Journées
"Systèmes de référence spatio-temporels"

Construction of the new high-precision Earth rotation series at a long time intervals

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Paris Observatory, France, 16 – 18 September 2013

INTRODUCTION

- **In the previous investigation** (Pashkevich, 2013) **the high-precision Rigid Earth Rotation Series (designated RERS2012)** dynamically adequate to the JPL DE406/LE406 ephemeris over 2000 and 6000 years were constructed.
- **RERS2012 include** about 4113 periodical terms (without attempt to estimate new sub-diurnal and diurnal periodical terms). **Discrepancies between the numerical solution and RERS2012** do not surpass:

  10 μas over 2000 years,

  2 mas over 6000 years.

Aims of the present research:

1) **Construction of the improved high-precision Rigid Earth Rotation Series (RERS2013)** dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years.
2) **Investigation of discrepancies** between the high-precision numerical solutions and the semi-analytical solutions **of the rigid Earth rotation problem with respect to the fixed ecliptic of epoch J2000,** by the least-squares method and by the spectral analysis methods.
3) **Comparison of the new Series RERS2013** with the previous solution RERS2012 (Pashkevich, 2013).

The problem is solved only for the relativistic (kinematical) case in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account.

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MATHEMATICAL MODEL

LAGRANGE DIFFERENTIAL EQUATIONS OF THE SECOND KIND:

In the previous investigation (Pashkevich, 2013)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\lambda}_i} - \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 0,1,2,3.$$

where $L = T + U, \;\; T = \frac{1}{2}(A\omega_{*1}^2 + B\omega_{*2}^2 + C\omega_{*3}^2), \;\; \bar{\omega}_* = \bar{\omega} + \bar{\sigma}$

$U -$ **the force function of the gravitational interaction of the Earth with the disturbing bodies (the Moon, the Sun and major planets).**

The force function U **is expanded in the spherical harmonics** and only the terms with coefficients **C_{j0} for j=2,…,5, C_{22}, C_{3k}, S_{3k} for k=1,2,3 and C_{41}, S_{41}** are used.

The orbital motions of the disturbing celestial bodies are defined by the JPL DE406/LE406 ephemeris.

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$\bar{\omega}_*$ is the relativistic angular velocity vector.

VECTOR OF THE GEODETIC ROTATION OF THE EARTH:

$$\bar{\sigma} = \sum_{j \neq \oplus} \frac{Gm_j}{c^2 \left| \bar{R}_j - \bar{R}_\oplus \right|^3} \left\{ (2\dot{\bar{R}}_j - \frac{3}{2}\dot{\bar{R}}_\oplus) \times (\bar{R}_j - \bar{R}_\oplus) \right\}.$$

Problem expressed in the Rodrigues – Hamilton parameters:

$$\lambda_0 = \cos\frac{\theta}{2}\cos\frac{\psi+\phi}{2}, \lambda_1 = \sin\frac{\theta}{2}\cos\frac{\psi-\phi}{2}, \lambda_2 = \sin\frac{\theta}{2}\sin\frac{\psi-\phi}{2}, \lambda_3 = \cos\frac{\theta}{2}\sin\frac{\psi+\phi}{2},$$

which are functions of the Euler angles ψ, θ and ϕ.

ITERATIVE ALGORITHM:

1. **Numerical solution of the rigid Earth rotation** is implemented with the quadruple precision of calculations. **The initial conditions** are computed by the semi-analytical solution of the rigid Earth rotation (RERS2012). **Discrepancies between the numerical solution and the semi-analytical solution** are obtained in Euler angles over all investigation time interval with one-day spacing.

2. **Investigation of the discrepancies** is carried out by the least squares method (LSQ) and by the spectral analysis (SA) method (**Pashkevich and Eroshkin,** Proceedings of “Journees 2005”). **The set of the frequencies of the semi-analytical solution** is used without change. **Only the coefficients** of the periodical terms and **the coefficients** of the Poisson terms are improved. **The secular, periodic and Poisson terms** representing the new high-precision rigid Earth rotation series **RERS2013-i** (where **i** is the number of iteration) are determined.

3. **Numerical solution of the rigid Earth rotation is constructed anew** with **the new initial conditions,** which are calculated by **RERS2013-i**.

4. **Steps 2 and 3 are repeated** till the assumed convergence level has been achieved.

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$$\left.\begin{aligned}
\Delta\psi &= \sum_{k=0}^{6}\psi_k\ t^k + \sum_j\sum_{k=0}^{4}[\psi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\psi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta\theta &= \sum_{k=0}^{6}\theta_k\ t^k + \sum_j\sum_{k=0}^{4}[\theta_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\theta_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta\varphi &= \sum_{k=0}^{6}\varphi_k\ t^k + \sum_j\sum_{k=0}^{4}[\varphi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\varphi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k
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$$\left.\begin{aligned} \psi_{\mathrm{RERS2012Xi}} &= \Delta\psi_{\mathrm{i-1}} + \psi_{\mathrm{RERS2012Xi-1}} \\ \theta_{\mathrm{RERS2012Xi}} &= \Delta\theta_{\mathrm{i-1}} + \theta_{\mathrm{RERS2012Xi-1}} \\ \varphi_{\mathrm{RERS2012Xi}} &= \Delta\varphi_{\mathrm{i-1}} + \varphi_{\mathrm{RERS2012Xi-1}} \end{aligned}\right\}$$

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Fig.1. Differences between **the numerical solutions** (for DE422/LE422) **and RERS2012** (dynamically adequate to the JPL DE406/LE406 ephemeris) **(over 2000 years)**

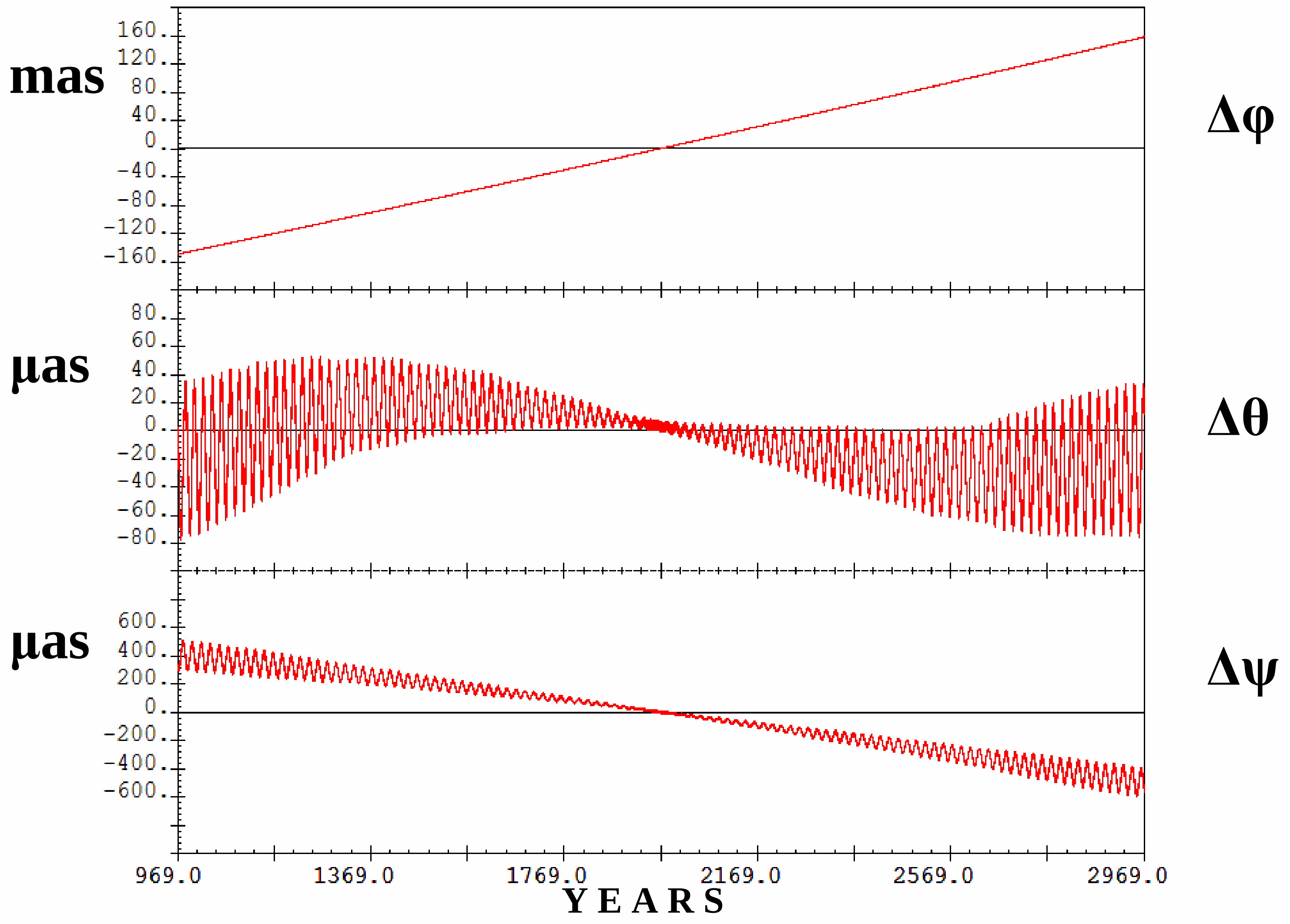


Fig.2. Differences between **the numerical solutions and RERS2012** after formal removal of the secular trends

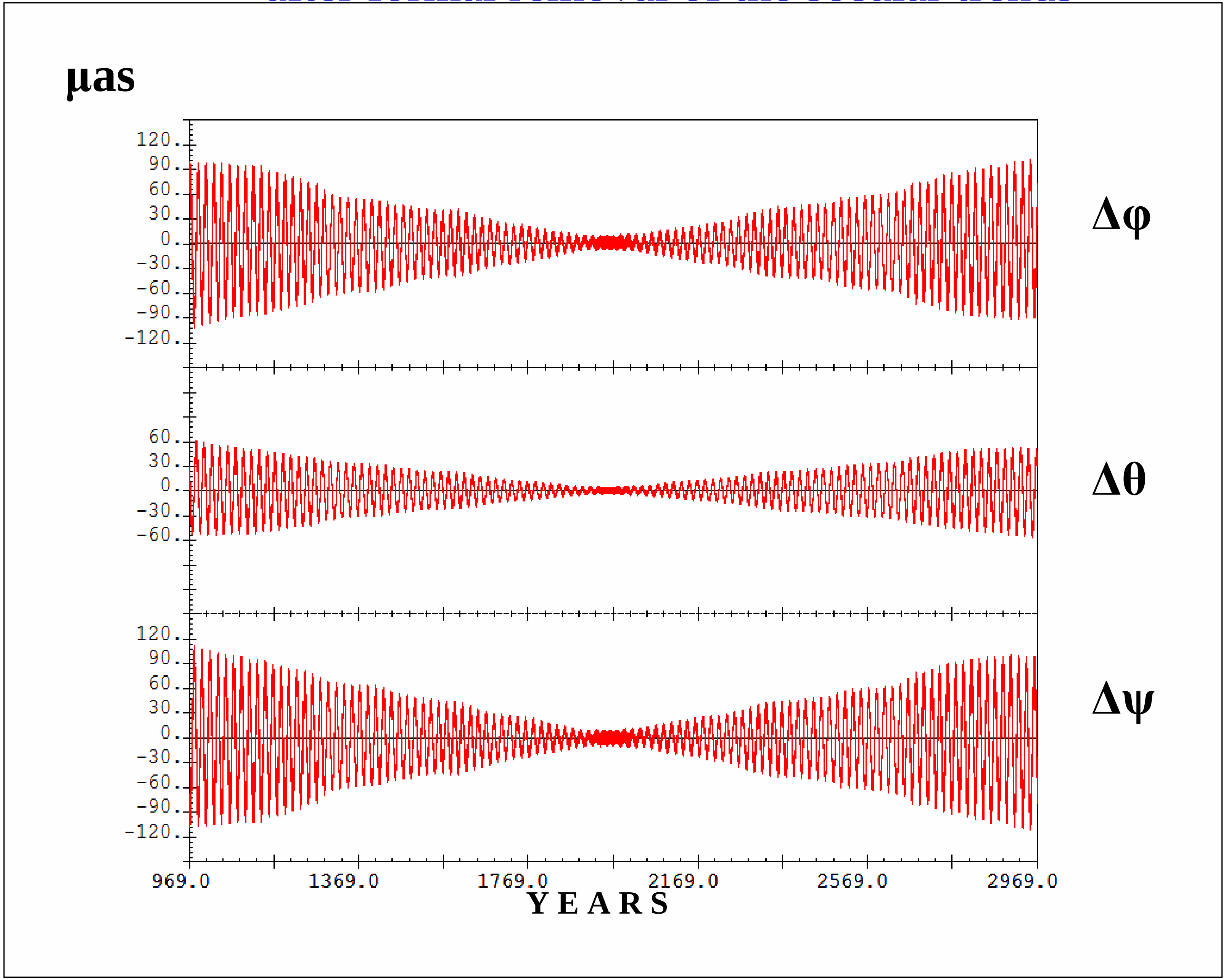


Fig.3. **The numerical solutions 2 minus RERS2013-1** after formal removal of the secular trends in the proper rotation angle

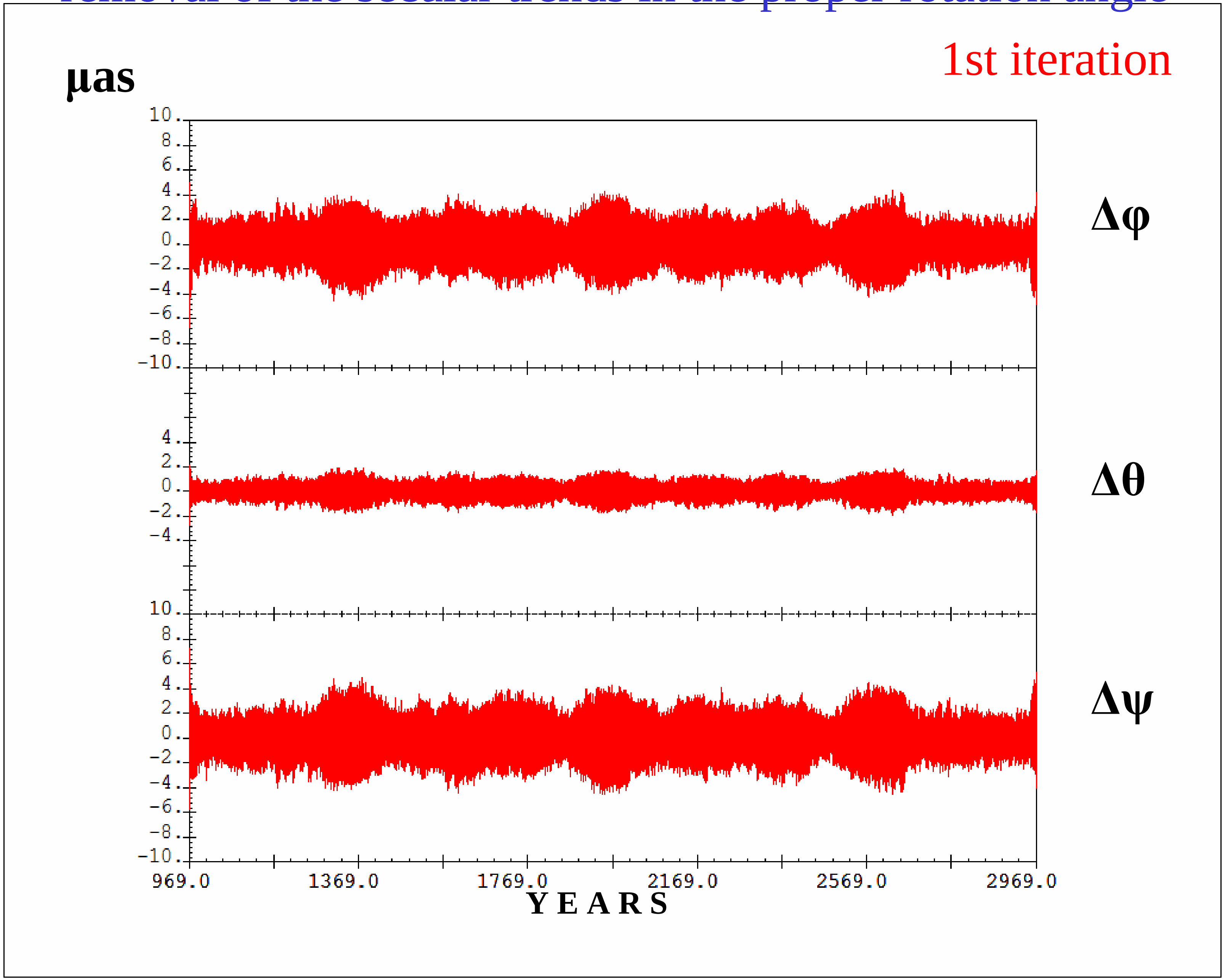


Fig.4. **The numerical solutions 3 minus RERS2013-2** after formal removal of the secular trends in the proper rotation angle

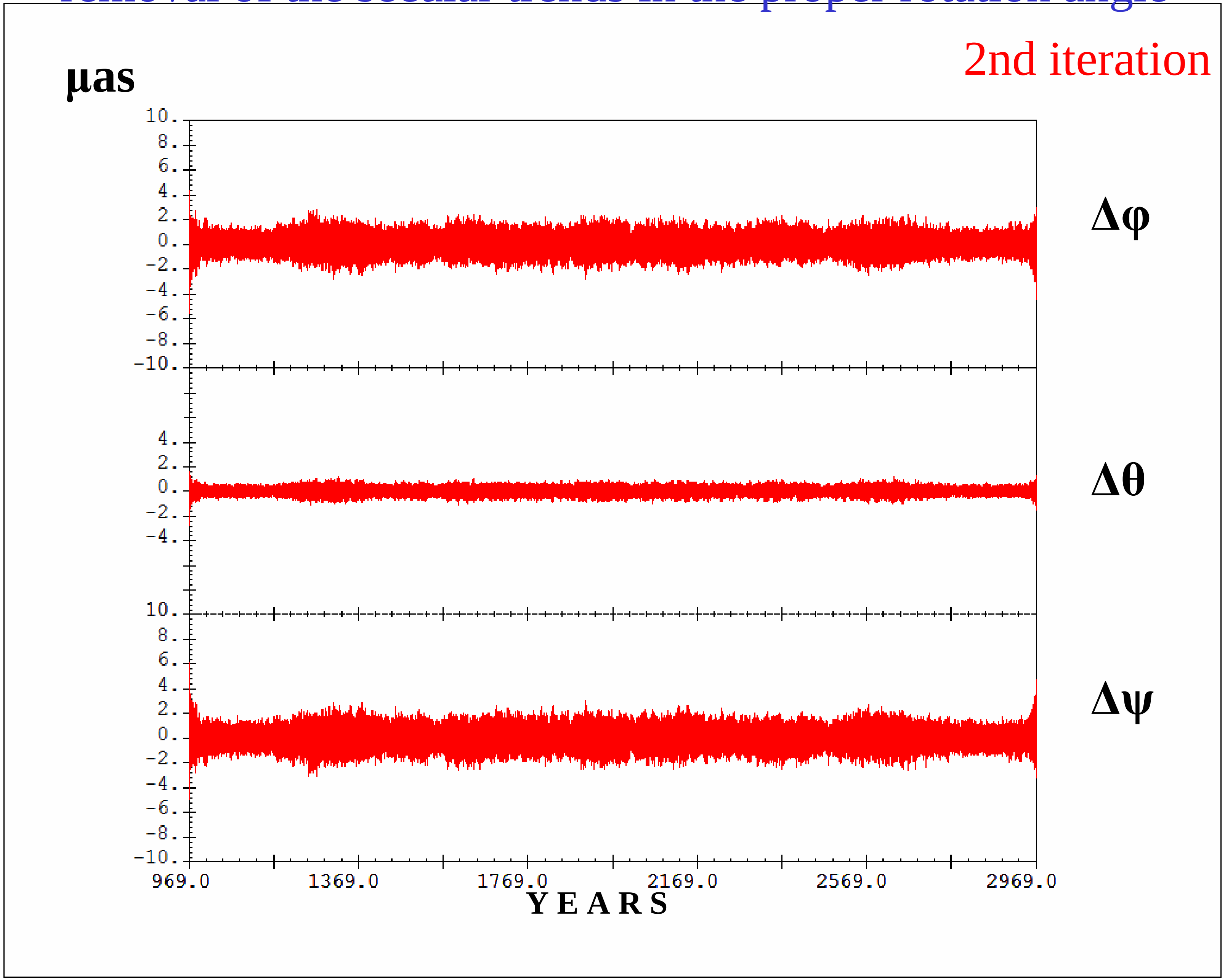


Fig.5. **The numerical solutions 3 minus RERS2013-2** after formal removal of the secular trends in the proper rotation angle and 4113 periodical terms

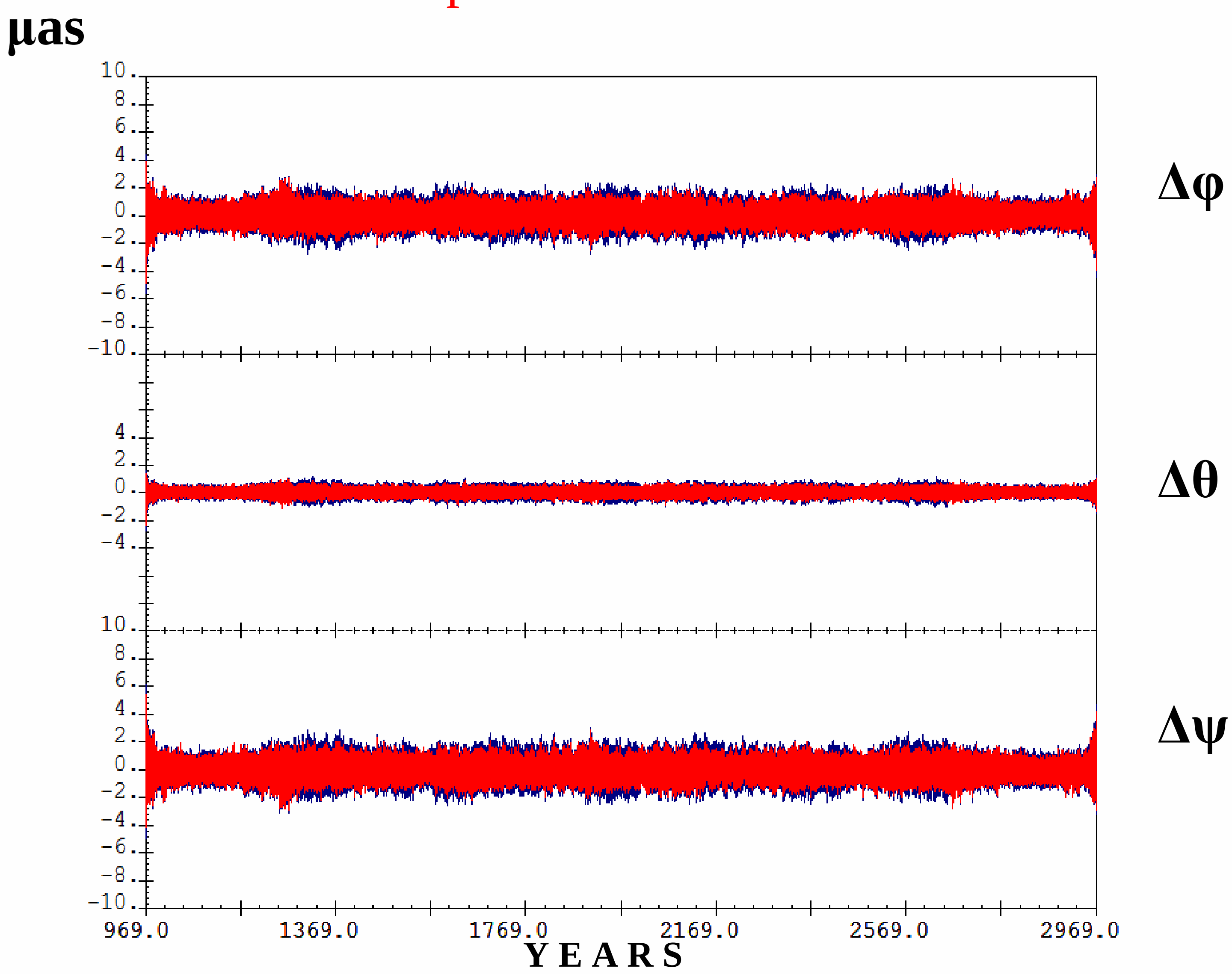


Fig.6. **The new improved high-precision Rigid Earth Rotation Series RERS2013** dynamically adequate to the DE422/LE422 have been constructed

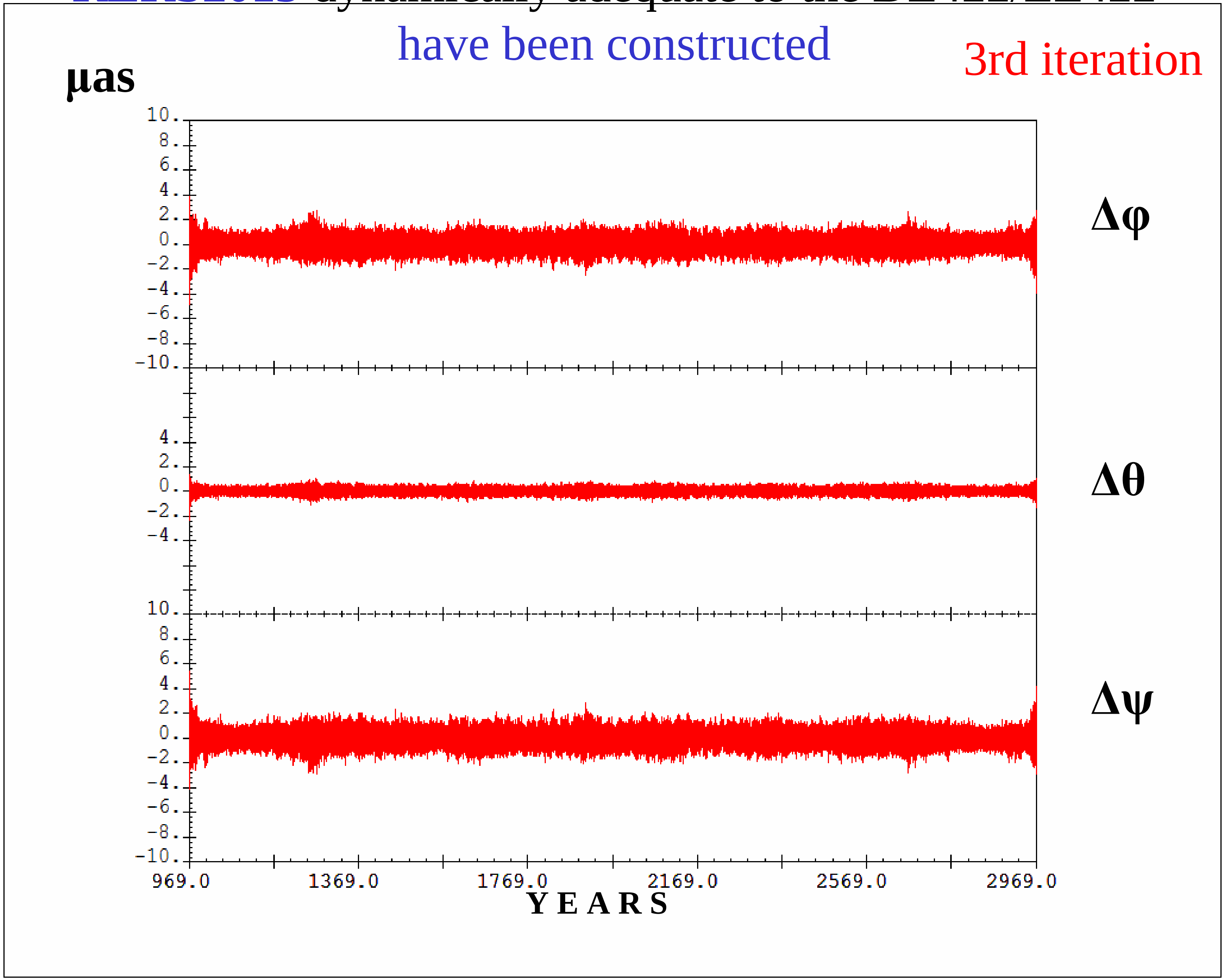


Fig.7. Differences between **the numerical solutions** (for DE422/LE422) **and RERS2012** (dynamically adequate to the JPL DE406/LE406 ephemeris)

(over 6000 years)

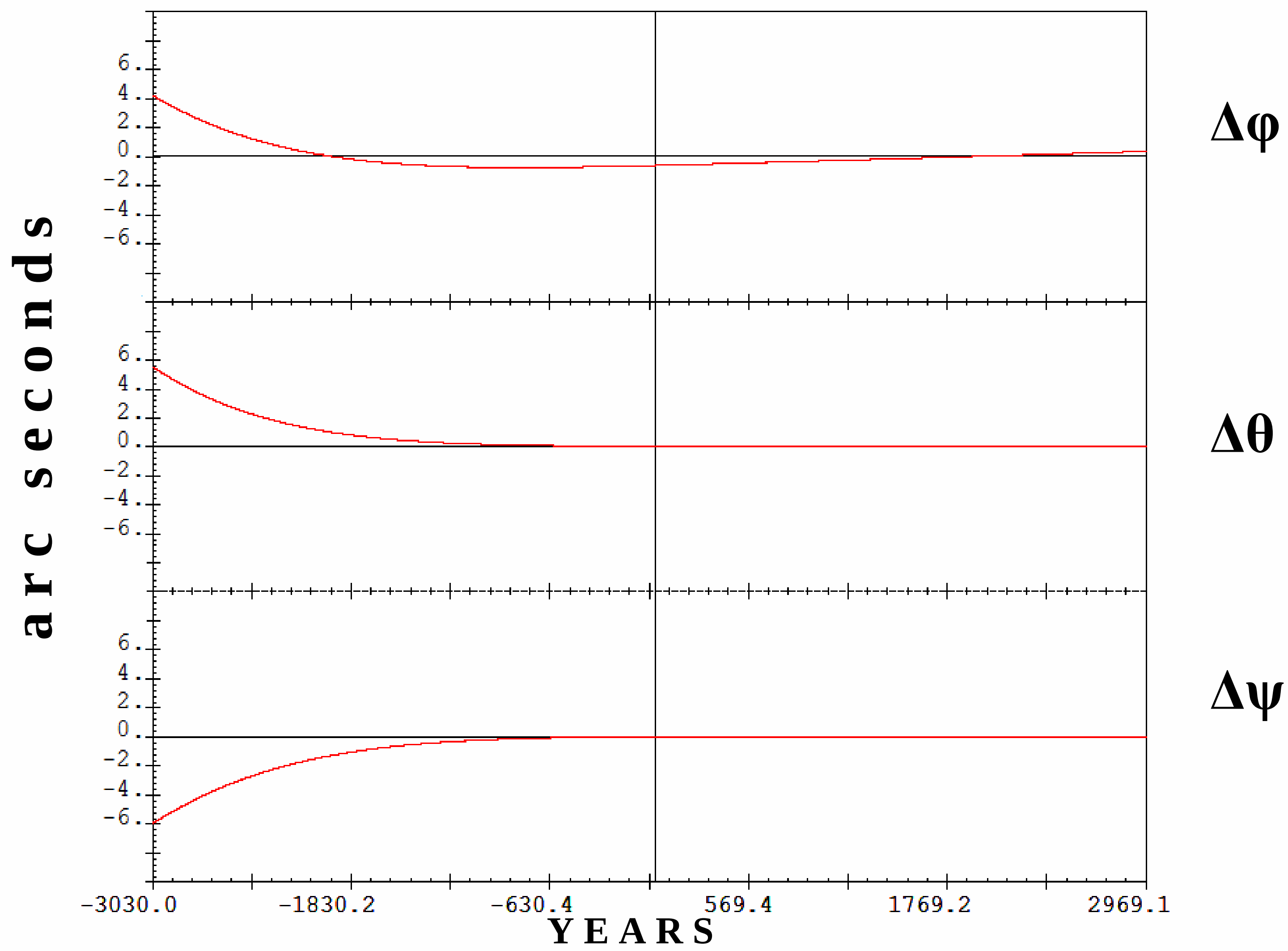


Fig.8. Differences between **the numerical solutions and RERS2012** after formal removal of the secular trends

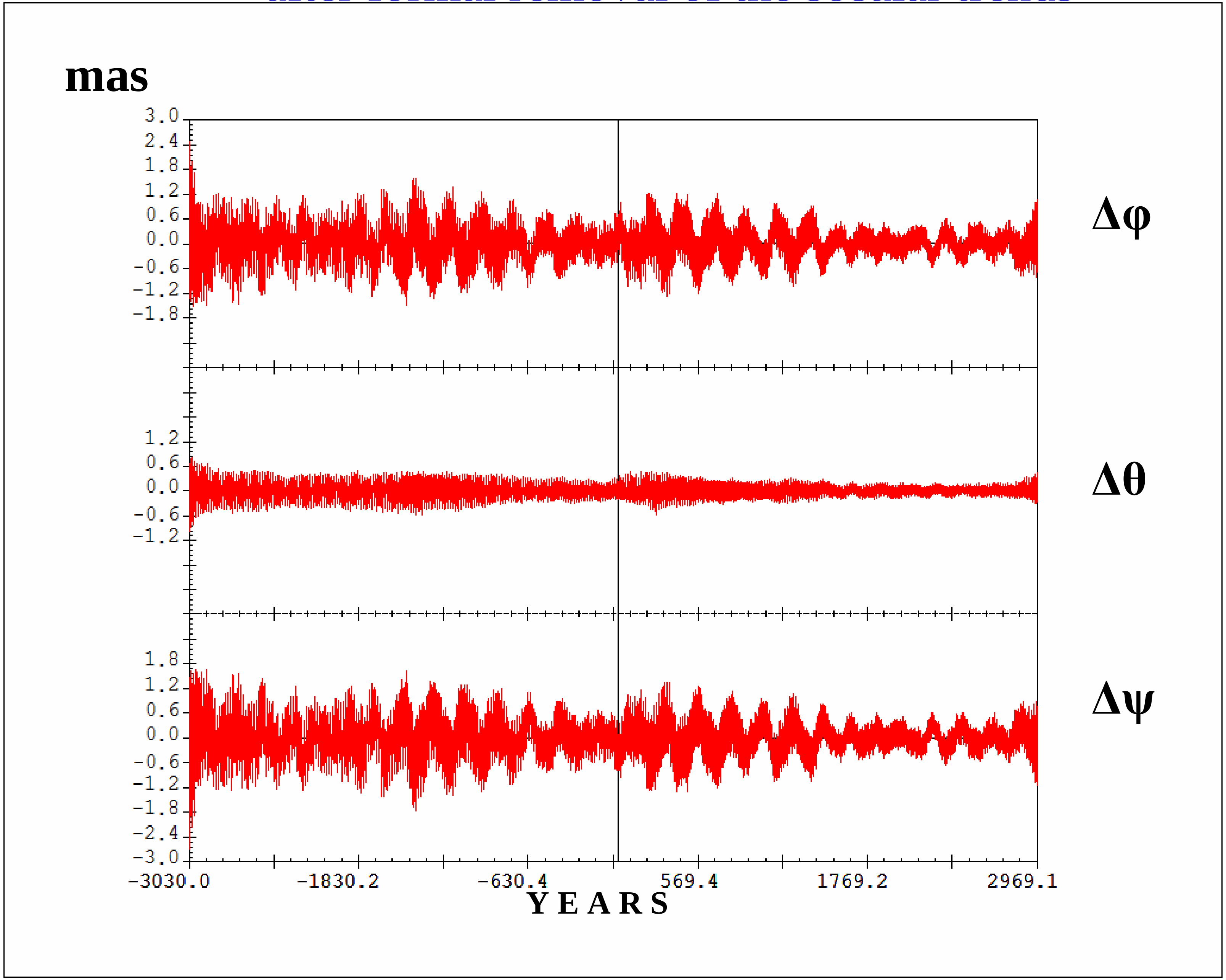


Fig.9. **The numerical solutions minus RERS2012** and **the numerical solutions 2 minus RERS2013-1** (after formal removal of the secular trends)

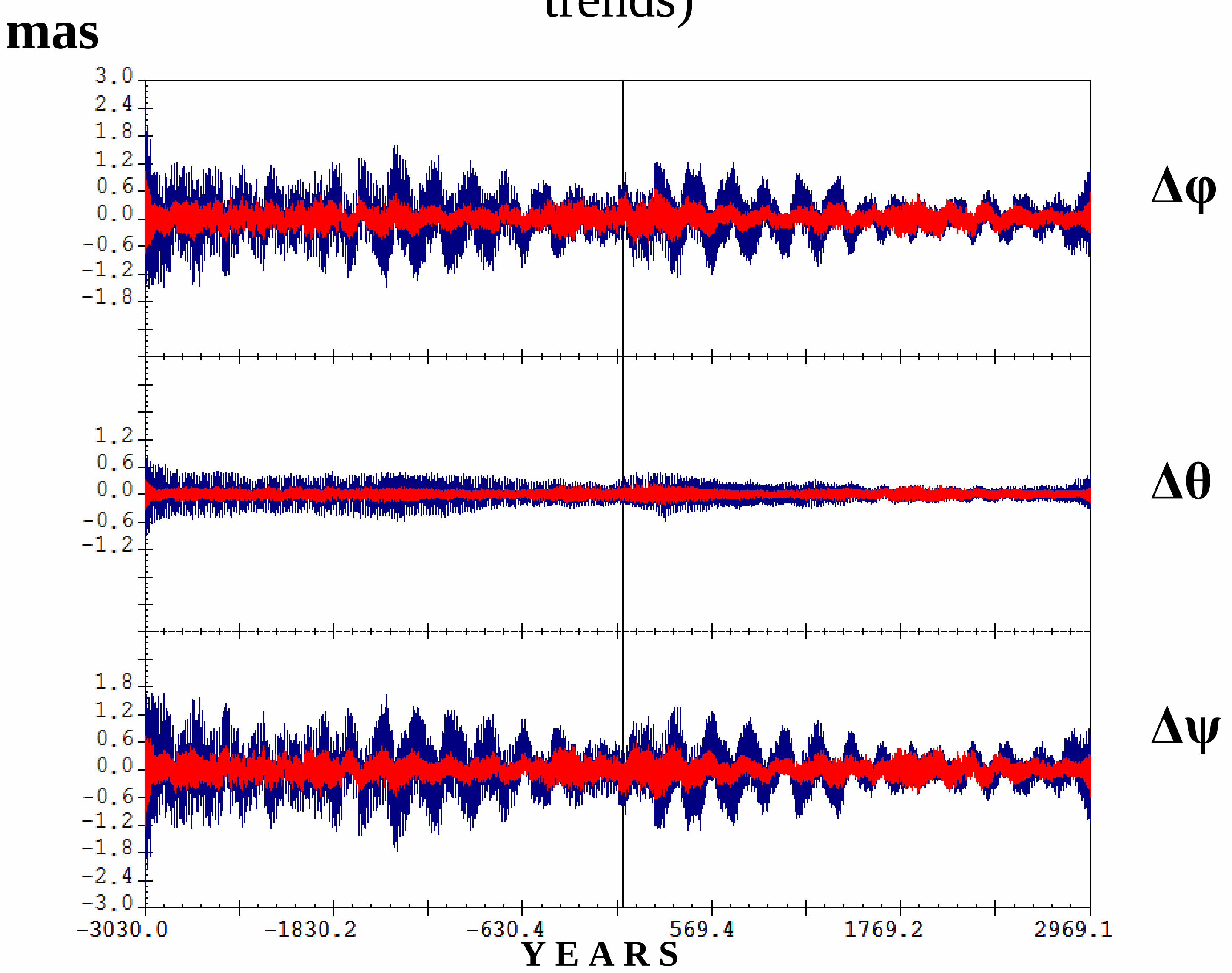


Fig.9. **The numerical solutions minus RERS2012** and **the numerical solutions 2 minus RERS2013-1** (after formal removal of the secular trends)

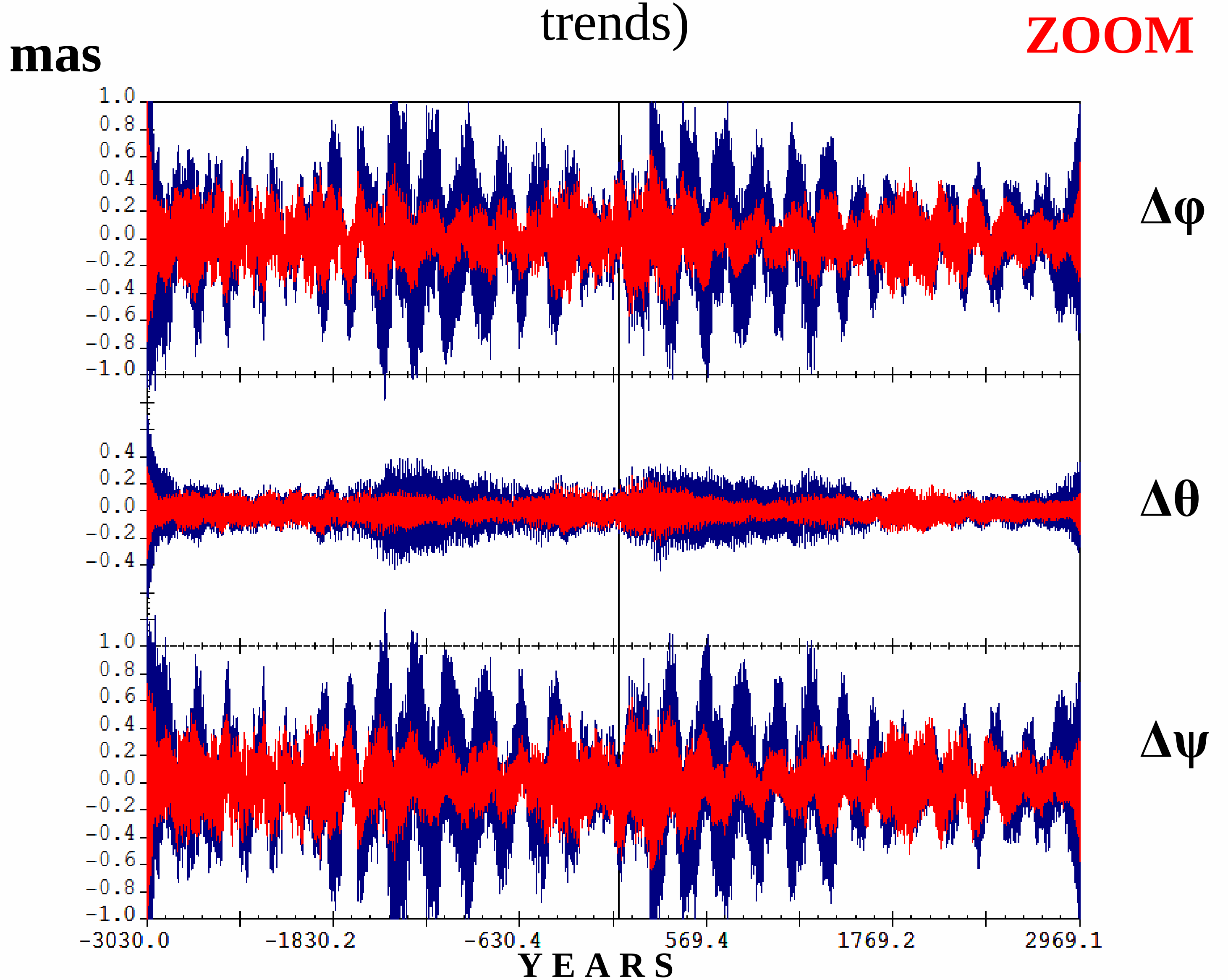


Fig.10. **The numerical solutions 2 minus RERS2013-1** after formal removal of the secular trends in the proper rotation angle

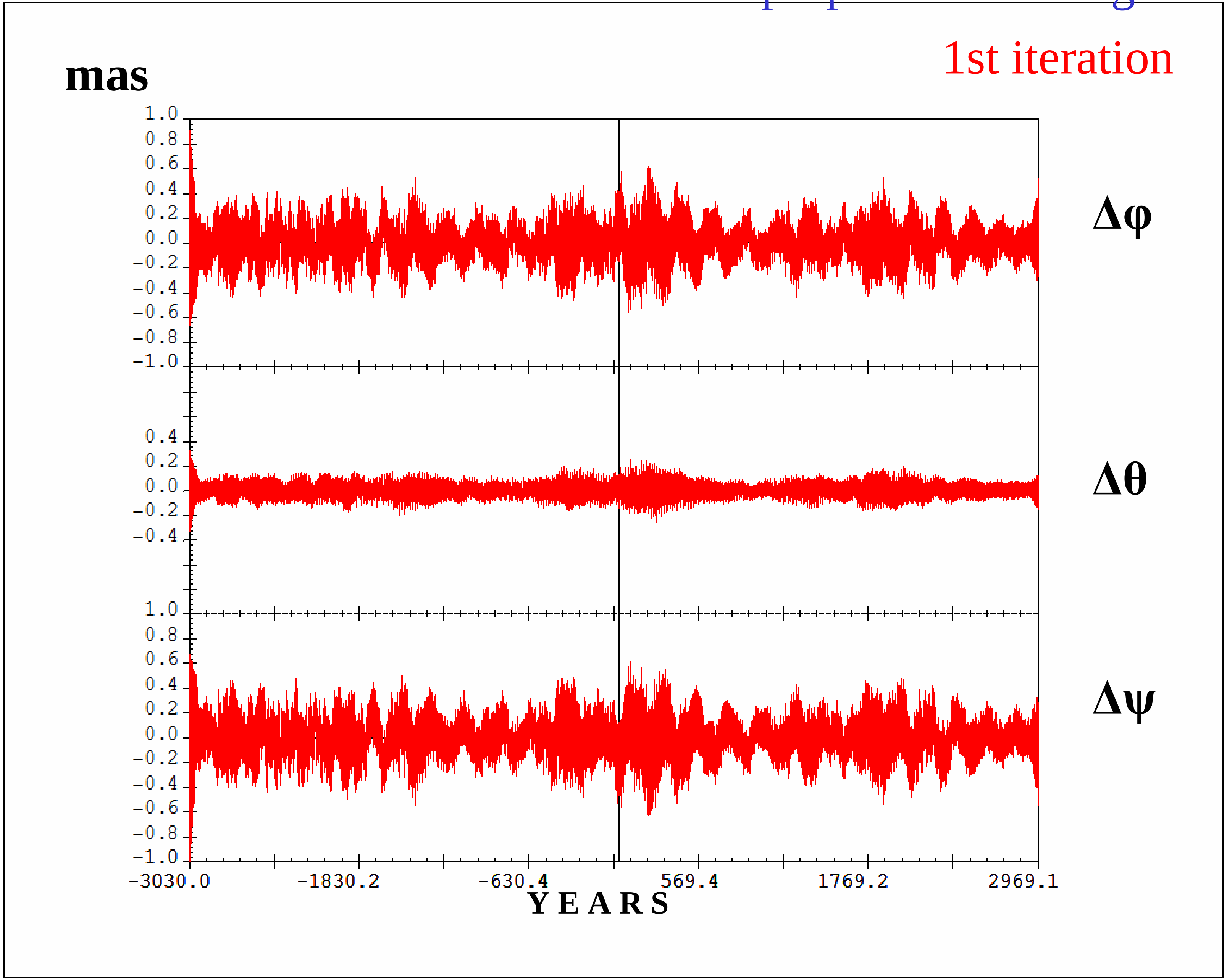


Fig.11. **The numerical solutions 2 minus RERS2013-1** and **the numerical solutions 3 minus RERS2013-2** (after formal removal of the secular trends in the proper rotation angle)

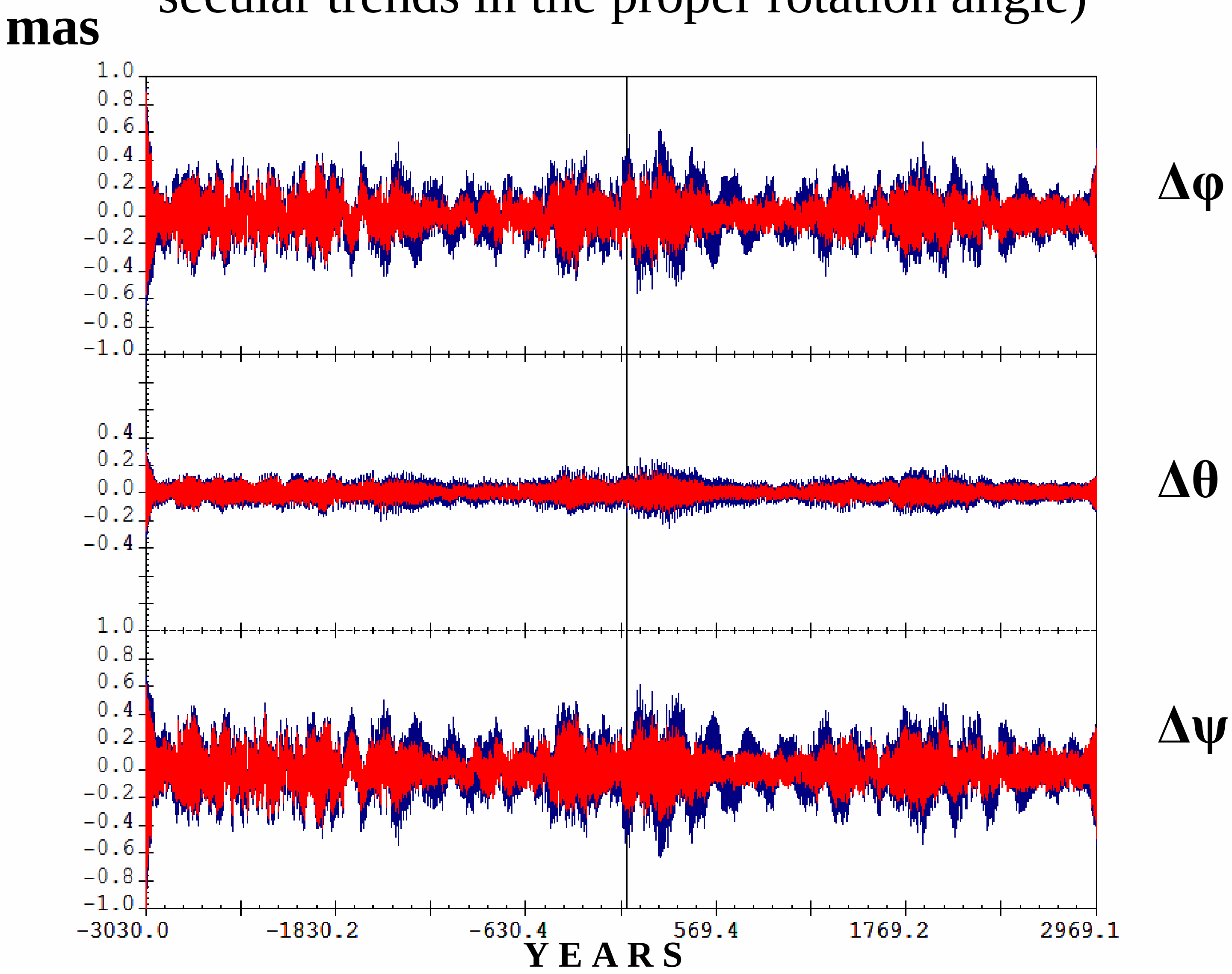


Fig.12. **The numerical solutions 3 minus RERS2013-2** after formal removal of the secular trends in the proper rotation angle

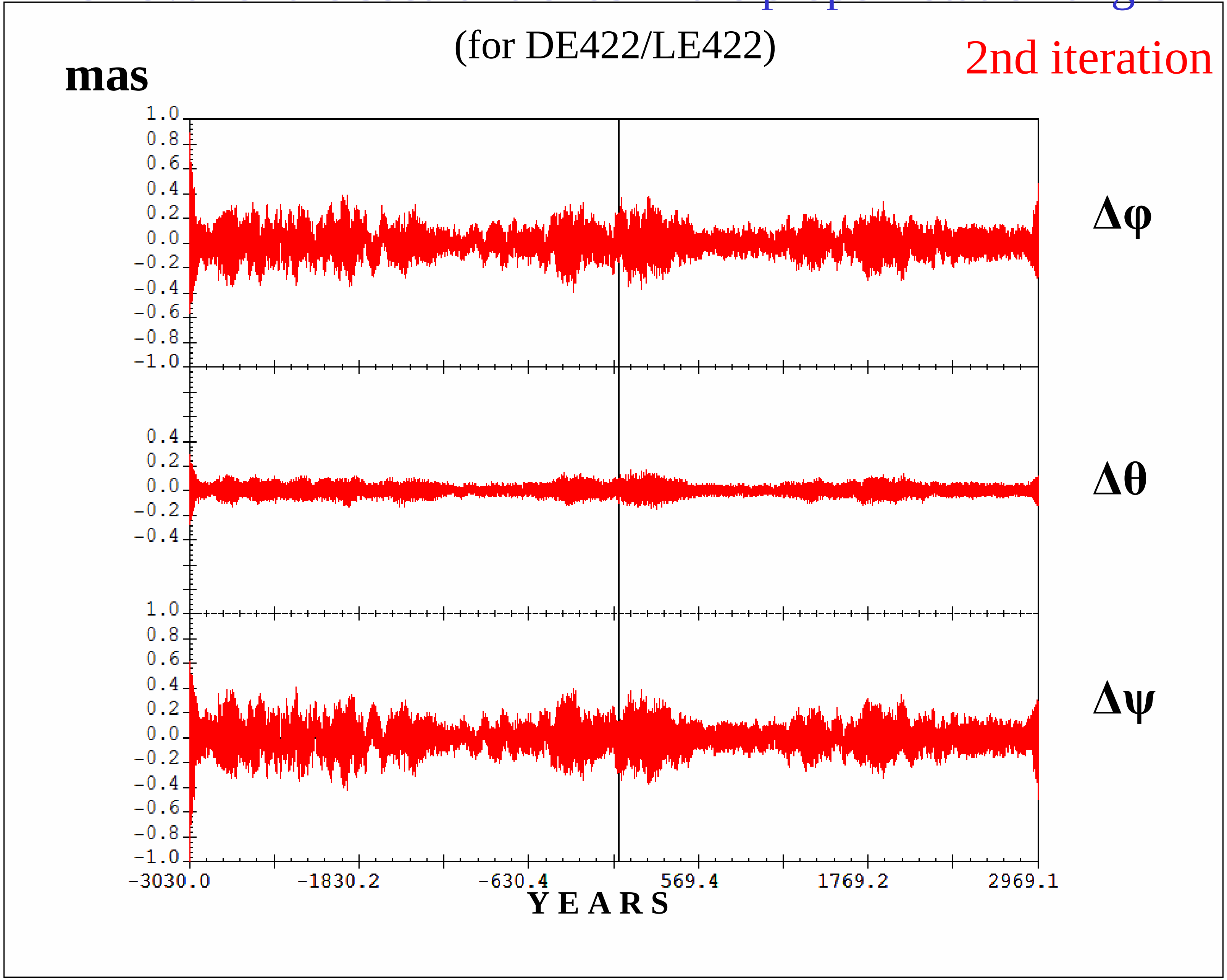


Fig.12a. **The numerical solutions 3 minus RERS2012-2** after formal removal of the secular trends in the proper rotation angle

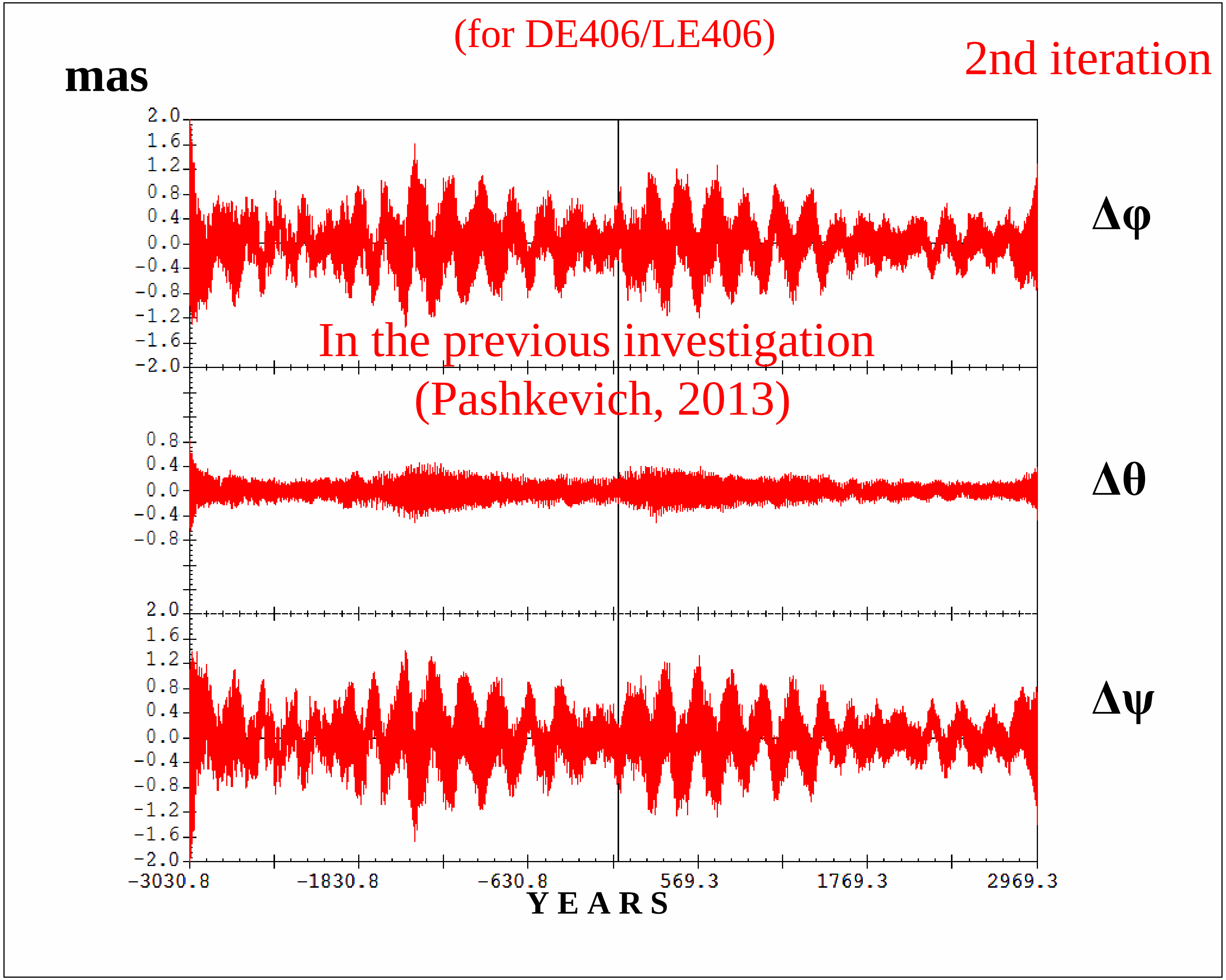


Fig.12. **The numerical solutions 3 minus RERS2013-2** after formal removal of the secular trends in the proper rotation angle

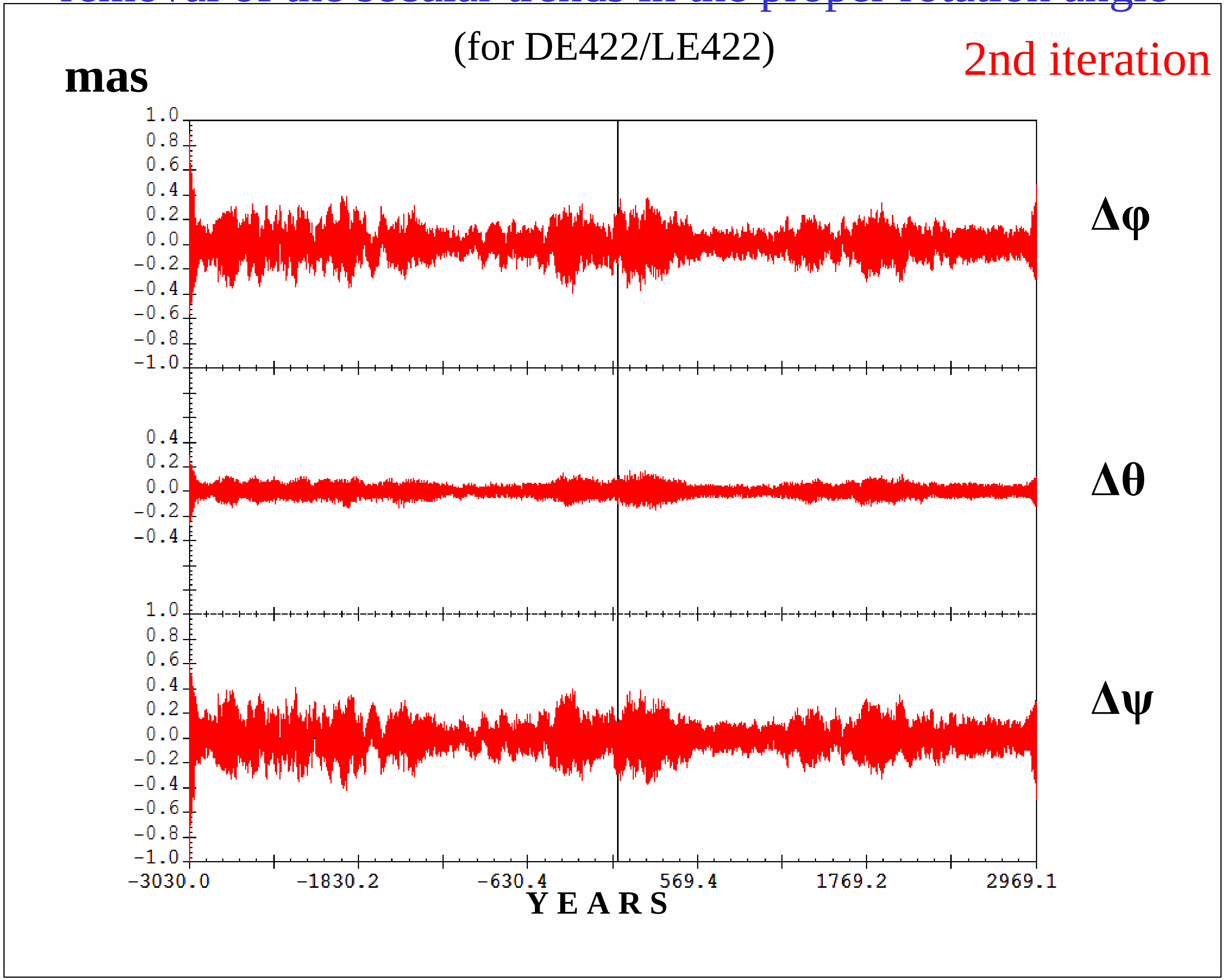


Fig.13. **The numerical solutions 3 minus RERS2013-2** after formal removal of the secular trends in the proper rotation angle and 4113 periodical terms

(for DE422/LE422)

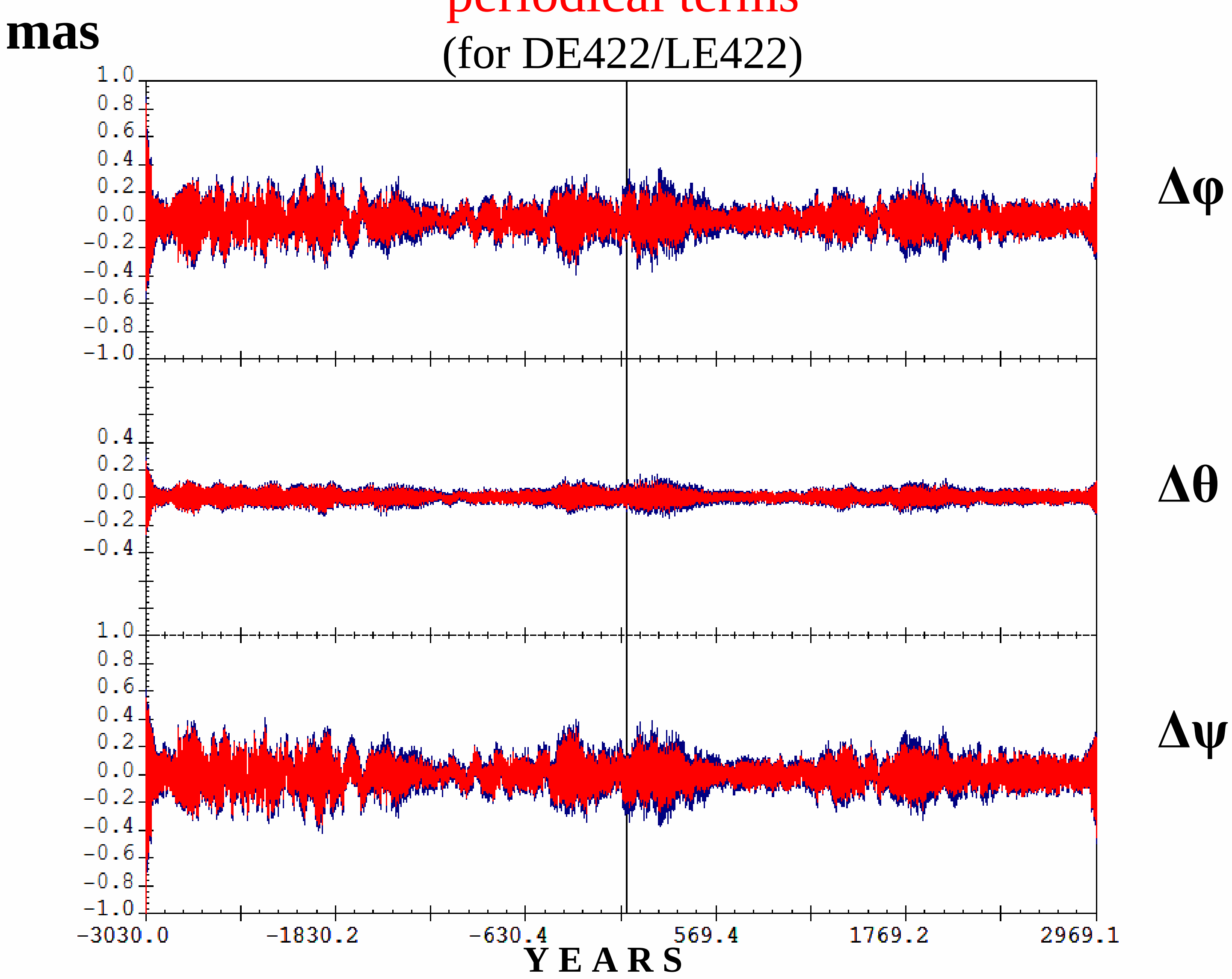
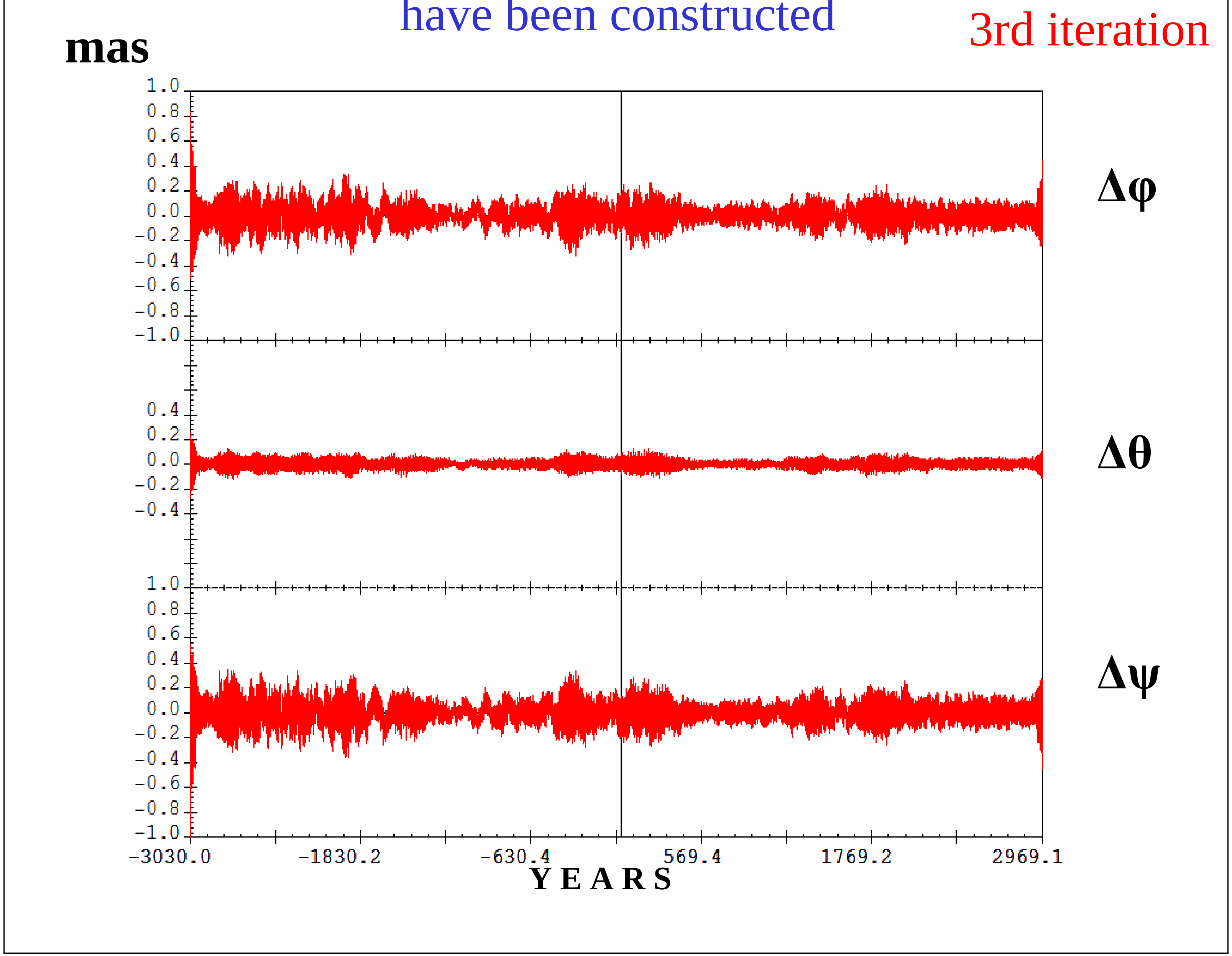


Fig.14. **The new improved high-precision Rigid Earth Rotation Series RERS2013** dynamically adequate to the DE422/LE422 have been constructed



CONCLUSIONS

- **The new improved high-precision Rigid Earth Rotation Series (RERS2013)** dynamically adequate to the DE422/LE422 ephemeris over 2000 and 6000 years have been constructed.
- **RERS2013 include** about 4113 periodical terms (without attempt to estimate new sub-diurnal and diurnal periodical terms). **Discrepancies between the numerical solution and RERS2013** do not surpass:

  4 μas over 2000 years,

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=> it means a good consistency of the **RERS2013** series with the DE422/LE422 ephemeris

=> the **RERS2013** series is more accurate than the **RERS2012** series, which is dynamically adequate to the DE406/LE406 ephemeris.

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A C K N O W L E D G M E N T S

This investigation was carried out at the **Central (Pulkovo) Astronomical Observatory of the Russian Academy of Sciences** and the **Space Research Centre of the Polish Academy of Sciences,** under a financial support of the **Cooperation between the Polish and Russian Academies of Sciences,** Theme No 34.

For the participation in the Journées 2013 was received the **Grant of the Paris Observatory from French Ministry of Research (MESR),** in the framework of the programme ACCES.