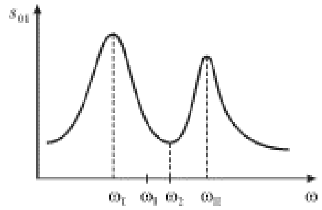

Approximation of Orbital Elements of Telluric Planets by Compact Analytical Series

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Modern Representations of Planetary/Lunar Coordinates

1. Numerical ephemerides

- DE-series (JPL NASA, the USA)
- EPM-series (IAA RAS, Russia)
- INPOP- series (IMCCE/Observatoire de Besançon, France)

Advantage: *high precision*

Disadvantages: *take up to several Gb;*

depend on the computer platform

2. Analytical/semi-analytical theories

- done at BDL/IMCCE, USNO, …

the latest theory is VSOP2013/TOP2013 (Simon et al. 2013)

Advantages: *compactness, computer platform independence*

Disadvatages: *it is relatively difficult to develop them;*

the theories still have to be fitted to numerical ephemerides

Modern Representations of Planetary/Lunar Coordinates (cont.)

3. Frequency analysis of planetary/lunar ephemerides, e.g.:

- Chapront (1995): planetary ephemerides
- Kudryavtsev (2007): lunar ephemeris, LE**A**-406 expansion
- the present study: planetary ephemerides, DE**A**-expansions

***Advantages**: compactness,*
computer platform independence,
accuracy of numerical ephemeris can be about reached

***Disadvantages**: the "close frequencies" problem,*
but it can be solved (diminished) by use of long-term numerical ephemerides (the longer, the better)
-> new spectral analysis methods are required

Spectral Analysis to Poisson Series

Let $f(t)$ be sampled over an interval [-T, T] with a small step.

We shall find representation of $f(t)$ in the form

$$f(t) \approx A_0 + A_1 t + \ldots + A_p t^p + \sum_{k=1}^{N} \left\{ \left[A_{k0}^c + A_{k1}^c t + \ldots + A_{kp}^c t^p \right] \cos \omega_k(t) + \left[A_{k0}^s + A_{k1}^s t + \ldots + A_{kp}^s t^p \right] \sin \omega_k(t) \right\}$$

where $\omega_k(t)$ are some pre-defined polynomilal arguments:

$$\omega_k(t) = \nu_k t + \nu_{k2} t^2 + \ldots + \nu_{kq} t^q \quad (\omega_0(t) \equiv 0).$$

- At the 1st step we numerically calculate the scalar products:

$$A_{lk}^c = \frac{1}{2T} \int_{-T}^{T} f(t) t^l \cos \omega_k(t) \chi(t) dt, \quad A_{lk}^s = \frac{1}{2T} \int_{-T}^{T} f(t) t^l \sin \omega_k(t) \chi(t) dt,$$

basis functions

$\chi(t) = 1 + \cos \frac{\pi}{T} t$ is the weighting function (the Hanning filter)

- At the 2nd step the basis must be orthogonalized

(! $N \sim 10^3$-10^4; arguments are high-degree time polynomials).

Expansion of Mean Longitude of Telluric Planets over 3000BC-3000AD

Planet	Maximum difference		
	DE**A**406 - DE406	DE**A**424 - DE424	VSOP2013(*) - INPOP10a
Mercury	0.014"	0.0016"	0.163"
Venus	0.035"	0.015"	0.061"
EMB	0.022"	0.019"	0.343"
Mars	0.068"	0.056"	1.743"

(*) For the 3000BC-3000AD time interval the accuracy of VSOP2013 representation by Chebyshev polynomials is only reported. However, it is of the same order as accuracy of the original solution.

Expansion of Mean Longitude of Telluric Planets over 3000BC-3000AD (cont.)

Planet	Number of Poisson terms		
	DEA406	DEA424	VSOP2013 (*)
Mercury	331	767	28251
Venus	450	648	29979
EMB	586	535	31440
Mars	948	770	32418

(*) The number of the 0^{th}-order Poisson terms only.

Expansion of Mercury Orbital Elements from DE424 over 3000BC-3000AD

Element	**Maximum Difference** DE**A**424 – DE424	**Number of terms**
a	2.1×10^{-9} a.e.	479
e	7.5×10^{-9}	623
i	0.0004"	282
Ω	0.0013"	569
π	0.0032"	900
λ	0.0016"	767

Perspectives

- Use of new DE431 ephemerides (publ. on 15 Aug. 2013); it covers 30,000 years: 13,000BC – 17,000AD

First use: **Mean Longitude of Mercury** (898 terms)

Time Interval	Maximum Difference	
	DE**A**431-DE431	VSOP2013-INPOP10a
900AD - 3,100AD	0.0015"	0.01"
4,000BC - 8,000AD	0.002"	0.12"
13,000BC-17,000AD	0.030"	n/a

- Use of VSOP2013 secular terms for mean longitudes of planets and largest asteroids and TOP2013 argument μ.