Modern (theoretical) astrometry: status – work in progress

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Astrometry: accuracies

The graph shows the accuracies of various star position measurements over time, with a logarithmic scale on the y-axis for arcsec. The x-axis represents the year.

- Hipparchus - 1000 stars
- The Landgrave of Hessen - 1000
- Tycho Brahe - 1000
- Flamsteed - 4000
- Argelander - 26000
- Bessel - 1 star
- Jenkins - 6000
- USNO - 100
- Hipparcos - 120 000
- PPM - 400 000
- FK5 - 1500
- UCAC2 - 58 million
- Tycho - 1 million
- Gaia - 1000 million

Errors of best star positions and parallaxes are also indicated.
Towards nano-arcsecond astrometry

Nearby Earth Astrometric Telescope (NEAT) proposed to ESA

- Concept: pair of spacecrafts flying in formation at 40m distance
- Aim: detection of Earth-like planets within 50 light-years
- Astrometric accuracy: 50 nas
Reference systems, frames and observables in GRT

- Relativistic reference system(s)
  - Relativistic equations of motion
  - Equations of signal propagation
  - Definition of observables
    - Relativistic models of observables
      - Astronomical reference frames
      - Observational data

Coordinate-dependent parameters
If 'good' local co-moving coordinates are introduced one can derive the observables from coordinate-quantities. A good example for that is the Klioner-formalism.
Problems of relativistic astrometry

- the space-time reference system (ST coordinates and metric tensor)

- trajectory of observer (e.g., GAIA’s world-line)

- trajectory of light-rays

- calculation of observables
Two different approximation schemes used

• the post Newtonian-approximation (slow motion, weak fields)
  expansion in terms of 1/c

• the post-Minkowskian-approximation (weak fields)
  only terms linear in G are kept
  all v/c-terms will be retained
Minkowski metric (inertial Cartesian coordinates)

\[ g_{00} = -1 \]
\[ g_{0i} = 0 \]
\[ g_{ij} = \delta_{ij} \]
Post-Newtonian metric for light-rays

\[ g_{00} = -1 + \frac{2w}{c^2} \]
\[ g_{0i} = 0 \]
\[ g_{ij} = \delta_{ij} \left( 1 + \frac{2w}{c^2} \right) \]
post-post Newtonian metric for light-rays

\[ g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} \]

\[ g_{0i} = \frac{4w^i}{c^3} \]

\[ g_{ij} = \delta_{ij} \left( 1 + \frac{2w}{c^2} + \frac{2w^2}{c^4} \right) + \frac{4}{c^4} q_{ij} \]
For some body at rest, rotating, vibrating, with arbitrary shape and composition, the outer metric, determined by two families of multipole moments, $M_L$ and $S_L$ (mass- and spin-moments) is known for both, post-Newtonian (PN) and post-Minkoskian (PM) case.

Magnitude of light deflection for grazing ray at giant planets

<table>
<thead>
<tr>
<th></th>
<th>Monopole</th>
<th>Quadrupole</th>
<th>Rotation</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jupiter</td>
<td>$16.3 \times 10^3 \mu as$</td>
<td>240 $\mu as$</td>
<td>0.2 $\mu as$</td>
<td>0.7 $\mu as$</td>
</tr>
<tr>
<td>Saturn</td>
<td>$5.8 \times 10^3 \mu as$</td>
<td>95 $\mu as$</td>
<td>0.04 $\mu as$</td>
<td>0.2 $\mu as$</td>
</tr>
</tbody>
</table>

for milli-arcsecond astrometry only monopole relevant

for micro-arcsecond astrometry monopole and quadrupole relevant
Quadrupole light deflection


An efficient expression for Gaia data reduction was derived by S. Zschocke, S. Klioner: “On the efficient computation of the quadrupole light deflection”, Class. Quantum Grav. 28 (2011) 015009
Magnitude of higher multipoles on light deflection (grazing ray, giant planets at rest)

\[ \text{quadrupole } J_2 \]

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<tr>
<td>( J_2 )</td>
<td>240 ( \mu \text{as} )</td>
<td>95 ( \mu \text{as} )</td>
</tr>
<tr>
<td>( J_4 )</td>
<td>9.5 ( \mu \text{as} )</td>
<td>4.8 ( \mu \text{as} )</td>
</tr>
<tr>
<td>( J_6 )</td>
<td>0.6 ( \mu \text{as} )</td>
<td>0.5 ( \mu \text{as} )</td>
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Monopole light deflection

post-Newtonian solution versus post-post-Newtonian solution

(grazing ray at Jupiter)

error up to 16 μas

error less than 0.04 μas

S. Klioner, S. Zschocke:
“Numerical versus analytical accuracy of the formulae for light propagation”,
Class. Quantum Grav. 27 (2010) 075015
Light propagation in field of a moving point-like mass

light trajectory $x_p(t)$

body's trajectory $x_A(t)$

point mass (mass $M$)

exact analytical post-Minkowskian solution (metric & light-ray):
Light propagation in field of a moving + spinning point-like mass

light trajectory
\( x_p(t) \)

body's trajectory
\( x_A(t) \)

spinning
point mass
(mass \( M \), spin \( S \))

exact analytical post-Minkowskian solution for light trajectory:
Light propagation in field of an extended body at rest

exact analytical post-Minkowskian solution for light trajectory:
S. Kopeikin, P. Korobkov, A. Polnarev, Class. Quantum Grav. 23 (2006) 4299
Light propagation in field of an uniformly moving extended body

- post-Minkowskian metric for uniformly moving extended body in terms of local multipoles found by S. Zschocke, S. Klioner, M. Soffel, manuscript in preparation (2013)

- exact analytical post-Minkowskian solution for light trajectory in terms of local multipoles: S. Zschocke, S. Klioner, M. Soffel, work in progress
gravito-magnetic effects of moving multipoles on grazing ray-light deflection at giant planets

\textit{monopole} \( J_0 \)

\textit{J2}

\textit{J4}

Jupiter
\[ 700 \text{ nas} \]
\[ 11 \text{ nas} \]
\[ 0.7 \text{ nas} \]

Saturn
\[ 200 \text{ nas} \]
\[ 4 \text{ nas} \]
\[ 0.5 \text{ nas} \]

First rough estimates; the gravito-magnetic contribution is only part
Light propagation in post-post-Newtonian (ppN) approximation

- already for micro-arcsecond astrometry one has to consider ppN terms for point-masses

- for nano-arcsecond astrometry one has to take into account ppN metric for extended bodies

- work on the ppn-metric for extended (moving) bodies is in progress


C. Xu, X. Wu, S. Klioner, S. Soffel, “2 PN light – ray metric in the gravitational N-body problem with higher multipoles”, to be published
\[
g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} \\
g_{0i} = -\frac{4w^i}{c^3} \\
g_{ij} = \delta_{ij} \left(1 + \frac{2w}{c^2} + \frac{2w^2}{c^4}\right) + \frac{4}{c^4} q_{ij} \\
\]

\[
\Delta q^{ij} = q^{ij}_\sigma + q^{ij}_w \\
\Delta q^{ij}_\sigma = -4\pi G \sigma^{ij} \\
\Delta q^{ij}_w = -w_{,i}w_{,j} \\
\]

\[s : \text{mass-energy density}\]
\[\sigma^{ij} : \text{internal stresses}\]
\[q^{ij}_\sigma : \text{(inside body)}\]
\[q^{ij}_w : \text{(outside body)}\]

\[\sigma^{ij} \rightarrow q^{ij}_\sigma \rightarrow q^{ij}_\sigma \rightarrow -q^{ij}_\sigma \rightarrow +q^{ij}_\sigma \]
Problems arise due to internal stresses outside the body one faces $q^{ij}\sigma$ terms depending upon the internal structure of the body.

We carefully studied the problem of a single extended, spherically symmetric body in harmonic coordinates (Schwarzschild problem).

In the exterior region all structure depend terms cancel exactly or can be eliminated by a harmonic gauge transformation.

Special solutions for $q^{ij}$ for bodies with full multipole structure have been found. More work needed to understand the role of internal stresses for that case.
Summary

– theory + technology for micro – arcsecond astrometry: developed

– many subtle problems for nano – arcsecond astrometry (fundamental accuracy limit for astrometry: microlensing, g-waves)

some progress:

– metric and light trajectory in the field of arbitrarily moving and spinning point-like masses is known

– metric and light trajectory in the field of arbitrarily shaped, rotating and oscillating extended bodies at rest is known

– metric of arbitrarily shaped, rotating and oscillating extended bodies in uniform motion is known