Corrections to the IERS amplitudes of variations of geopotential coefficients due to frequency dependence of Love numbers

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ABSTRACT: A new algorithm is proposed for calculating amplitudes of the variations of geopotential coefficients for frequency dependence of Love numbers. Unlike the approach presently recommended by the IERS Conventions (2010), the new algorithm uses the representation of the Earth tide-generating potential (TGP) in the standard HW95 format and takes into account the phase of tidal waves. Corrections of up to 2 × 10^{-12} to the published by the IERS Conventions (2010) amplitudes of the corresponding variations of the geopotential coefficients are suggested.

I. INTRODUCTION AND GENERAL DEFINITIONS

The main effect of the solid Earth tides on the Earth gravitational potential ("Step 1") can be described through variations $\Delta\overline{C}^{ST}_{nm}$, $\Delta\overline{S}^{ST}_{nm}$ in the instant values of the normalized standard geopotential coefficients of degree *n* and order *m* (Eanes et al. 1983)

$$\Delta\overline{C}^{ST}_{nm} - i\Delta\overline{S}^{ST}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{\mu_j}{\mu_E}\left(\frac{R_E}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j}$$

where k_{nm} are frequency independent complex Love numbers; R_E, μ_E are, respectively, the Earth's equatorial radius and gravitational parameter; μ_j, r_j, Φ_j and λ_j are, respectively, the gravitational parameter, geocentric distance, geocentric latitude and east longitude (from Greenwich) of the Moon (*j*=2) and Sun (*j*=3) at epoch *t*; $\overline{P}_{nm}$ are the normalized associated Legendre functions.

Anelasticity of the Earth's mantle leads to the **frequency dependence of the Love numbers**. Therefore, as the "Step 2", the IERS Conventions (2010) recommend to compute some additional corrections to the gravitational coefficients due to deviations of the degree 2 Love numbers from their nominal values. The recommended formulae use the tide heights values from the Earth TGP expansion by Cartwright and Tayler (1971), Cartwright and Edden (1973). However, these tide heights values are not employed by the modern HW95 format (Hartmann and Wenzel 1995), which now is a common standard for representation of modern TGP catalogs. The known factors for conversion of amplitudes of tidal terms from Hartmann and Wenzel's conventions to Cartwright-Tayler-Edden's ones (IERS Conventions 2010) do not take into account the phase of the tidal waves. The latter are zero in all tide terms in the TGP catalog by Cartwright and Tayler (1971), Cartwright and Edden (1973), but the phases are **not** zero in many waves in the modern TGP catalogs, including several waves used in computing the tidal corrections at the "Step 2".

The following sections present amplitudes of the tidal corrections to geopotential coefficiients due to the frequency dependence of Love numbers. They use the Earth TGP development represented in the HW95 format and take into account the phases of tidal waves.

II. NEW FORMULAE FOR AMPLITUDES OF THE CORRECTIONS

The Earth TGP development written in the HW95 format represents the TGP value $V(t)$ at point $P(r,\Phi,\lambda)$ on the Earth surface as

$$V(t) = \sum_{n=1}^{n_{\max}}\left(\frac{r}{R_E}\right)^n\sum_{m=0}^{n}\overline{P}_{nm}(\sin\Phi)$$
$$\times\sum_{f(n,m)}\left(\overline{C}^*_{nmf}(t)\cos\omega^*_{nmf}(t)+\overline{S}^*_{nmf}(t)\cos\omega^*_{nmf}(t)\right),$$

where $\overline{C}^*_{nmf}(t)=\overline{C}^*_{0nmf}+\overline{C}^*_{1nmf}t$, $\overline{S}^*_{nmf}(t)=\overline{S}^*_{0nmf}+\overline{S}^*_{1nmf}t$,

and $\overline{C}^*_{0nmf}, \overline{C}^*_{1nmf}, \overline{S}^*_{0nmf}, \overline{S}^*_{1nmf}$ are numerical coefficients,

ω^*_{nmf} are arguments based on the development frequencies $f(n,m)$.

Then the in-phase $A^{(ip)}_{2mf}$ and out-of-phase $A^{(op)}_{2mf}$ amplitudes are:

$$A^{(ip)}_{20f} = \frac{R_E}{\mu_E}\left(\delta k^R_{20f}\overline{C}^*_{20f}+\delta k^I_{20f}\overline{S}^*_{20f}\right),$$

$$A^{(op)}_{20f} = \frac{R_E}{\mu_E}\left(-\delta k^R_{20f}\overline{S}^*_{20f}+\delta k^I_{20f}\overline{C}^*_{20f}\right),$$

$$A^{(ip)}_{21f} = \frac{R_E}{\mu_E}\left(\delta k^R_{21f}\overline{S}^*_{21f}-\delta k^I_{21f}\overline{C}^*_{21f}\right),$$

$$A^{(op)}_{21f} = \frac{R_E}{\mu_E}\left(\delta k^R_{21f}\overline{C}^*_{21f}+\delta k^I_{21f}\overline{S}^*_{21f}\right)$$

$$A^{(ip)}_{22f} = \frac{R_E}{\mu_E}\delta k^R_{22f}\overline{C}^*_{22f}, \quad A^{(op)}_{22f} = -\frac{R_E}{\mu_E}\delta k^R_{22f}\overline{S}^*_{22f}.$$

III. UPDATED VALUES FOR AMPLITUDES

Table 1 *Values for in-phase amplitudes* $A^{(ip)}_{21f}$ *, units:* 10^{-12}

f, deg/h	Doodson No.	IERS Conventions (2010)	This study
15.07749	166,455	0.1	0.2
15.08214	166,554	-20.6	-20.5

Table 2 *Values for in-phase amplitudes* $A^{(ip)}_{22f}$ *, units:* 10^{-12}

f, deg/h	Doodson No.	IERS Conventions (2010)	This study
28.43973	245,655	-0.3	0.2
28.98410	255,555	-1.2	0.8

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Ref: Kudryavtsev S.M. (2013), *Celest. Mech. Dyn. Astron.*, v.115, 353-364

JOURNÉES 2013: Systèmes de Référence Spatio-Temporels, 16-18 September 2013, Observatoire de Paris, Paris, France