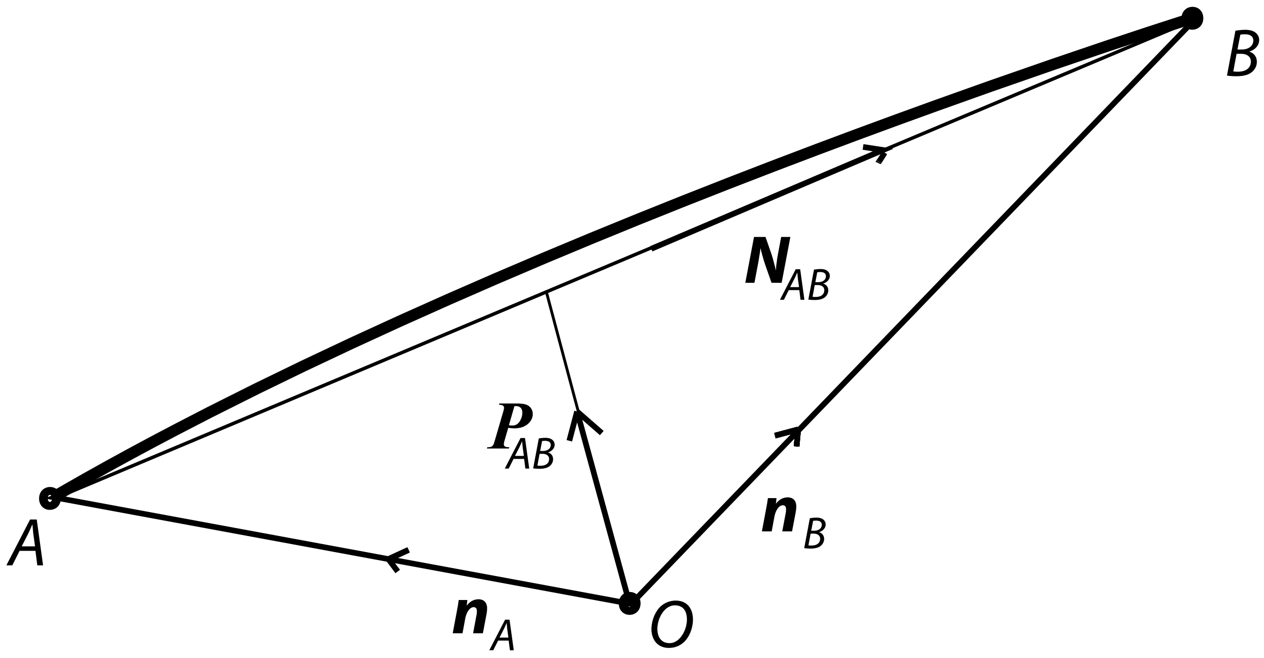

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Asymptotic expressions of the $\mathcal{T}^{(n)}$ in a conjunction $\longrightarrow$ enhanced terms:

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(cf. Ashby & Bertotti 2010)

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These expressions are reliable for $n = 1, 2, 3$ for configurations such that

$$\left|\mathcal{T}^{(n)}(\mathbf{x}_A, \mathbf{x}_B)\right| \ll \left|\mathcal{T}^{(n-1)}(\mathbf{x}_A, \mathbf{x}_B)\right|, \quad \text{with } \mathcal{T}^{(0)}(\mathbf{x}_A, \mathbf{x}_B) = \frac{1}{c}|\mathbf{x}_B - \mathbf{x}_A|$$

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Condition (C) is met in the solar system. For $r_B = 1$ au and $r_A \geq r_B$, one has

$$\frac{2m_\odot}{r_A + r_B}\frac{r_A r_B}{r_c^2} \leq 9.12 \times 10^{-4} \times \frac{R_\odot^2}{r_c^2}.$$

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$\Longrightarrow$ Our results may be applied to the solar system experiments.

Application to solar system experiments

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$$\Downarrow$$

$$\gamma \text{ at the level } 10^{-8} \quad \Longleftrightarrow \quad \mathcal{T} \text{ at the level } 0.7\text{ps}$$

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$r_c/R_\odot$	$\lvert\mathcal{T}^{(1)}_S\rvert$	$\mathcal{T}^{(1)}_{J_2}$	$\mathcal{T}^{(2)}_{enh}$	$\mathcal{T}^{(2)}_{\kappa}$	$\mathcal{T}^{(3)}_{enh}$
1	10	2	-17616	123	31.5
2	5	0.5	-4404	61.5	2
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Application to solar system experiments

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- the 1rst-order mass quadupole effect: $\mathcal{T}_{J_2}^{(1)} \sim \frac{(1+\gamma)m_\odot}{c} J_{2\odot} \frac{R_\odot^2}{r_c^2}$, with $J_{2\odot} \approx 2 \times 10^{-7}$.

Application to solar system experiments

2. Deflection of light in a LATOR-like experiment: $r_A \approx r_B \approx 1$ au

For a ray passing near the Sun

$$\Delta\chi^{(3)} \sim \left| \left(\widehat{\underline{\mathbf{l}}}_B^{(3)}\right)_{enh} - \left(\widehat{\underline{\mathbf{l}}}_A^{(3)}\right)_{enh} \right| \sim \frac{16(1+\gamma)^3 m^3}{r_c^3} \frac{r_A r_B}{(r_A + r_B)^2} \frac{r_A r_B}{r_c^2}$$

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For $r_B \approx 1$ au and $r_c = R_\odot$,

$$\Delta\chi^{(3)} \approx 12\,\mu\text{as} \qquad \text{(see Hees } et\ al \text{ in this meeting)}$$

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