# ABOUT HOMOGENEITY IN COMBINED CATALOGS

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**Aims:**

The IAU recommendations regarding the ICRF realizations require the construction of radio sources catalogs obtained using very-long-baseline interferometry (VLBI) methods. The improvement of these catalogs is a necessary procedure for the further densification of the ICRF over the celestial sphere. [1], [2].

The different positions obtained from several catalogs using common sources to the ICRF make it necessary to critically revise the different methods employed in improving the ICRF from several radio sources catalogs. In this sense, a revision of the analytical and the statistical methods is necessary in line with their advantages and disadvantages.

We define homogeneity as applied to our problem in a dual sense: the first deals with the spatial distribution of the data over the celestial sphere. The second has a statistical meaning, as we consider that homogeneity exists when the residual between a given catalog and the ICRF behaves as a unimodal pure Gaussian. We use a nonparametrical method, which enables us to homogeneously extend the statistical properties of the residual over the entire sphere.

A combination of catalogs can only be homogeneous if we configure the weights carefully. In addition, we provide a procedure to detect inhomogeneities, which could introduce deformities, in these combined catalogs.

An inappropriate use of analytical adjustment methods provides erroneous results. Analogously, it is not possible to obtain homogeneous-combined catalogs unless we use the adequate weights.

## Q1. Compiling an accurate catalog from other catalogs

If we want to obtain a Gaussian residual by supposing Y is independent of Z, we have

$$\sigma_U^2=\alpha_1^2\sigma_Y^2+\alpha_2^2\sigma_Z^2$$

Taking

$$\alpha_1=\frac{\sigma_U^2}{2\sigma_Y^2},\alpha_2=\frac{\sigma_U^2}{2\sigma_Z^2}$$

It can be shown that the previous properties are satisfied, because we have obtained a normal residual (at least in a radius equal to the typical deviation and centered in the mean).

## Homogeneity and errors in combined catalogs

Let $\{x_i\}$, $i=1,2,\dots n$ be the ICRF-Ext2 [3] positions, and let $\{x^1_i\}$, $\{x^2_i\}$ $i=1,2,\dots n$ in two other catalogs. Let $\{y_i\}$, $i=1,2,\dots n$ and $\{z_i\}$, $i=1,2,\dots n$ be the residuals for Catalog1-ICRF or Catalog2- ICRF. Suppose that we have $y_i=m^{[1]}(x_i)+\varepsilon_i^{[1]}$, $z_i=m^{[2]}(x_i)+\varepsilon_i^{[2]}$, where each $\varepsilon_i^{[j]}$, $j=1,2$ is a normal random variable with $N(0,\sigma_1^2)$ $N(0,\sigma_1^2)$ respectively and $m^{[1]}$, $m^{[2]}$ models of adjustment. In general, any linear combination of gaussian random variables (RV henceforth) is not a pure Gaussian, but a Gaussian mixture. In the particular case of being a pure Gaussian, we say that we have a homogeneous combined catalog.

The usual procedure employed to obtain a densified catalog from others that are referred to a main catalog is based on finding a new RV, U and a model $m$ from the RV Y, Z and the models $m^{[1]}$, $m^{[2]}$ so that the residual U-$m$(X) verifies that it is a pure Gaussian RV with null expectation and that $\sigma_U^2<\sigma_X^2+\sigma_Y^2$

Two questions arise from this point:

Q1. There are many ways to consider the models $m^{[1]}$, $m^{[2]}$, and the weights. If we have chosen the models, which are the optimum weights to account for the two previous conditions?

Q2 We consider the word homogeneity applied to our problem in a dual sense: the first deals with the spatial distribution of the data over the celestial sphere. The second has a statistical meaning, as we consider that homogeneity exists when the residual between a given combined catalog and the ICRF behaves as a unimodal pure Gaussian. If we have a catalog obtained from another two catalogs, is it possible to know if its construction has been homogeneous?

## Q2. A compiled catalog and its homogeneity

To obtain a new compiled catalog, two or more catalogs are linearly combined so that the errors generally propagate to the final catalog as a Gaussian mixture. We now deal with the inverse problem: How can we find the weights and the variances of a sum of Gaussians? This questions is answered in [5]

## The use of JPL and USNO catalogs to build a combined catalog

In this study, we considered only the sources which have at least 15 observations in two sessions. We have not included some reference sources in our calculus that present oddly high residuals. All values are given in µas.

With respect to the study of the residuals, we have chosen to carry out a preliminary kernel nonparametric (KNP henceforth) [7] adjustment for the $\Delta\alpha\cos\delta$ and $\Delta\delta$ in both catalogs and a vectorial spherical harmonics (VSH henceforth) of first order for the adjustment model. Then, we apply our mixed method [4], [5]. The existence of deformations has required the use of a correction for each catalog given by

$$\min_{C^i}\int_{S^2}\left\{\left[\left(\Delta\alpha\cos\delta\right)^{(i)}-m_\alpha^{[i]}\left(\alpha,\delta\right)\right]^2+\left[\left(\Delta\delta\right)^{(i)}-m_\delta^{[i]}\left(\alpha,\delta\right)\right]^2\right\}dS$$

where $C^i$ are the coefficients of the models $m_\alpha^{[i]}$ and $m_\delta^{[i]}$ with i=1 (USNO) and i=2 (JPL). The results for the coefficients of the VSH of first order are listed in the Table 1. This must be considered in future studies.

Next, we consider only the rotations. We subtract the corrections provided by the rotations to the initial position to obtain the intermediate catalogs USNO[1] and JPL[1]: Cat[1]=Cat-correction, where these corrections depend only on the rotations. The adjustment itself is given by $cat_i^1$-(ICRF-Ext2) $=m^{[i,1]}+\varepsilon^{[i,1]}$ i=1,2

We use now the term WRMS that denotes weighted root mean squared. In our case, the function uses the weights assigned by the KNP adjustment. With regard to the WRMS in the entire sphere where we have used numerical integration and a KNP adjustment, a summary of the data may be seen in Table 2.

**Table 1:** Rotation and deformation parameters for the USNO and JPL.

| | USNO | JPL |
|---|---|---|
| $\varepsilon_x$ | 105.90 | 34.54 |
| $\varepsilon_y$ | -1.19 | 6.87 |
| $\varepsilon_z$ | 13.04 | -82.02 |
| $d_{1,-1}$ | 0.21 | -19.73 |
| $d_{1,0}$ | 35.09 | -29.33 |
| $d_{1,1}$ | 10.18 | 10.28 |

**Table 2:** Mean Squared Errors over the sphere after rotation adjustment.

| | USNO Initial | USNO final | JPL initial | JPL final | Combined catalog |
|---|---|---|---|---|---|
| $\mu_\alpha$ | 7.4 | 7.4 | -1.4 | -1.4 | |
| $\sigma_\alpha$ | -61.1 | 36.4 | 99.5 | 53.7 | 35.6 |
| $\mu_\delta$ | -14.7 | -14.7 | -8.3 | -8.3 | |
| $\sigma_\delta$ | 113.9 | 41.5 | 87.3 | 77.0 | 41.0 |

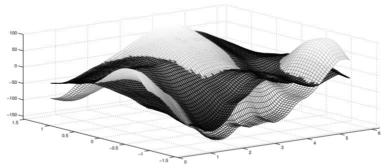

**Figure 1:** Differences in (ICRF-Ext2)-USNO for $\Delta\alpha\cos\delta$ (in µas). The clear surface represents the initial differences, the dark surface represents the differences after the correction given by the rotations.

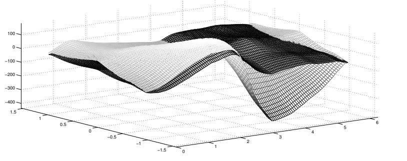

**Figure 2:** Differences in (ICRF-Ext2)-USNO for $\Delta\delta$ (in µas). The clear surface represents the initial differences, the dark surface represents the differences after the correction given by the rotations

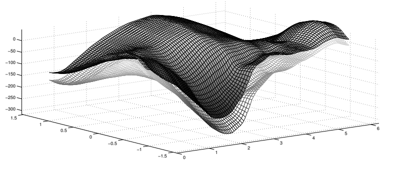

**Figure 3:** Differences in (ICRF-Ext2)-JPL for the $\Delta\alpha\cos\delta$ (in µas). The clear surface represents the initial differences, the dark surface represents the differences after the correction given by the rotations.

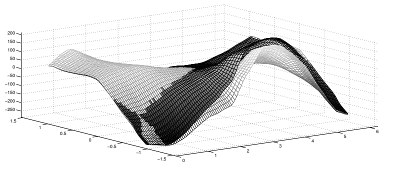

**Figure 4:** Differences in (ICRF-Ext2)-JPL for the $\Delta\delta$ (in µas). The clear surface represents the initial differences; the dark surface represents the differences after the correction given by the rotations.

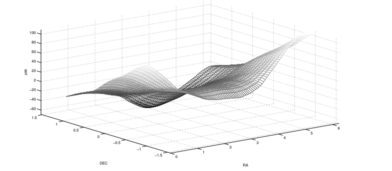

**Figure 5:** Residuals in the final combined catalog for the $\Delta\alpha\cos\delta$ (in µas).

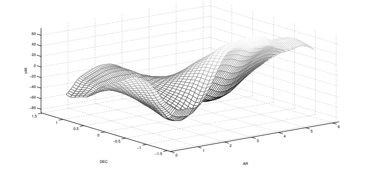

**Figure 6:** Residuals in the final combined catalog for the $\Delta\delta$ (in µas).

## Conclusions

To obtain a homogeneous combined catalog, it is necessary to use adequate weights. Reciprocally, it is possible to determine if a combined catalog has been obtained homogeneously or not.

A mixed method using nonparametrical regression to build an intermediate catalog presents many advantages to obtain accurate parameters:

a) a more robust approach in the sense that the computation of the coefficients is carried out by means of functional scalar products over the sphere. The homogeneous grid selection preserves the functional orthogonality in the process of discretization.
b) There is no need to choose an a priori model.
c) The approximation of the density function by means of a KNP (which generalizes the assignment of weights).

The mixed method with an adequate weights selection has provided us a combined catalog with little final residuals.

We have also proposed a method to decide if a given candidate source may be coherently accepted as a new reference source with the obtained results for the combined catalog. The five sources proposed using our method have been included in the ICRF2, which includes two new defining sources and supports the reliability of our methods.

## Method to densify the reference

Finally, we propose a procedure to densify the reference frame. We work with the candidate sources of the ICRF-Ext2, but it could be also applied to other sources. For this purpose, we apply a simple test to discover if a source belongs to a determined population. The steps to follow are

**Step 1:** We compute $cat_1^C$-(ICRF-Ext2)-$m_{G1}^{[1]}$, where the C refers to the candidate sources and G1 to the rotations provided by the USNO catalog (in $\Delta\alpha\cos\delta$ and $\Delta\delta$). Each function of residuals is represented by a surface. The valid surface levels from the point of view given in the previous guidelines are the ones included in the interval (-29,44) µas for $\Delta\alpha\cos\delta$ and (-55,27) µas for $\Delta\delta$.

**Step 2:** We compute $cat_2^C$-(ICRF-Ext2)-$m_{G2}^{[2]}$, where C refers to the candidate sources and G2 to the rotations provided by the JPL catalog. The resulting intervals are (-55,52) µas for $\Delta\alpha\cos\delta$ and (-85,68) µas for $\Delta\delta$.

**Step 3:** We include the candidate sources in our contour map and we take the ones that are included in the intersection of the areas determined by all the contour lines as preliminary new reference sources.

**Step 4:** Finally, we substitute the selected sources in the final residual function associated with the combined catalog.

The selected sources are:

ICRF J000613.8-062335
ICRF J011343.1+022217
ICRF J014922.3+055553
ICRF J102444.8+191220
ICRF J180024.7+384830

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