Aberration in proper motions for Galactic stars

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Outline

- Acceleration of the SSB and aberration in proper motions
- Theoretical expressions
- More suitable formulas for short periodic stars
- Discussion
Introduction

- **Origin:** acceleration of the SSB, known as secular/Galactic aberration (drift) for extragalactic sources

- **Dipolar proper motions and its impact on ICRF and EOP**


- **Rotation of the ICRS:** \(0.1 \mu\text{as yr}^{-1} < \omega_{x,y,z} < 1 \mu\text{as yr}^{-1}\) depending on the distribution of sources

- Sec. 7. Time dependent aberration of stars in the Galaxy

Basic assumption: central Newtonian force toward the GC

![Diagram of apparent proper motions of stars](image)

\[ \Delta \mu_s = \frac{-V_c^2 R \sin \zeta}{c(R^2 + \rho^2 - 2R\rho \cos \zeta)} \]
Theoretical Expressions

Planetary aberration in positions

\[ \Delta p_{PL} = \frac{1}{c} p \times [(v^E - v^{PL}) \times p] \]

Higher hierarchy reference system

Aberration in proper motions

\[ \Delta \dot{p}^S = \frac{1}{c} p \times [(a^B - a^S) \times p] \]

Apparent proper motions in Galactic longitude and latitude

\[ \Delta \mu_\ell \cos b = \Delta \dot{p}^S \cdot e_\ell = \frac{1}{c} p \times [(a^B - a^S) \times p] \cdot e_\ell \]

\[ \Delta \mu_b = \Delta \dot{p}^S \cdot e_b = \frac{1}{c} p \times [(a^B - a^S) \times p] \cdot e_b, \]

Galactic aberration constant for the SSB

\[ A^B = \frac{a^B}{c} = \frac{V_0^2}{c R_0} \approx 5 \, \mu \text{as yr}^{-1} \]

Session 1 poster by Z. Malkin.

and define:

\[ \gamma = \frac{a^S}{a^B} \]

\[ = - A^B \left( 1 - \gamma \frac{R_0}{d} \right) \sin \ell \]

\[ = - A^B \left( 1 - \gamma \frac{R_0}{d} \right) \cos \ell \sin b \]
Simplified rotation curve of the Galactic disk

\[ \gamma = \frac{R}{d} \]

\[ \Delta \mu_\ell \cos b = -A^B \left[ 1 - \left( \frac{R_0}{d} \right)^2 \right] \sin \ell \]

\[ \Delta \mu_b = -A^B \left[ 1 - \left( \frac{R_0}{d} \right)^2 \right] \cos \ell \sin b. \]
Simplified rotation curve of the Galactic disk

\[ \gamma = \frac{R_0 d}{d_0^2} \]

\[ \Delta \mu_\ell \cos b = -A^B \left[ 1 - \left( \frac{R_0}{d_0} \right)^2 \right] \sin \ell \]

\[ \Delta \mu_b = -A^B \left[ 1 - \left( \frac{R_0}{d_0} \right)^2 \right] \cos \ell \sin b \]
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The light travel time issue

- Approximation in the basic equation \( \Delta \hat{p}^S = \frac{1}{c} p \times [(a^B - a^S) \times p] \) (*)
- How to deal with non-rectilinear acceleration during the light travel time from the star to the observer?

Define the ratio of the orbital period and the light time \( \tau \).

\( \tau \gg 1 \) means (*) is appropriate.

Flat rotation curve:

\[
\tau = \frac{P}{T_L} = 2\pi \left( \frac{d}{r} \right) \left( \frac{c}{V_S} \right)
\]

Within the bulge region

\( \tau \simeq 320 \) \( (d < d_0 = 0.3 \text{ kpc}) \)
Example: $S2$ star near the central black hole.

<table>
<thead>
<tr>
<th>Table A1. Elements of the Keplerian orbit of S0-2.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Element of the orbit</strong></td>
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<tr>
<td>Central mass</td>
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<tr>
<td>Semimajor axis</td>
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<tr>
<td>Separation of pericentre</td>
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<td>Eccentricity</td>
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<td>Period</td>
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<td>Pericentre passage</td>
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<td>Inclination</td>
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<tr>
<td>Angle of line of nodes</td>
</tr>
<tr>
<td>Angle of node of pericentre</td>
</tr>
</tbody>
</table>

*Note: The table is grabbed from table 4 of Schödel et al. (2003).*

Light time from S2 to the SSB:

$$T_L \simeq 8000 \, pc \times 3.26 \, yr \, pc^{-1} = 26000 \, yr,$$

Apparent proper motions:

$$\Delta \mu_\alpha \cos \delta \simeq a^s T_L \frac{\Delta x}{d} \frac{1}{R_0},$$

$$\Delta \mu_\delta \simeq a^s T_L \frac{\Delta y}{d} \frac{1}{R_0}$$

unrealistic large proper motions
A more suitable expression

- The previous equation may be inappropriate for short periodic stars
- Integer multiple of periods: the aberrational effect is zero
- Define: \( P_f = \text{remainder of } T_L \text{ divided by } P \)
- Effective aberration in proper motions: 
  \[
  \Delta \dot{p}^S = \frac{1}{c} \frac{p}{P} \times \left( \frac{a^B - \frac{v_2^S}{T_L}}{T_L} \right) \times p
  \]
- Apparent proper motions:
  \[
  \Delta \mu_\alpha \cos \delta = \frac{1}{\kappa} \frac{v_{2,x} - v_{1,x}}{r}, \quad \Delta \mu_\delta = \frac{1}{\kappa} \frac{v_{2,y} - v_{1,y}}{r}
  \]
- True proper motions at \( t_1 \):
  \[
  [\Delta \mu_\alpha \cos \delta]^{\text{true}} = \frac{1}{\kappa} \frac{v_{1,x}}{r}, \quad [\Delta \mu_\delta]^{\text{true}} = \frac{1}{\kappa} \frac{v_{1,y}}{r}
  \]
Observed proper motions at $t_1$:

\[
\begin{align*}
[\Delta \mu_\alpha \cos \delta]_{1}^{\text{obs}} &= \frac{1}{\kappa} \frac{v_{2,x}}{r} = [\Delta \mu_\alpha \cos \delta]_{2}^{\text{true}} \\
[\Delta \mu_\delta]_{1}^{\text{obs}} &= \frac{1}{\kappa} \frac{v_{2,y}}{r} = [\Delta \mu_\delta]_{2}^{\text{true}}.
\end{align*}
\]

$P_f << T_L$:

- The effect of aberration will have no effect on the shape of the orbit
- Only change the observed phase of stars on the orbit

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**Systematic effect on the Gaia stellar reference frame**

- Global rotation: larger than the effect of Galactic aberration (μas level)
- Small distortion also exits.
- Important in the GaiaCRF?
Discussion

- Non-disk stars (halo, bulge)
- More realistic model of Galactic kinematics is necessary.
- Observable? (Embedded in stellar proper motions.)

Kovalevsky (2003): “Applying this correction corresponds to changing from the Barycentric Celestial Reference Frame (BCRS) to a Galactocentric Celestial Reference Frame (GalCRS).”

The nomenclature issue for relevant effects.

Thank you!