Modern (theoretical) astrometry: status – work in progress

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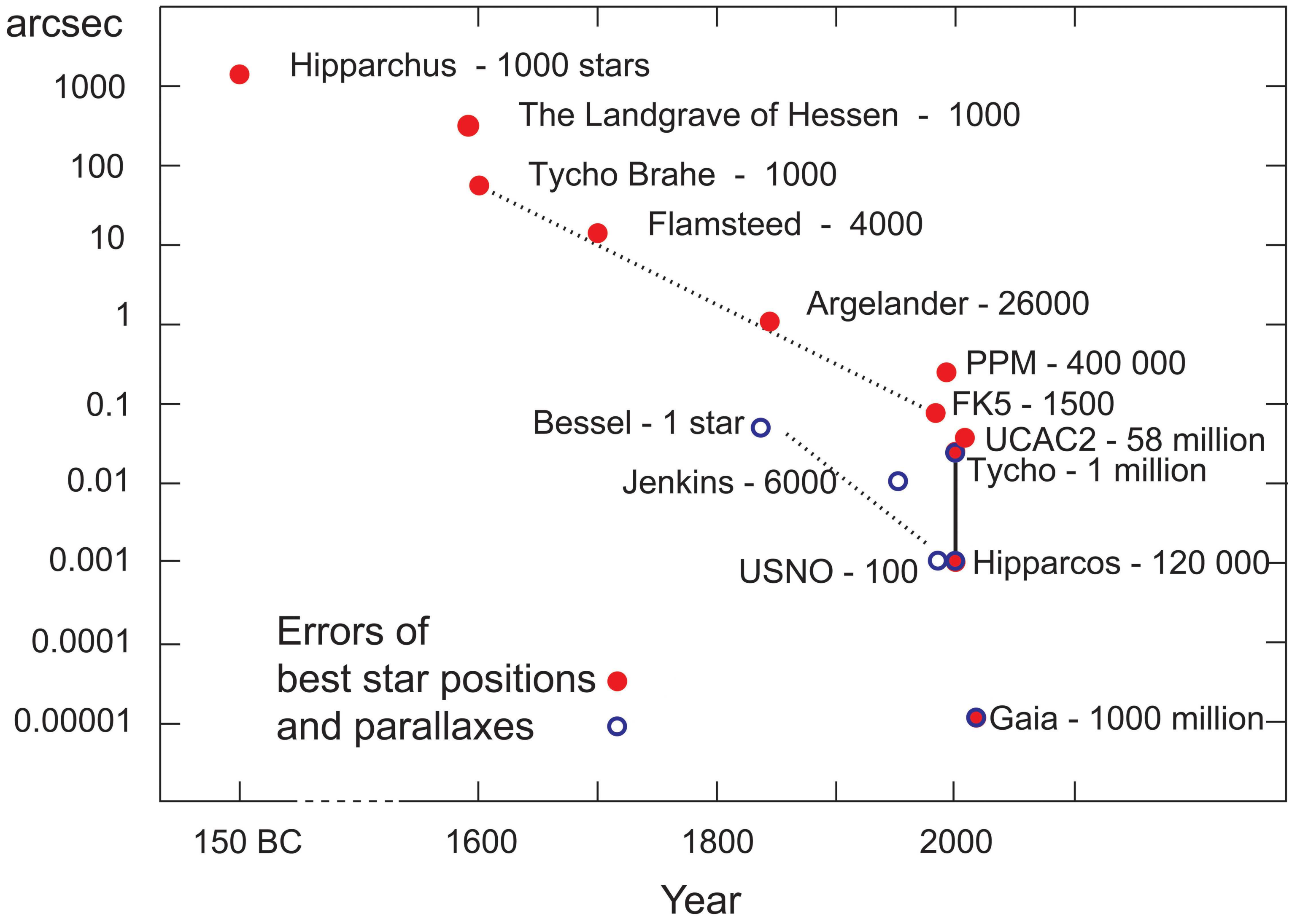
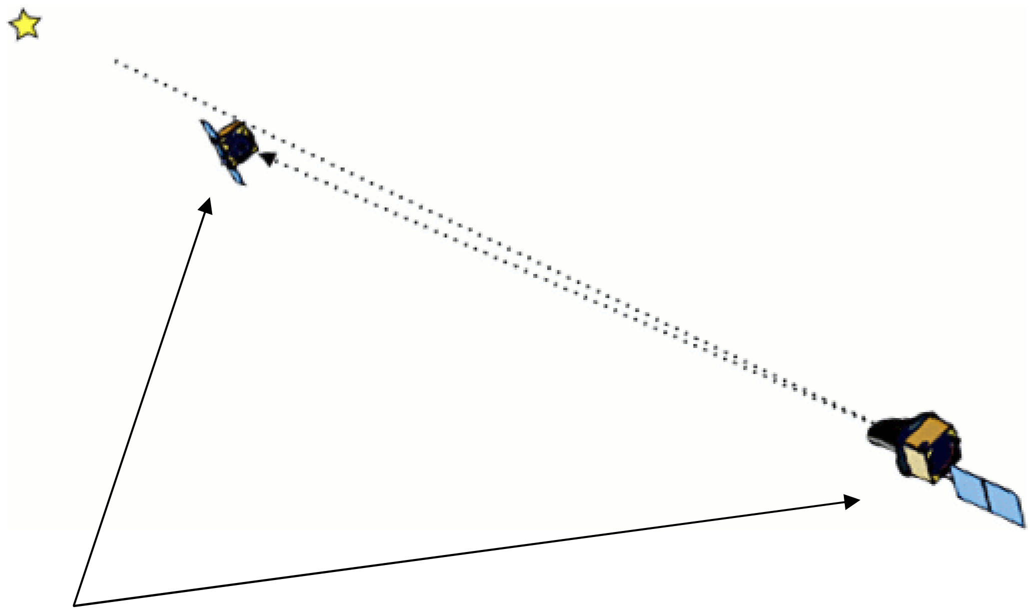
Astrometry: accuracies



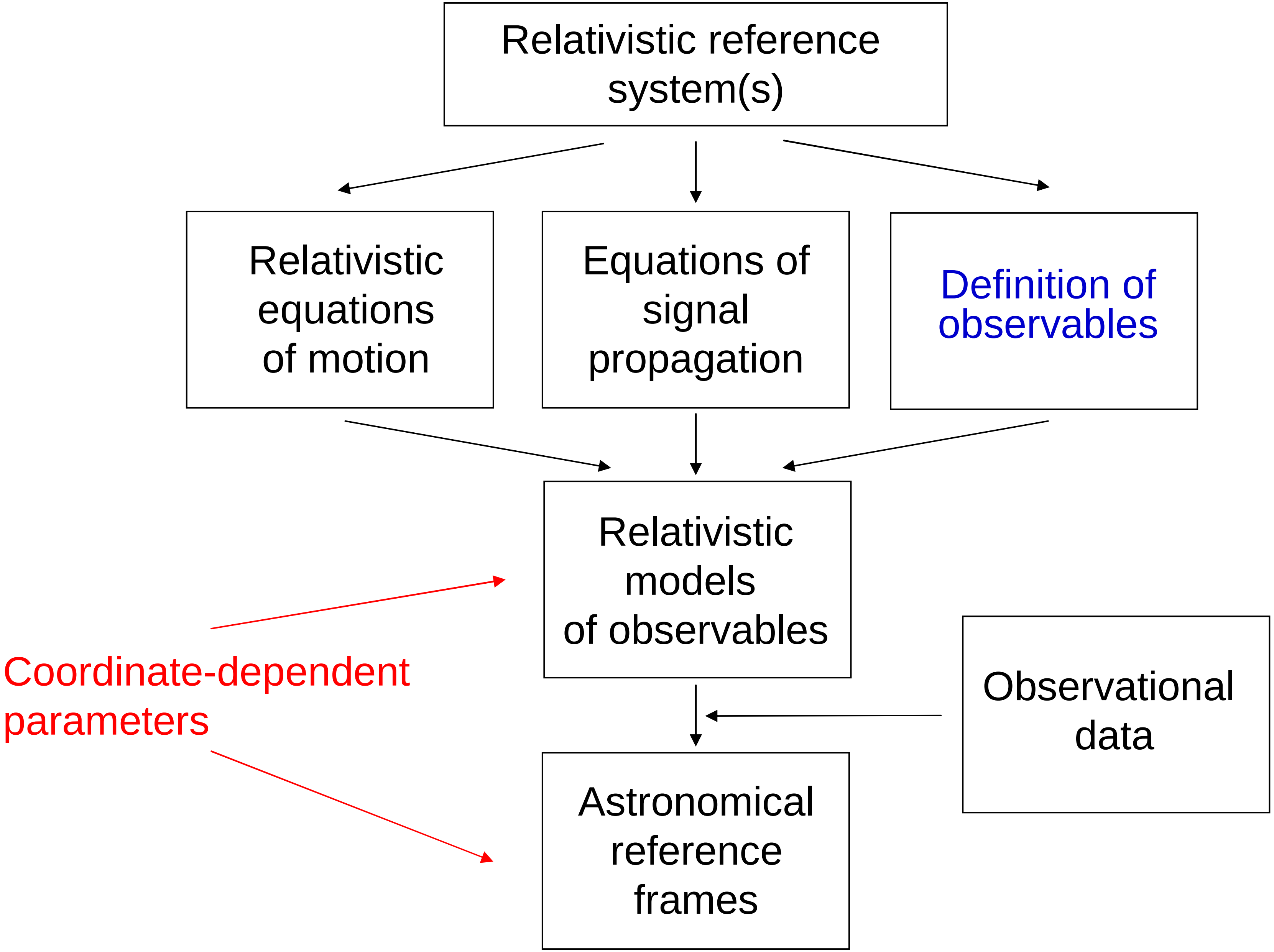
Towards nano - arcsecond astrometry

Nearby **E**arth **A**strometric **T**elescope (NEAT) proposed to ESA

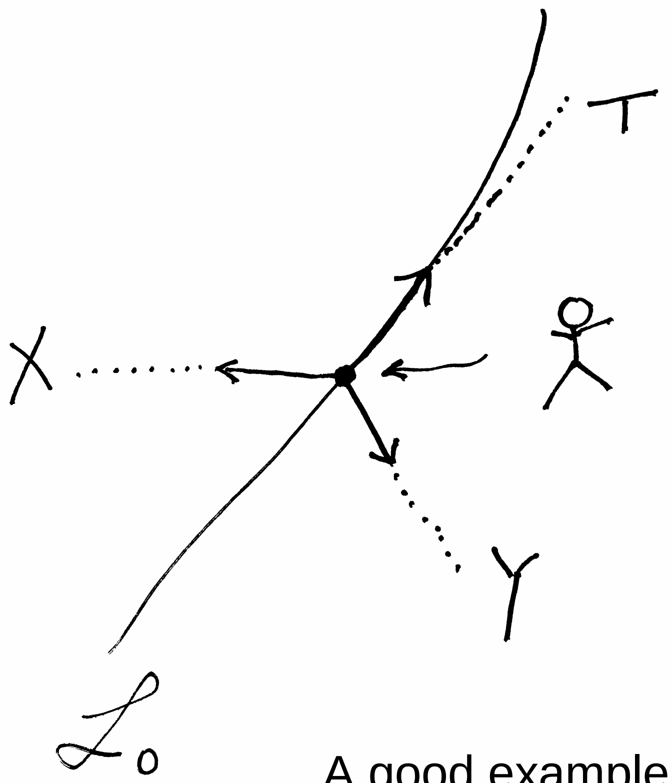


- concept: pair of spacecrafts flying in formation at 40m distance
- aim: detection of Earth-like planets within 50 light-years
- astrometric accuracy: 50 nas

Reference systems, frames and observables in GRT



If ‚good‘ local co-moving coordinates are introduced one can derive the observables from coordinate-quantities



A good example for that is the Klioner-formalism

Problems of relativistic astrometry

- the space-time reference system
  (ST coordinates and metric tensor)

- trajectory of observer (e.g., GAIA‘s world-line)

- trajectory of light-rays

- calculation of observables

Two different approximation schemes used

- the post Newtonian-approximation (slow motion, weak fields)

  expansion in terms of 1/c

- the post-Minkowskian-approximation (weak fields)

  only terms linear in G are kept
  all v/c-terms will be retained

Minkowski metric (inertial Cartesian coordinates)

$$g_{00} = -1$$

$$g_{0i} = 0$$

$$g_{ij} = \delta_{ij}$$

Post-Newtonian metric for light-rays

$$g_{00} = -1 + \frac{2w}{c^2}$$
$$g_{0i} = 0$$
$$g_{ij} = \delta_{ij} \left(1 + \frac{2w}{c^2}\right)$$

post-post Newtonian metric for light-rays

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4}$$

$$g_{0i} = -\frac{4w^i}{c^3}$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2w}{c^2} + \frac{2w^2}{c^4}\right) + \frac{4}{c^4} q_{ij}$$

For some body at rest,

rotating, vibrating, with arbitrary shape and composition,

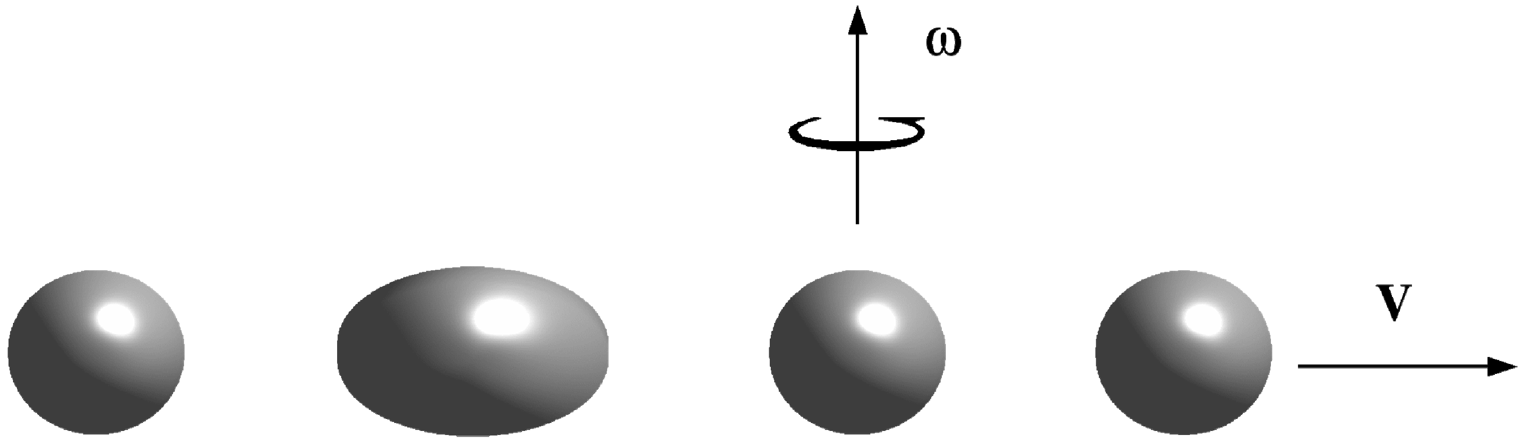
the outer metric, determined by two families of multipole moments,

M_L and S_L (mass- and spin-moments)

is known for both, post-Newtonian (PN)
and post-Minkoskian (PM)
case

Blanchet, L., Damour, T., 1989, Ann.Inst.Henri Poincare **50**, 377 (PN)
Damour, T., Iyer, B., 1991, Phys.Rev. **D 43**, 3259 (PM)

Magnitude of light deflection for grazing ray at giant planets



	Monopole	Quadrupole	Rotation	Translation
Jupiter	$16.3\times10^{3}\,\mu as$	$240\ \mu as$	$0.2\ \mu as$	$0.7\ \mu as$
Saturn	$5.8\times10^{3}\,\mu as$	$95\ \mu as$	$0.04\ \mu as$	$0.2\ \mu as$

for milli-arcsecond astrometry only monopole relevant

for micro-arcsecond astrometry monopole and quadrupole relevant

Quadrupole light deflection

first investigations by O. Ivanitskaya (1979), R. Epstein, I. Shapiro (1980) , G. Richter, R. Matzner (1982) , S. Cowling (1984)

an expression suitable for practical data reduction was first obtained by

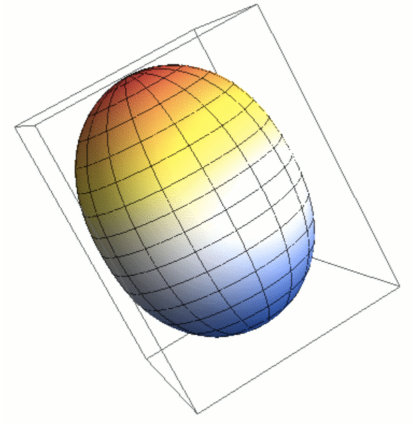
S. Klioner: “Influence of the quadrupole field and rotation of objects on light propagation”, Sov. Astron. J. **35** (1992) 523

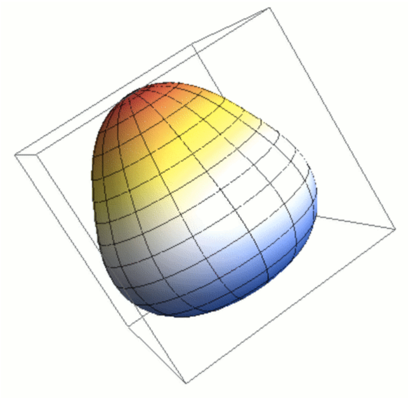
an efficient expression for Gaia data reduction was derived by

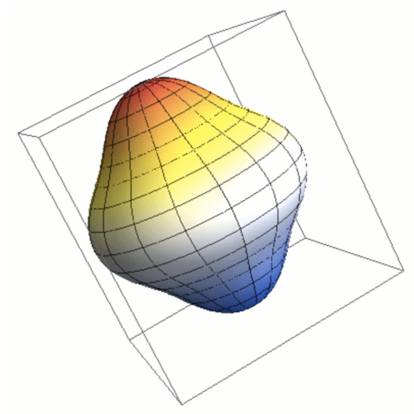
S. Zschocke, S. Klioner: “On the efficient computation of the quadrupole light deflection”, Class. Quantum Grav. **28** (2011) 015009

Magnitude of higher multipoles on light deflection (grazing ray, giant planets at rest)

quadrupole J_2 J4 J6







	J_2	J4	J6
Jupiter	240 μas	9.5 μas	0.6 μas
Saturn	95 μas	4.8 μas	0.5 μas

S. Klioner, Astron. J. **125** (2003) 1580

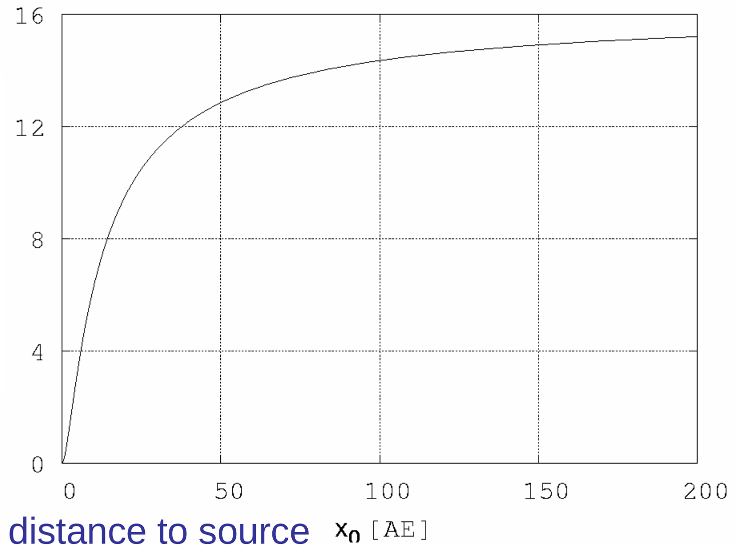
C. Le Poncin-Lafitte, P. Teyssandier, Phys. Rev. D **77** (2008) 044029

Monopole light deflection

post-Newtonian solution versus **post-post-Newtonian solution**

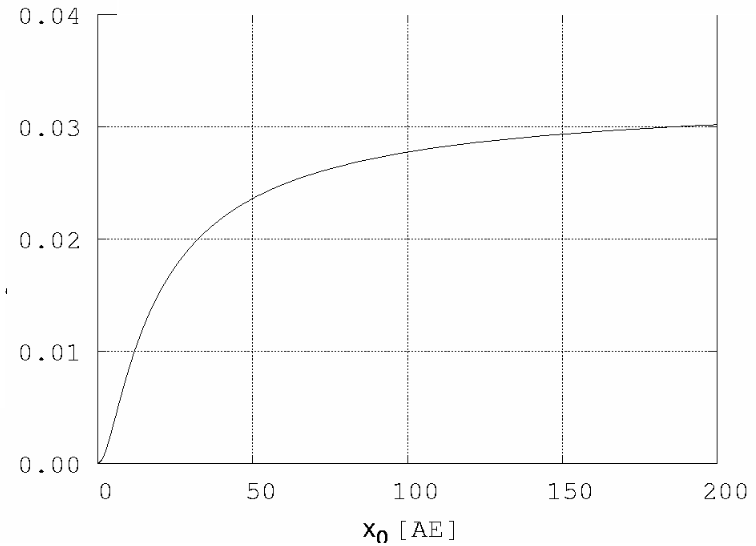
(grazing ray at Jupiter)

error up to 16 μas



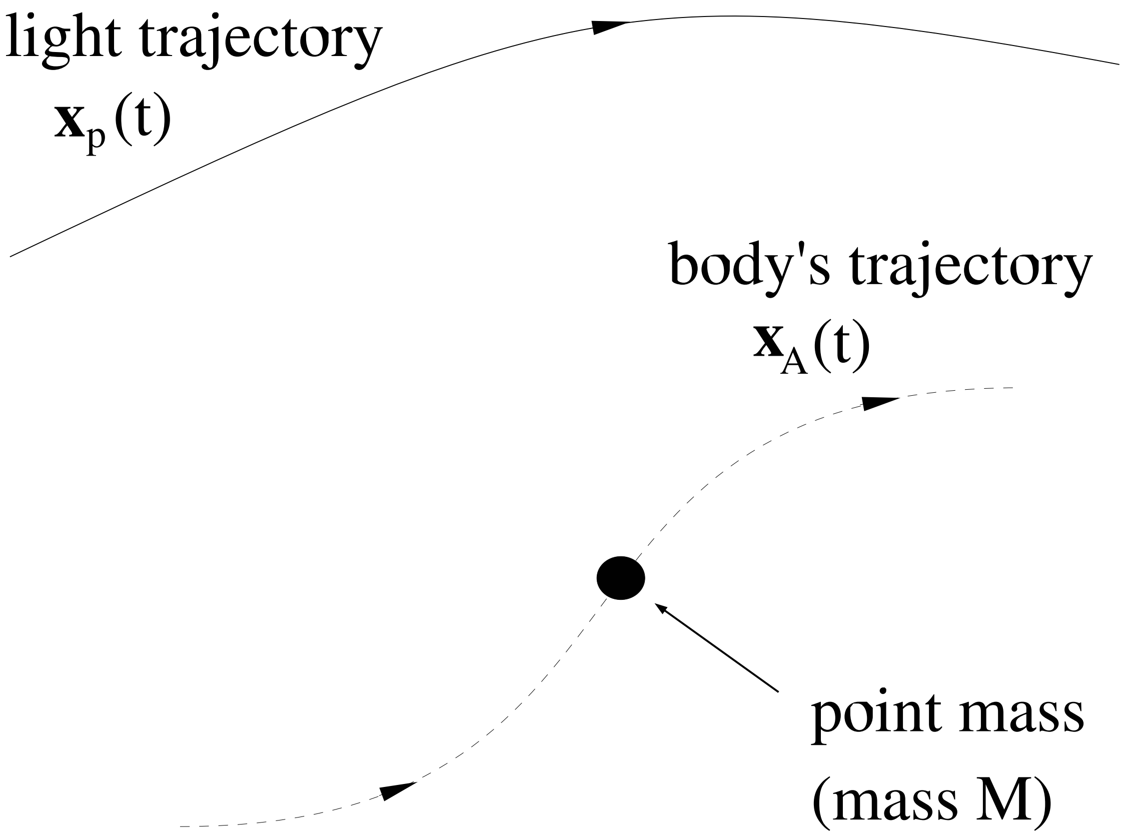
distance to source

error less than 0.04 μas



S. Klioner, S. Zschocke:
“Numerical versus analytical accuracy of the formulae for light propagation”,
Class. Quantum Grav. **27** (2010) 075015

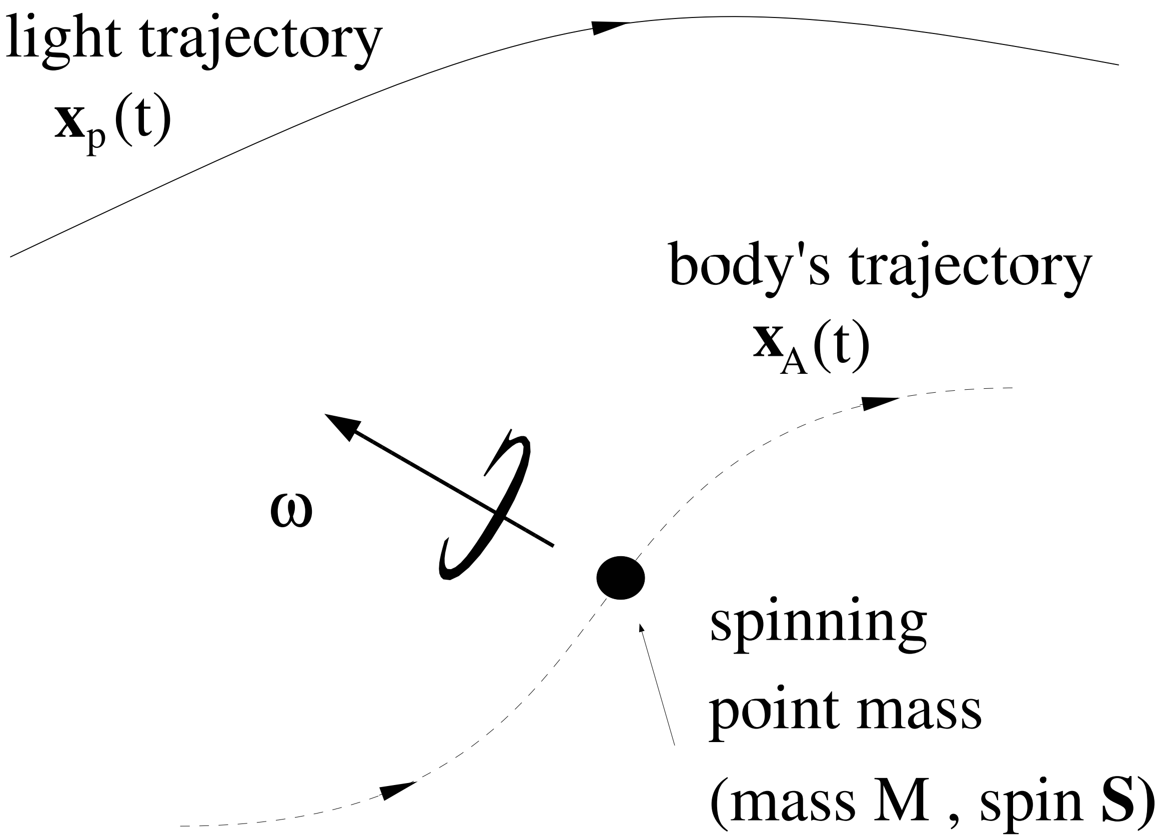
Light propagation in field of a moving point - like mass



exact analytical post - Minkowskian solution (metric & light-ray):

S. Kopeikin, G. Schäfer, Phys. Rev. **D 60** (1999) 124002

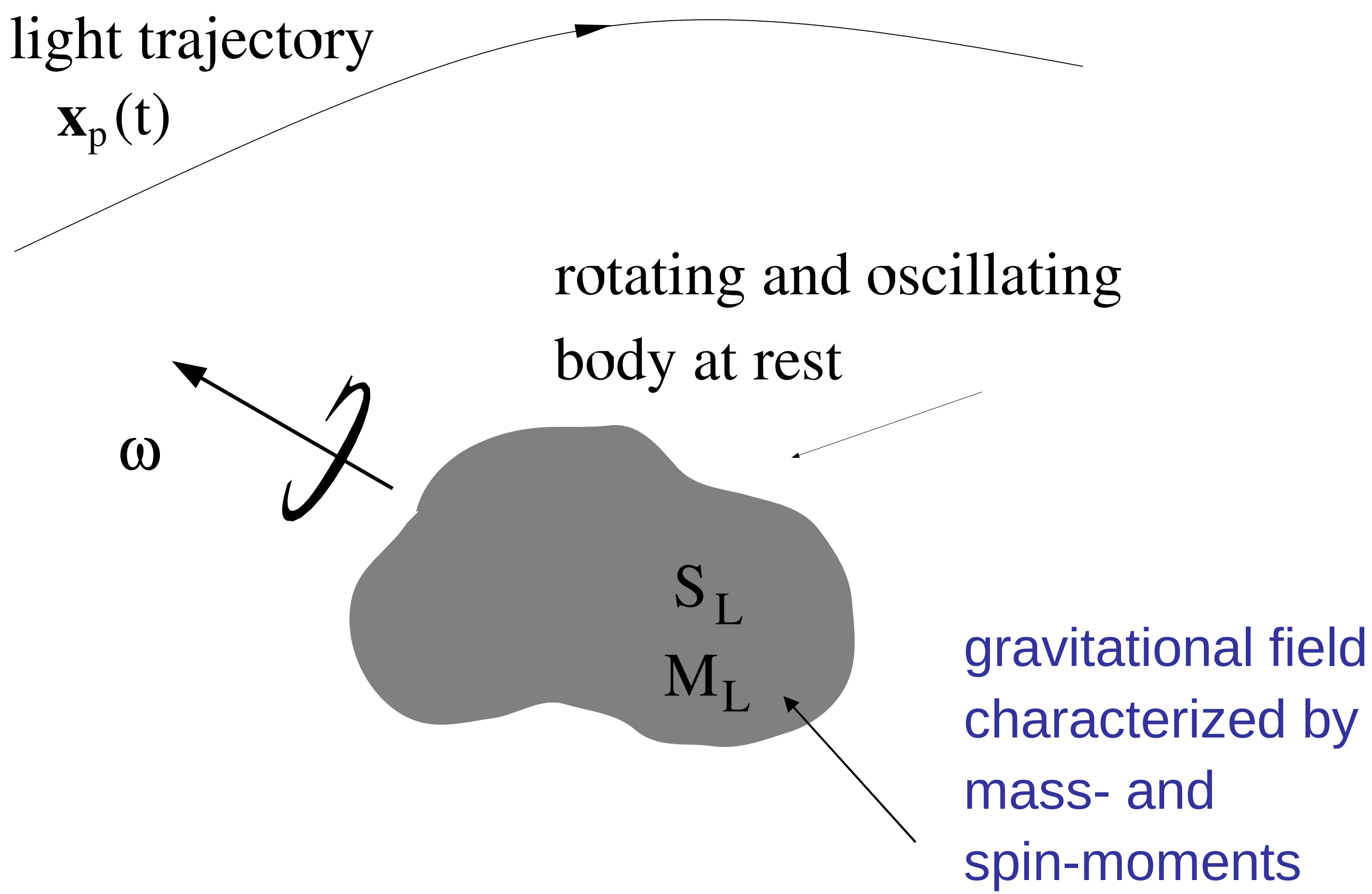
Light propagation in field of a moving + spinning point - like mass



exact analytical post - Minkowskian solution for light trajectory:

S. Kopeikin, B. Mashhoon, Phys. Rev. **D 65** (2002) 064025

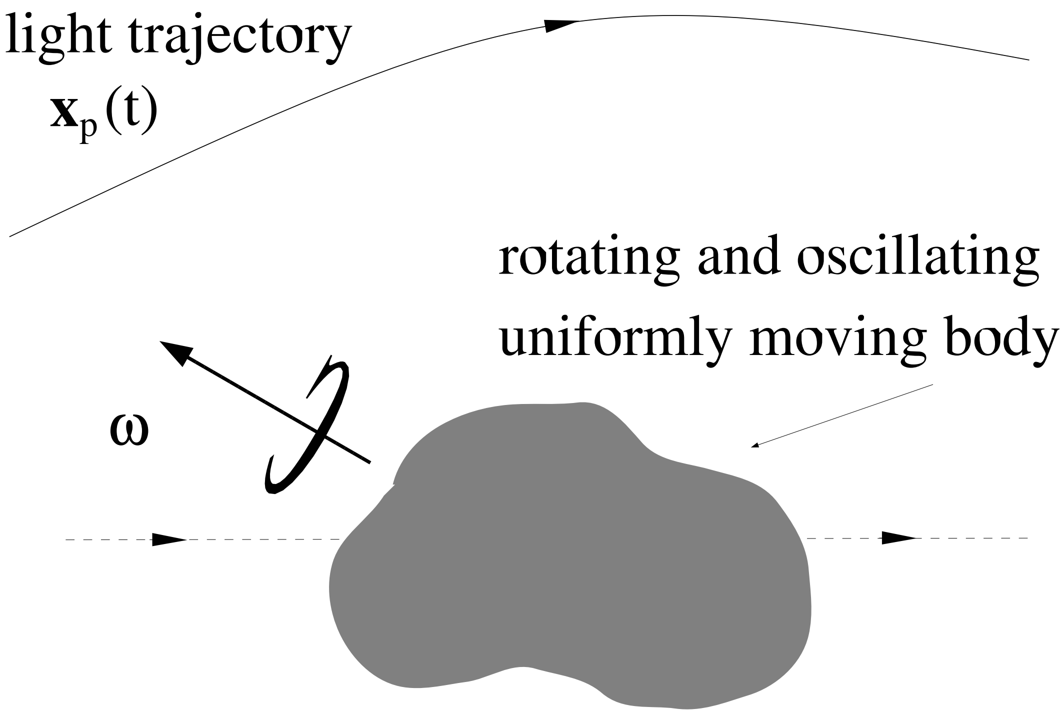
Light propagation in field of an extended body at rest



exact analytical post - Minkowskian solution for light trajectory:

S. Kopeikin, P. Korobkov, A. Polnarev, Class. Quantum Grav. **23** (2006) 4299

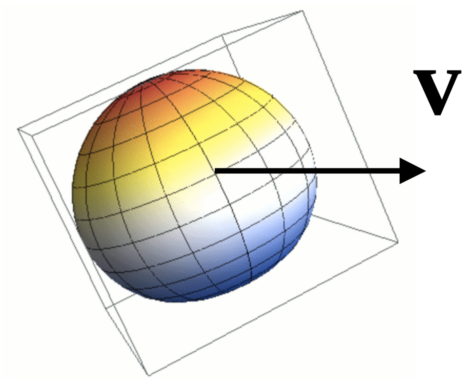
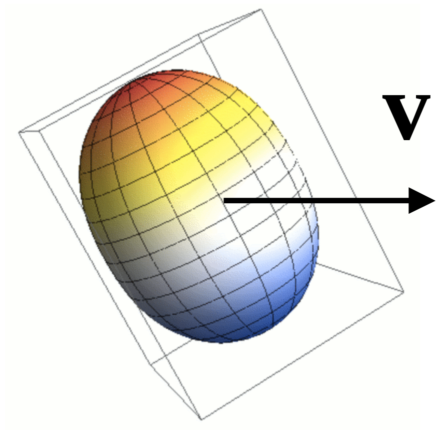
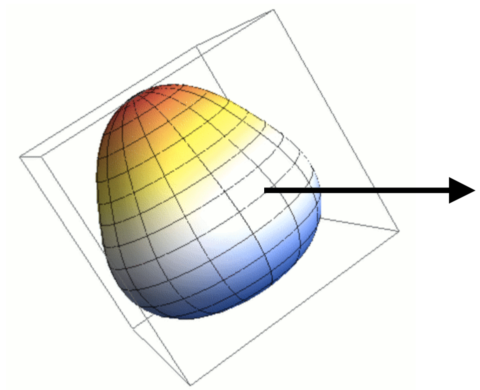
Light propagation in field of an uniformly moving extended body



– post - Minkowskian metric for uniformly moving extended body in terms of local multipoles found by S. Zschocke, S. Klioner, M. Soffel, manuscript in preparation (2013)

- exact analytical post - Minkowskian solution for light trajectory in terms of local multipoles: S. Zschocke, S. Klioner, M. Soffel, work in progress

gravito-magnetic effects of moving multipoles on grazing ray-light deflection at giant planets

	monopole J_0	J2	J4
			
Jupiter	700 *nas*	11 *nas*	0.7 *nas*
Saturn	200 *nas*	4 *nas*	0.5 *nas*

First rough estimates; the gravito-magnetic contribution is only part

Light propagation in post-post-Newtonian (ppN) approximation

- already for micro – arcsecond astrometry one has to consider ppN terms for point-masses

- for nano – arcsecond astrometry one has to take into account ppN metric for <u>extended bodies</u>

- work on the ppn-metric for extended (moving) bodies is in progress

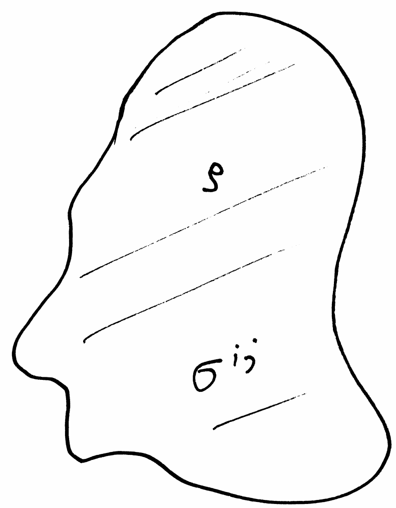
C. Xu, Y. Gong, X. Wu, S. Klioner, S. Soffel, “Second order post-Newtonian equations of light propagation in multiple systems”, arXiv: 0510074

C. Xu, X. Wu, S. Klioner, S. Soffel, “2 PN light – ray metric in the gravitational N – body problem with higher multipoles”, to be published

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4}$$
$$g_{0i} = -\frac{4w^i}{c^3}$$
$$g_{ij} = \delta_{ij}\left(1 + \frac{2w}{c^2} + \frac{2w^2}{c^4}\right) + \frac{4}{c^4}\,q_{ij}$$

$$\Delta q^{ij} = q_\sigma^{ij} + q_w^{ij}$$

$$\Delta q_\sigma^{ij} = -4\pi G\,\sigma^{ij}$$
$$\Delta q_w^{ij} = -w_{,i}w_{,j}$$



ρ : mass-energy density

σ^{ij} : internal stresses

$\rho \to w \to {}^{-}q_w^{ij}$ (inside body)

$ \to {}^{+}q_w^{ij}$ (outside body)

$\sigma^{ij} \to q_\sigma^{ij} \to {}^{-}q_\sigma^{ij}$

$\phantom{\sigma^{ij} \to q_\sigma^{ij}} \to {}^{+}q_\sigma^{ij}$

Problems arise due to internal stresses

outside the body one faces q^{ij}_{σ} terms depending upon the internal structure of the body

we carefully studied the problem of a single extended, spherically symmetric body in harmonic coordinates (Schwarzschild problem)

in the exterior region all structure depend terms cancel exactly or can be eliminated by a harmonic gauge transformation

special solutions for q^{ij} for bodies with full multipole structure have been found

more work needed to understand the role of internal stresses for that case

Summary

- theory + technology for micro – arcsecond astrometry: developed
- many subtle problems for nano – arcsecond astrometry (fundamental accuracy limit for astrometry: microlensing, g-waves)

some progress:

- metric and light trajectory in the field of arbitrarily moving and spinning point-like masses is known
- metric and light trajectory in the field of arbitrarily shaped, rotating and oscillating extended bodies at rest is known
- metric of arbitrarily shaped, rotating and oscillating extended bodies in uniform motion is known

FIN