AN ACCURATE AND STABLE MIXED METHOD TO OBTAIN COEFFICIENTS IN VSH DEVELOPMENTS OF RESIDUALS FROM ICRF2- CATALOG DIFFERENCES

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Objectives:

- Gaia catalog contains positions of quasars and positions and proper motions of stars in visible wavelengths.
- The update of methods which are useful in the treatment of systematic errors in the celestial sphere is pertinent.
- The study of the residuals in positions and proper motions was carried out by several authors, such as [3] and [6], through the use of a model of adjustment using infinitesimal rotations. Further, in [1] the adjustment using vectorial spherical harmonics (VSH henceforth) for the global differences was introduced, in addition to this model. An exposition about VSH may be seen in [5]. VSH were already used by Wahr in his doctoral thesis [9] about nutation for a non rigid Earth model.
- To study the global differences, we proceed in two different ways:
    1. Direct computation of the coefficients of the considered model with a wide and selected number of common stars to both catalogues and calculation of the coefficients of the model, using the discrete least squares method. The presence of bias in the data suggests the use of an alternative procedure: The previous computation of a residual adjustment over the whole sphere by means of a kernel non parametric adjustment (KNP henceforth).
    2. The hypothesis that the function of adjustment is square integrable over the sphere, allows the application of the method of least squares in its continuous version: The function is replaced by a KNP intermediate adjustment to obtain the coefficients for a selected parametrical model whose coefficients we want to estimate. This procedure was used in [1] for the comparation Hipparcos-FK5 obtaining, for the first time, value of $d_{1,0}$ that must be taken into account, and in [2] for the determination of spin between FK5 and Hipparcos, obtaining results fully compatible to those of [3] and [9]).
- Mignard & Klioner [3] have made a detailed study of VSH in positions and proper motions, providing interesting theoretical and practical results. The authors emphasize the importance of considering, in the adjustment of the residuals of the positions, a particular term of the adjustment in declination, comparable to the one obtained in [1] by us).

Problems in a discrete least squares method. A simple example.

Let us take, a simple random sampling of abscises. We consider the values of the exact function over these points and then we perturb these values with a random noise with a standard deviation 0.5 To avoid problems in the frontier (the chosen frontier is not periodical) we consider two different cases. Te first uses 200 points in the [-1.5, 1.5] interval and the second uses 156 points in the [-1,1] interval. It is easy to see how the discrete least squares method provides inaccurate results. That is why we propose an alternative mixed method.

This alternative method that overcomes the errors that, as it is easy to see, may occur is the Mixed Method in the continuous least squares context.

Let f be a one dimensional function of $L^2([-1,1])$ given as a finite sum of Legendre polynomials. f is generally only known over some discrete points not homogeneously distributed and such values are affected by a random noise. We can obtain reliable results taking an estimation of f, defined over an homogeneously distributed grid. In this case, the functional orthogonality is preserved when we discretize the integral formulas and so, we can obtain the coefficients using a local polynomial regression method of 6th order [9]. The obtained values are trivially obtained . A 96% of the power of the function is recovered (being its value 3.892).

	ε_x	ε_y	ε_z	$d_{1,0}$	$d_{1,1}$	$d_{1,-1}$
[4]	-18.1	-14.6	-18.5	-64	-1.3	-18.3
[1]	-21.4	-18.6	-20.4	-60.1	-0.9	-25.7
VSH(1)	-20.2	-20	-18.5	-61.8	-6.3	-15.2
VSH(2)	-18.5	-13.6	-18.26	-61.8	-2.0	-18.9

Table 1: Global orientation and glide between the FK5 and the Hipparcos catalog, in mas, (1991.25). Results for [4] and [1] were obtained using the common stars and VSH have been obtained using a Mixed Method over the whole sphere. Coupled equations in VSH(1) line and decoupled in VSH(2).

Coefficients of VSH developments

The case of the vectorial spherical harmonics developments is shown in detail in [1] We have seen two different approaches to the problem as well as the obtained results for a simple example. Following the continuous line of work, it is necessary to perform certain mathematical hypothesis of regularity, regarding the type of function that we want to find. With the usual method (VSH(1)) we get coupled equations, which is not relevant for first order (see numerical result in VSH(1) in table 2). Nevertheless, we implement the problem more directly.

Let us consider the vectorial field $\Delta\mathbf{X}\equiv\mathbf{V}(\alpha,\delta)=V^{\alpha}(\alpha,\delta)\mathbf{e}_{\alpha}+V^{\delta}(\alpha,\delta)\mathbf{e}_{\delta}=(\Delta\alpha\cos\delta)\,\mathbf{e}_{\alpha}+\Delta\delta\,\mathbf{e}_{\delta}$ being $V^{\alpha}(\alpha,\delta)$, $V^{\delta}(\alpha,\delta)$ the scalar fields of the residuals and $\mathbf{e}_{\alpha}$, $\mathbf{e}_{\delta}$ the unitary vectors in the tangent plane and in the directions of the right ascension and declination, respectively.

On the other hand, and provided that we are in the surface of the unitary sphere, the only vectorial spherical harmonics involved are the spheroidal spherical harmonics $\mathbf{S}_{l,m}$ and the spherical toroidal harmonics $\mathbf{T}_{l,k}$. We suppose that the field $\mathbf{V}$ accepts a development:

$$\mathbf{V}(\alpha,\delta)=\sum_{k\geq 1}\sum_{l=-k}^{k}\left[t_{l,m}\mathbf{T}_{l,m}+s_{l,m}\mathbf{S}_{l,m}\right] \qquad t_{l,m}=\frac{\int_{S^2}\mathbf{V}\mathbf{T}_{k,l}}{\left\|\mathbf{T}_{k,l}\right\|^2} \quad s_{l,m}=\frac{\int_{S^2}\mathbf{V}\mathbf{S}_{k,l}}{\left\|\mathbf{S}_{k,l}\right\|^2}$$

For the calculation, in points regularly spaced over the sphere, of $\mathbf{V}$ we can use the kernel regression method [7] or a polynomial kernel regression. The first one is more economic computationally and it is accurate enough for the problem that we are dealing to. It is important to emphasize that, once the adjustment has been established for $\mathbf{V}$ over the set of points, this same set can be used in the numerical integration of the numerators. Thus we can easily calculate the estimations for higher orders of harmonics. The results up to order one are listed in Table 2 in the VSH(2) line and are only slightly different from the VSH(1) results.

Conclusions

The discrete least squares methods may lead to instability and inaccuracy. In contrast, a greater accuracy and efficiency may be reached using the continuous least squared formulation discretized using our proposed Mixed Method. In the Legendre polynomials example this Mixed Method recovers the 96% of the power and the variance of the real residual (so, the entire signal+noise has been recovered). The power that has not been recovered is due to numerical truncations or discretizations.

The Mixed Method does not need to fix a priori an order for the adjustment. The stop criterion is to retrieve a % of the power (96% as is the case, it seems sufficient), so it is a non-linear adjustment Greedy method. The discrete least squares method is highly inefficient, and with each addition of a new order of the adjustment, all the coefficients must be recalculated again. Finally, the power theoretically recovered is false. There is no indication of a relationship between the recovered power of the function and the obtained coefficients.

In conclusion the Mixed Method proposed is better in efficiency and stability, which also involves an accurate recovery of the function and, as a complement, an estimate of the noise in the data that is perfectly determined.

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