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On the spatial distribution of Main Belt Asteroids.

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ABSTRACT

There are over 600,000 minor planets in our solar system. Since the discovery of the first body (1) Ceres, the orbital, dynamical and physical properties of these objects have constantly been investigated. Their spatial distribution has been extensively studied in osculatory, mean, and proper elements. We investigate the distribution of main belt asteroids in the (inclination, longitude of the ascending node) space with respect to the ecliptic-equinox J2000. We identify and confirm a sinusoidal behaviour of this distribution, which disappears when the inclination is given with respect to Jupiters orbital plane, or with respect to the invariable plane (IP). We show that this behaviour can be explained by planetary secular effects, mainly due to Jupiter. Furthermore, we identify three different orbital behaviours that explain the density distribution in this space.

Key words: Secular perturbations – Minor planets, asteroids: general – Celestials mechanics.

1. INTRODUCTION

Since about two centuries, the dynamical and physical aspects of minor planets and asteroids have been extensively investigated. Several dynamical groupings have been identified and explained. Groupings in proper elements (a, e, i) define asteroid families. Recently, Carruba et al. (2013) investigated these populations in the multi-domain defined by proper elements, the geometric albedo from Wide-field Infrared Survey Explorer (WISE) mission, as well as the Sloan Digital Sky Survey-Moving Object Catalog data, fourth release (SDSS-MOC4) multi-band photometry data.
Here we present the analysis of a not so common distribution, the Main Belt Asteroid (MBA) populations in the osculatory element domain inclination vs. longitude of the ascending node (i, Ω). We show (Fig. 1) that a particular sinusoidal distribution is observed and that it is due to secular effects of the planets, mainly Jupiter.

2. MODEL AND ARGUMENTS

One should ask two questions:
$i)$ Is this distribution an artefact due to an observational bias?
$ii)$ does the area of maximal density correspond to a particular dynamical grouping?

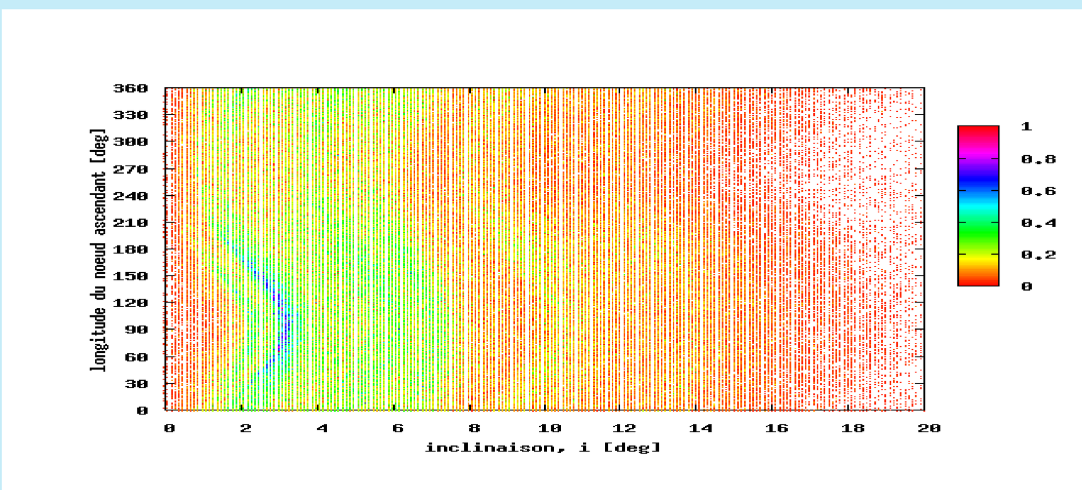


Fig.1 Density distribution of the asteroids (333.841 numbered MBAs) in term of orbital plans with respect to the ecliptic.

The answer to both question is *"No"*. In fact, these "waves" are also observed in the distribution defined by mean i vs mean Ω (for which it is shifted by an angle of $180°$ in the longitude of the ascending node).

Though, when the elements are given with respect to the solar system's invariable plane (IP) (defined by Souami & Souchay, 2012) or with respect to Jupiter's or Saturn's orbital planes, this sinusoidal behaviour tends to disappear and the distribution is flattened.

Some people have looked into the problem, for example Michkovitch (1947) suggested the secular perturbations to be at the origin of a similar distribution for the longitude of the perihelium, Scheirich (2005) observed the distribution but did not provide an explanation.

Here, using a simplified secular model following the approach of [Murray & Dermott (1999), Chap. 7], we provide an answer to the problem.

The procedure is as follows, we consider the truncated Hamiltonian to the second order in both eccentricities and inclinations.
For a massless asteroid, of semi-major axis a, and mean motion n; the perturbing funcion is written:

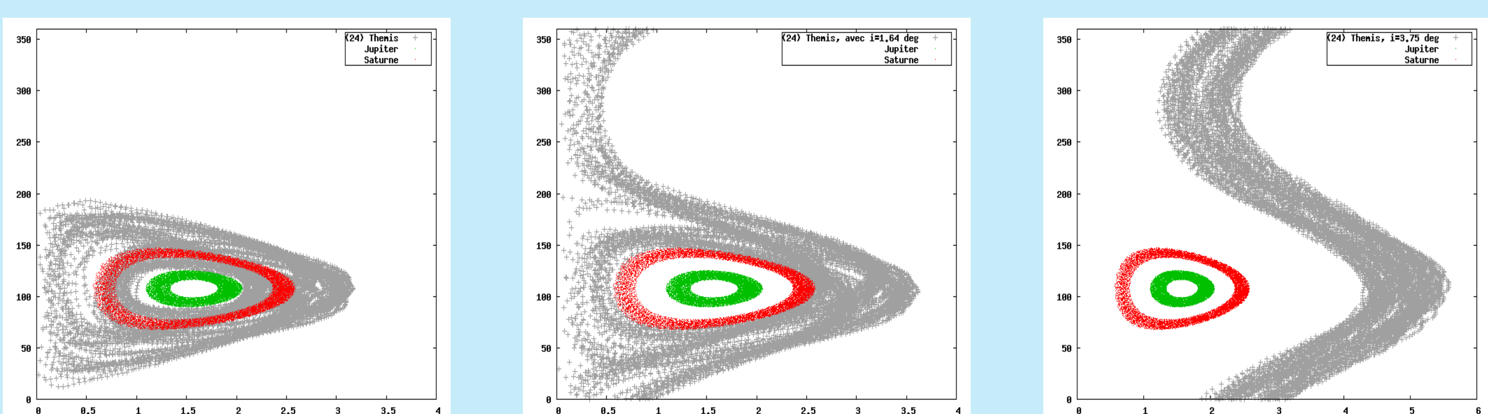
$$\mathcal{R} = na\Big[\frac{1}{2}A(h^2+k^2) + \frac{1}{2}B(p^2+q^2) + \sum_{j=1}^{2} A_j(hh_j + kk_j) + \sum_{j=1}^{2} B_j(pp_j + qq_j)\Big], \quad (1)$$

where p and q are the components of the inclination vector; h and k those of the eccentricity vector.

the perturbing function given be Eq.1 is uncoupled in eccentricity and inclination, thus we can restrict ourselves to the secular terms on the inclination, where:

$$B = -\frac{n}{4}\sum_{i=1}^{2}\frac{m_i}{m_c}\alpha_i\bar{\alpha}_i b_{3/2}^{(1)}(\alpha_i) \quad , \quad B_i = \frac{n\,m_i}{4\,m_c}\alpha_i\bar{\alpha}_i b_{3/2}^{(1)}(\alpha_i), \quad (2)$$

The indices $i=1$, and $i=2$ are used for Jupiter and Saturn, respectively. The coefficients α et $\bar{\alpha}$, as well as the Laplace coefficients are given by their classical expressions in the literature ([Murray & Dermott (1999)] for example).



(a) (24) Themis, $i = 0°, 75$ (b) $i = 1°, 64$ (c) $i = 3°, 75$

Fig.2 The secular motion in (i, Ω) plane over 10.000 years, for the asteroid (24) Themis (in grey): $a = 3,14$ au, $e = 0,13$ et $i = 0°, 75$ (Fig. 2(a)) . The other figures 2(b), 2(c) are for a fictitious Themis fictifs of inclination $1°, 64$ et $3°, 75$, respectively; green for Jupiter et red Saturn.

The solution to the secular problem induced by the effects of both Jupiter and Saturn, is given by the following equations:

$$p(t) = I_{free}\sin(Bt+\gamma) + p_0(t), \quad (3)$$
$$q(t) = I_{free}\cos(Bt+\gamma) + q_0(t), \quad (4)$$

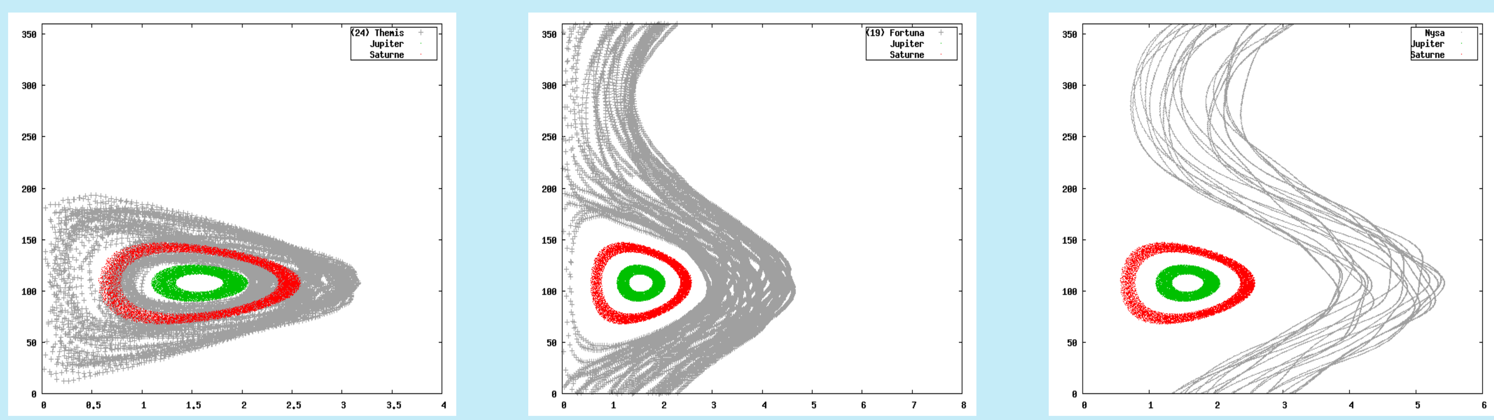
It is written as the sum of the forced solution $(p_0(t), q_0(t))$ and the free one (which is periodic and has an amplitude I_{free}).
The free motion depends upon the initial conditions of the inclination vector (of amplitude I_{free}, and phase γ). This is clearly observed for the asteroid (24) Themis (Fig.2(a)), and two fictitious Themis (Figs:2(b),2(c)) for which we vary the value of the initial inclination (I_{free}). The forced motion, depends upon the semi-major axis as it is a function of the Laplace coefficients (Eq. 5)

$$p_0(t) = -\sum_{i=1}^{2}\frac{\mu_i}{B-f_i}\sin(f_i t+\gamma_i) \quad , \quad q_0(t) = -\sum_{i=1}^{2}\frac{\mu_i}{B-f_i}\cos(f_i t+\gamma_i), \quad (5)$$

where $\mu_i = \sum_{j=1}^{2} B_j I_{ji}$.
The forced motion is associated with a forced inclination I_{forced}, and node Ω_{forced}L

$$I_{\text{forced}} = \sqrt{p_0^2+q_0^2} \quad , \quad \tan\Omega_{\text{forced}} = \frac{p_0}{q_0}, \quad (6)$$

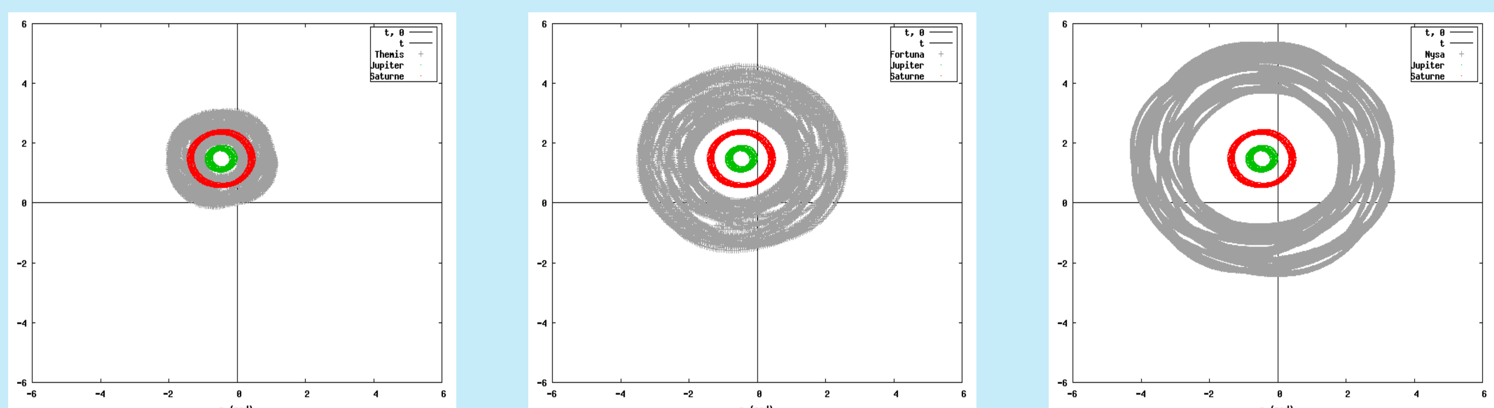
We have identified three different secular dynamics of the asteroids, depending on the initial conditions (I_{free}). This is shown figure 3 for the three real asteroids (24) Themis (Fig.3(a)), (19) Fortuna (Fig.3(b)), and (44) Nysa (Fig.3(c)). The corresponding motion in the (p, q) plane is shown figure 4.



(a) (24) Themis, $i = 0°, 75$ (b) (19) Fortuna, $i = 1°, 57$ (c) (44) Nysa, $i = 3°, 70$

Fig. 3 Motion over 10.000 years, in (i, Ω) plane: in gray for the asteroids, in green for Jupiter, and in red for Saturn.

The three identified dynamics are:
- libration for small I_{free} values, i.e. the case of (24) Themis for example (Fig.4(a)).
- homogenous or regular circulation, i.e. the case of (19) Fortuna for example (Fig.4(b)).
- heterogenous circulation, i.e. the case of (44) Nysa for example (Fig4(c)). What we recall here by heterogenous circulation is a circularion for which the body spends most of his time in a forced region.



(a) (24) Themis, $i = 0°, 75$ (b) (19) Fortuna, $i = 1°, 57$ (c) (44) Nysa, $i = 3°, 70$

Fig. 4 Motion over 10.000 years, in (q, p) plane: in gray for the asteroids, in green for Jupiter, and in red for Saturn.

3. CONCLUSIONS

- We have explained with a simple secular model the observed distribution of minor planets in the (i, Ω) plane, with respect to the ecliptic-equinox J2000.0;
- This distribution is mainly due to secular effects of Jupiter.
- The results are the same when considering a more complete model, with all the other planets of the solar system which we have integrated using a Gauss-Radau integrator (Eggl & Dvorak, 2010).
- We were able to distinguish three different dynamical behaviours that depend on the initial inclination.
- The superposition of the three dynamics explains the observed distribution, though one question remains open: what are the initial conditions that would lead to the observed distribution?

4. REFERENCES

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