

Sofiane Khelifa

Centre of Space Techniques, PO Box 13, 31200, Arzew, Algeria

Correspondence to: Sofiane Khelifa (khelifa_sofiane@yahoo.fr)

Abstract

Using wavelet transform and Allan variance, we have analyzed three different solutions (IGN, INA and LCA) of DORIS station coordinates, in order to compare the spectral characteristics of their residual noise. The temporal correlations between the three solutions, two by two and station by station, in each component (North, East and Vertical) reveal a high correlation in the horizontal components. In the North component, the correlation average is about 0.88, 0.81 and 0.79 between, respectively, IGN-INA, IGN-LCA and INA-LCA solutions, and in the East component it is about 0.84, 0.82 and 0.76, respectively. However, the correlations in the Vertical component are moderate with an average of 0.64, 0.57 and 0.58 for, respectively, IGN-INA, IGN-LCA and INA-LCA solutions. After removing the trends and seasonal components (annual and semi-annual signals) from the analysed time series, the Allan variance analysis shows that the three solutions are dominated by a white noise in the all three components (North, East and Vertical). The wavelet transform analysis, using the VisuShrink method with soft thresholding, reveals that the noise level is less important in the LCA solution compared to IGN and INA solutions. Indeed, the standard deviation of the noise in the three components is in the range of 5-11 mm, 5-12 mm and 4-9 mm for, respectively, IGN, INA, and LCA solutions.

1. Wavelet Transform Analysis

❖ The **Continuous Wavelet Transform (CWT)** can be defined as the projection of the signal $X(t)$ on the basis of wavelet function $\psi_{u,s}$:

$$CWT(u,s) = \langle X, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} X(t) \overline{\psi_{u,s}(t)} dt \quad \psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right) \quad u, s \in \mathbb{R}; s \neq 0$$

Where: u, s translation and scale factor, respectively, and $\overline{\psi_{u,s}}$ the complex conjugate of $\psi_{u,s}$.

❖ The **original signal can be reconstructed** from its wavelet coefficients $CWT(u,s)$

$$X(t) = \frac{1}{C_\psi} \int_{\mathbb{R}} \int_{\mathbb{R}} CWT(u,s) \psi_{u,s}(t) \frac{ds}{s^2} \quad C_\psi = \int_0^{+\infty} \frac{|FT(\psi(t))|^2}{t} dt < +\infty$$

Where: C_ψ is the standardization coefficient and FT is the Fourier transform.

❖ Taken $s = 2^m$ and $u = n 2^m$ ($n, m \in \mathbb{Z}$), The **Discrete Wavelet Transform (DWT)** is formalized as:

$$DWT_{m,n} = \langle X, \psi_{m,n} \rangle = 2^{-\frac{m}{2}} \int_{-\infty}^{+\infty} X(t) \overline{\psi}(2^{-m}t - n) dt$$

❖ The **DWT** allows the computation of the wavelet coefficients in the context of **multi-resolution analysis** which allows, by successive filtering, to produce a series of signals corresponding to an increasingly fine resolution of the signal. The signal is separated in two components: one representing the **approximation** of the signal (represented by its low-frequency) and the other representing its **details** (represented by its high-frequency).

❖ **Denosing steps**

1. Choose a wavelet and compute the wavelet coefficients at level j of the signal $(X)_{t=1,N}$;

2. Determination of the threshold λ (VisuShrink): $\lambda = \hat{\sigma} \sqrt{2 \log(N)}$

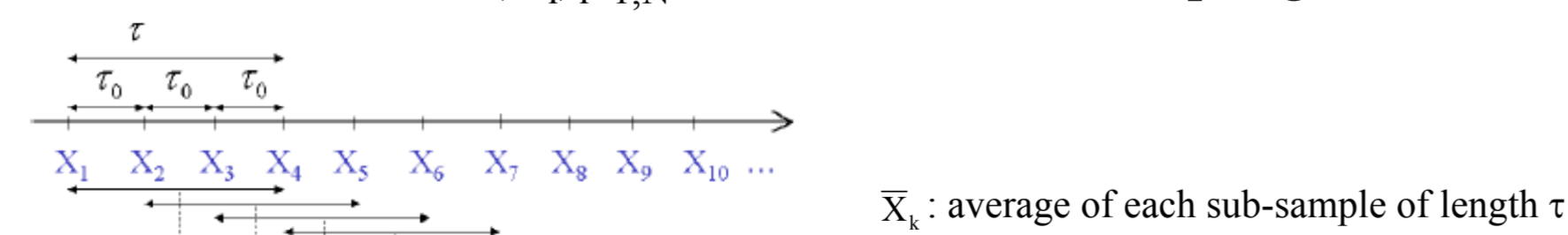
3. Thresholding of the wavelet coefficients: Soft thresholding: $T_\lambda^{Soft}(W) = \begin{cases} W - \lambda & \text{if } |W| \geq \lambda \\ W + \lambda & \text{if } |W| \leq -\lambda \\ 0 & \text{if } |W| \leq \lambda \end{cases}$

4. Reconstruct the signal from the threshold wavelet coefficients.

2. Allan Variance Analysis

❖ The **Allan variance** of coordinate residuals, for a given time interval (τ), is computed by averaging the coordinate residuals over that interval and computing the variance of differences between adjacent averaged values. The Allan variance of a time series $(X_i)_{i=1,N}$ with k items and sampling time τ is defined by:

$$\hat{\sigma}_X^2(\tau) = \frac{1}{2} \langle (\overline{X}_{k+1} - \overline{X}_k)^2 \rangle$$



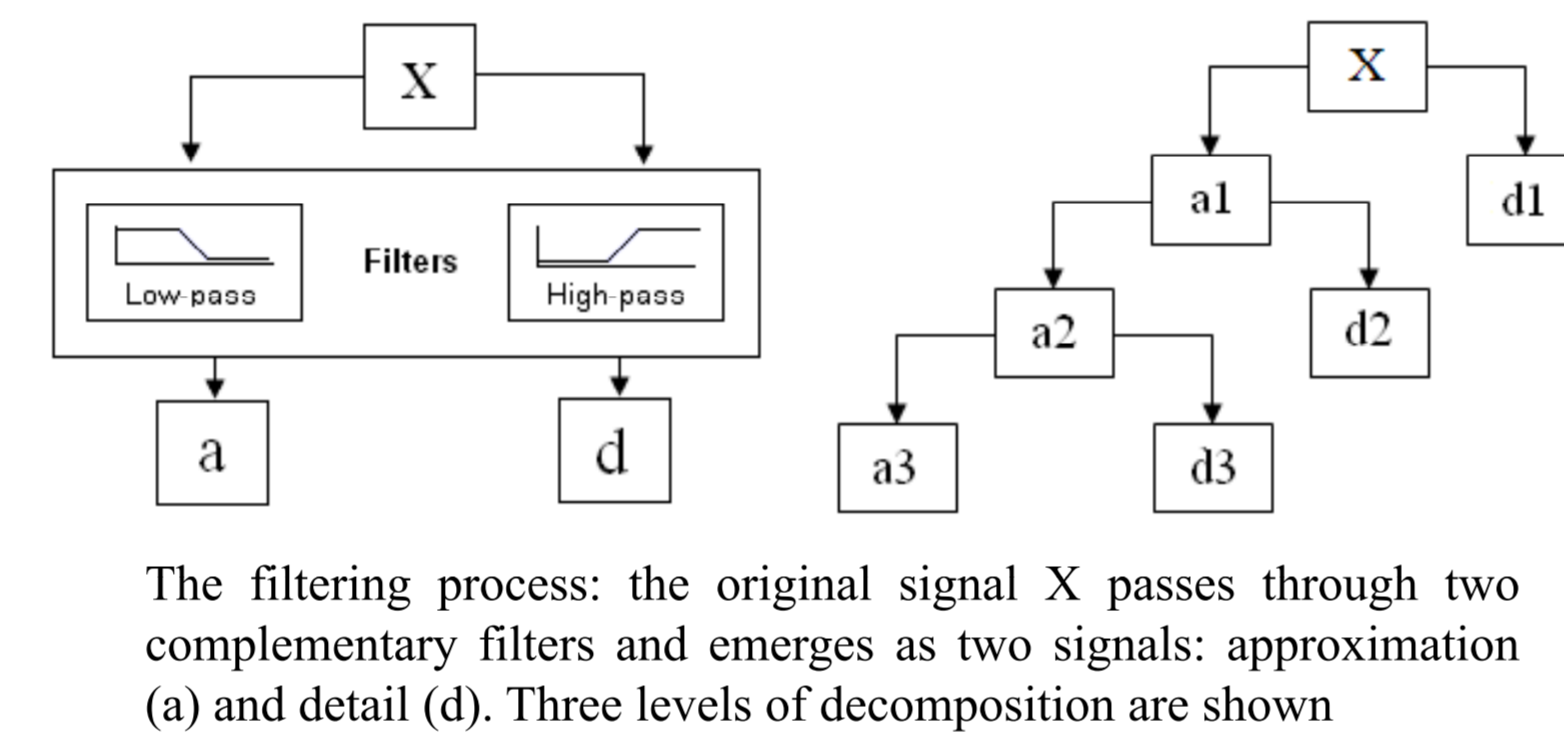
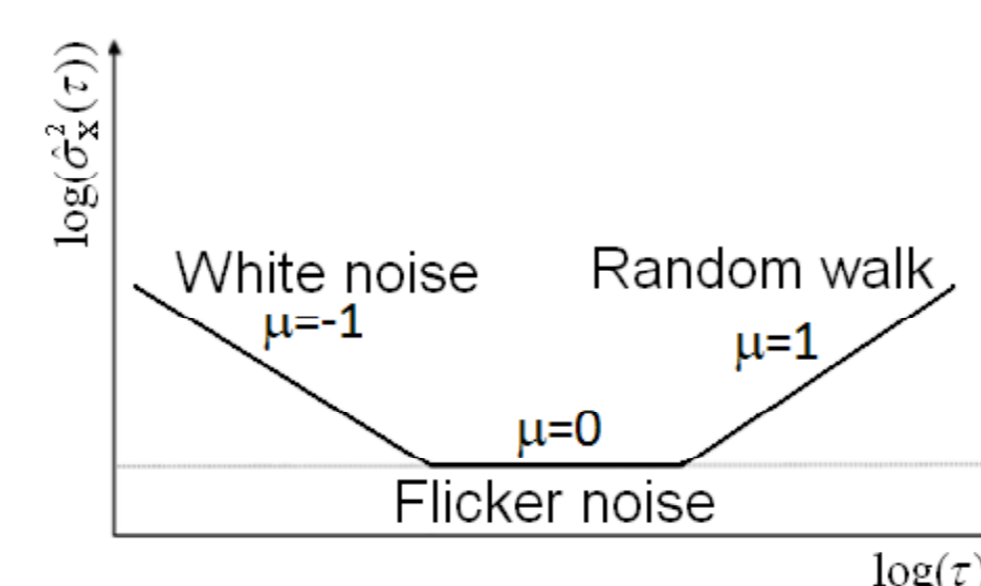
❖ The **noise type** of the time series can be determined from the slope μ of the Allan variance graph according to the sampling time τ :

$$\log(\hat{\sigma}_X^2(\tau)) = \mu \log(\tau) \quad \text{for } \tau = \tau_0, 2\tau_0, 4\tau_0, \dots$$

- **White noise**: random errors affecting the measurements (independent of time);

- **Flicker noise**: due to local tectonics, instruments defects, analysis strategy;

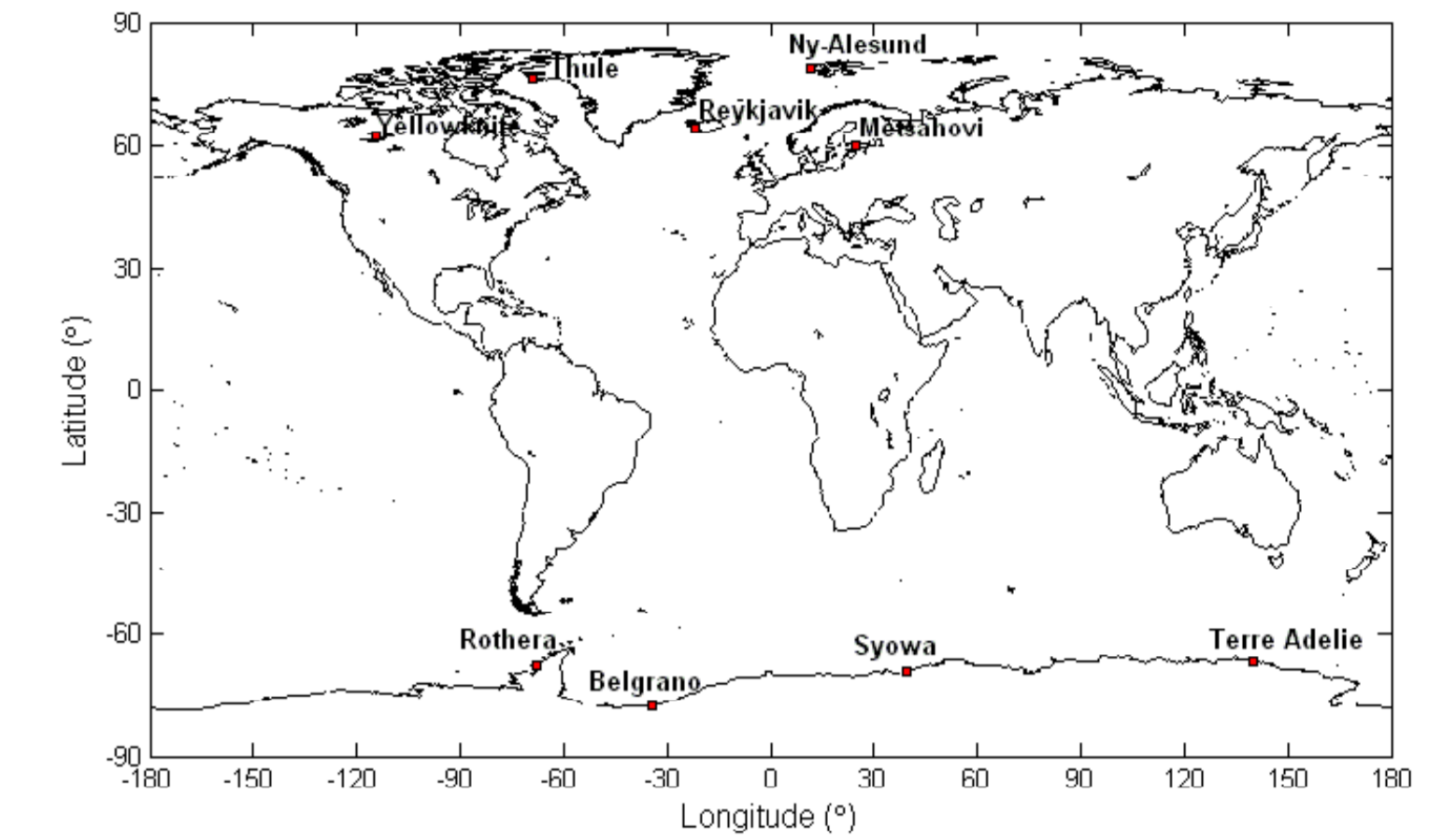
- **Random walk**: caused by the gaps in a time series.



The filtering process: the original signal X passes through two complementary filters and emerges as two signals: approximation (a) and detail (d). Three levels of decomposition are shown

3. Data Description

- Solutions of weekly position residuals in STCD format of 09 high latitude DORIS stations;
- Provided from three Analysis Centers :
IGN-JPL (solution [ign11wd01](#)), INASAN (solution [ina10wd01](#)) and CNES-CLS (solution [lca11wd02](#));
- IGN and INA solutions are computed using GIPSY-OASIS II software, while LCA solution is obtained by GINS-DYNAMO software
- IGN and LCA solutions are referred to ITRF2008 and INA solution is referred to ITRF2005

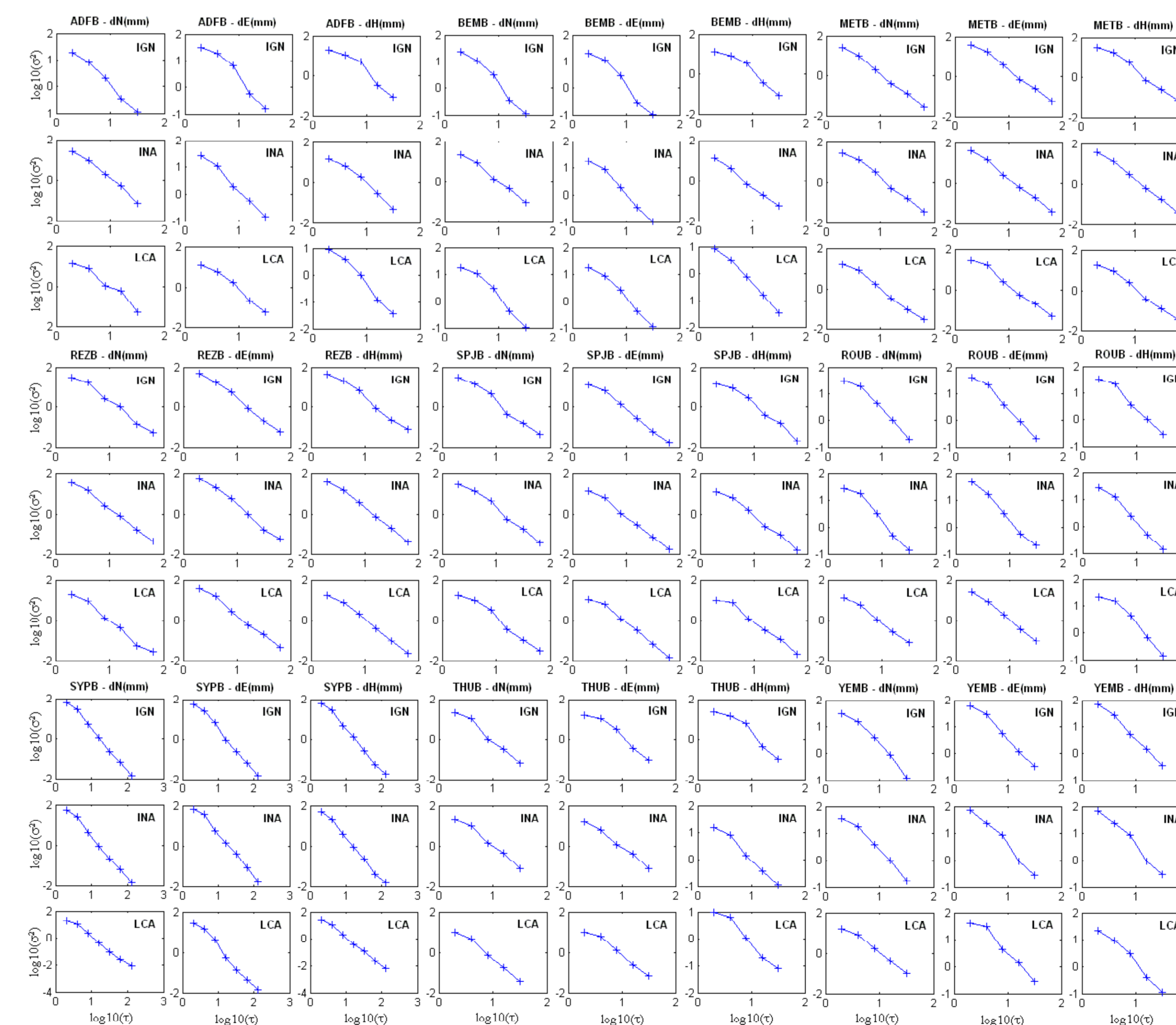


4. Analysis Results

Correlation between the three solutions on the East, North and Vertical components

Acronym	Site	Common data span	Correlation - East			Correlation - North			Correlation - Vertical		
			IGN-INA	IGN-LCA	INA-LCA	IGN-INA	IGN-LCA	INA-LCA	IGN-INA	IGN-LCA	INA-LCA
ADFB	Terre Adelle	2008.2-2010.7	0.81	0.77	0.72	0.80	0.86	0.83	0.34	0.38	0.21
BEMB	Belgrano	2007.1-2010.8	0.45	0.58	0.38	0.93	0.91	0.90	0.50	0.30	0.52
METB	Metsahovi	2005.4-2010.8	0.96	0.92	0.90	0.97	0.94	0.94	0.68	0.78	0.57
REZB	Reykjavik	2004.8-2010.8	0.95	0.93	0.89	0.98	0.98	0.98	0.61	0.26	0.35
SPJB	Ny-Alesund	2005.0-2010.8	0.95	0.91	0.91	0.98	0.97	0.97	0.88	0.80	0.79
ROUB	Rothera	2007.9-2010.8	0.81	0.86	0.80	0.77	0.67	0.78	0.46	0.58	0.66
SYPB	Syowa	2001.1-2010.8	0.82	0.60	0.59	0.89	0.67	0.63	0.78	0.65	0.68
THUB	Thule	2008.1-2010.8	0.94	0.94	0.95	0.67	0.50	0.28	0.73	0.74	0.80
YEMB	Yellowknife	2007.6-2010.8	0.83	0.85	0.74	0.92	0.77	0.80	0.82	0.63	0.65

Noise type of the North, East and Vertical components for the three solutions IGN, INA and LCA



Standard deviation (STD) of the noise in the components North, East and Vertical for the three solutions IGN, INA and LCA

Station	Component	STD (mm)		
		IGN	INA	LCA
ADFB	North	6.5	6.9	5.6
	East	8.3	7.0	5.2
	Vertical	7.1	5.6	4.1
BEMB	North	6.8	7.0	5.9
	East	6.8	5.7	5.8
	Vertical	7.2	5.8	4.5
METB	North	6.8	7.4	5.9
	East	9.1	9.6	7.8
	Vertical	7.9	8.1	6.3
REZB	North	7.7	8.3	6.3
	East	10.1	10.2	8.8
	Vertical	10.1	8.4	6.4
SPJB	North	7.2	7.3	6.0
	East	5.4	5.4	5.0
	Vertical	6.3	5.2	4.9
ROUB	North	7.8	7.5	5.8
	East	9.0	9.8	7.0
	Vertical	9.0	7.4	5.9
SYPB	North	9.6	9.9	6.5
	East	9.5	10.1	7.7
	Vertical	10.0	9.2	6.9
THUB	North	6.5	6.6	4.8
	East	6.3	6.1	4.7
	Vertical	8.5	6.1	5.2
YEMB	North	8.2	8.5	5.8
	East	10.3	11.8	9.1
	Vertical	10.5	10.1	6.7