Rotational-oscillatory motion of the deformable Earth in the short time intervals

Moscow Aviation Institute, Russia

Authors: Markov Yu. G., Rykhlova L.V., Filippova A.S.

Presented by: Filippova A.S.

Used sources

- IERS Annual Reports, http://www.iers.org
- *B. Luzum, N. Capitaine et all* - The IAU 2009 system of astronomical constants – Celest. Mech. And Dyn. Astr. 2011
- *L.D. Akulenko, Yu.G. Markov, V.V. Perepelkin* – Modeling of the Earth's Rotary-Oscillatory Motion within a Short Time Interval - Dokl. Phys., 2011;
- *L.D. Akulenko, Yu.G. Markov, V.V. Perepelkin, L.V. Rykhlova* – Fluctuations in the Angular Momentum of the Atmosphere and Intraday Irregularities in the Earth's Rotation - Astron. Rep., 2011;
- *Yu.G. Markov, L.V. Rykhlova, I.N. Sinitsyn* – Development of Methods for Constructing Models for Intra-Year Irregularity in the Earth's Rotation - Astron. Rep., 2012;
- *L.D. Akulenko, Yu.G. Markov, V.V. Perepelkin, L.V. Rykhlova, A.S. Filippova* – Rotational–Oscillatory Variations in the Earth Rotation Parameters within Short Time Intervals - Astron. Rep., 2013.

Model requirements

- Be in qualitatively and quantitatively agreement with observation data
- Contain small amount of parameters
- Parameters should be material and stable, prone to small distributions

Target settings

- A mathematic model of the oscillations of the Earth's pole and variations of the length of the day is built with the aid of celestial mechanics approach
- Numerical simulation is made according to the IERS data
- Forecast given for different time intervals

Euler-Liouville equation

$$\frac{d(A^* + \delta A)p}{dt} + N_p(B^* + \delta B)q + \sigma_p(A^* + \delta A)p = \delta J_{qr} r^2 + M_p,$$

$$\frac{d(B^* + \delta B)q}{dt} - N_q(A^* + \delta A)p + \sigma_q(B^* + \delta B)q = -\delta J_{pr} r^2 + M_q,$$

$$\frac{d(C^* + \delta C)r}{dt} + (B^* - A^*)pq + (\delta J_{qr} - \delta J_{pr} q)\mathrm{r}_0 = M_r.$$

- Gravitational-tidal moments with small parameter introduced

$$M_{p,q}^{S} = M_{p,q}^{S,0} + \varepsilon_{p,q} M_{p,q}^{S,1} + \ldots$$

$$M_{p,q}^{L} = M_{p,q}^{L,0} + \varepsilon_{p,q} M_{p,q}^{L,1} + \ldots$$

$$M_r^{S,\mathrm{L}} = M_{r,0}^{S,L} + \varepsilon_r M_{r,1}^{S,L} + \ldots$$

Differential equation of EOP

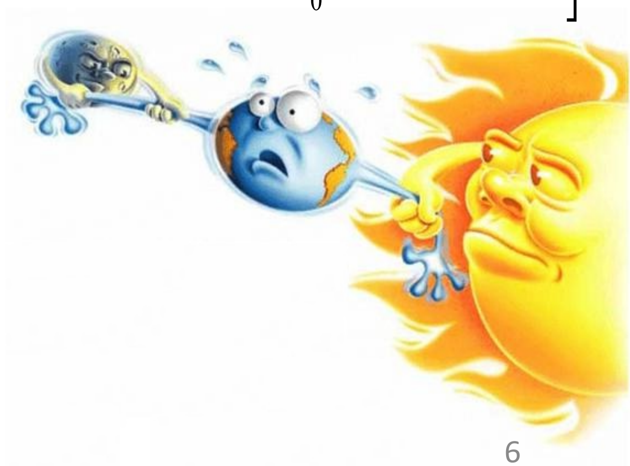
- For the coordinates x_p, y_p, $l.o.d.(t)$, UT1-TAI

$$\dot{x}_p + N_x y_p + \sigma_x x_p = \kappa_q r_0^{\,2} + M_p^{\;S,L} + \varepsilon_p \left[2 r_0 \delta r(t) k_q + r_0^{\,2} \sum_{i=1}^{N} A_i \cos(2\pi \vartheta_i \tau + \alpha_i) + \Delta M_p^{\;S,L}(\dot{\Omega}, \dot{I}) \right]$$

$$\dot{y}_p - N_y x_p + \sigma_y y_p = -\kappa_p r_0^{\,2} + M_q^{\;S,L} + \varepsilon_q \left[-2 r_0 \delta r(t) k_p + r_0^{\,2} \sum_{i=1}^{N} B_i \cos(2\pi \vartheta_i \tau + \beta_i) + \Delta M_q^{\;S,L}(\dot{\Omega}, \dot{I}) \right]$$

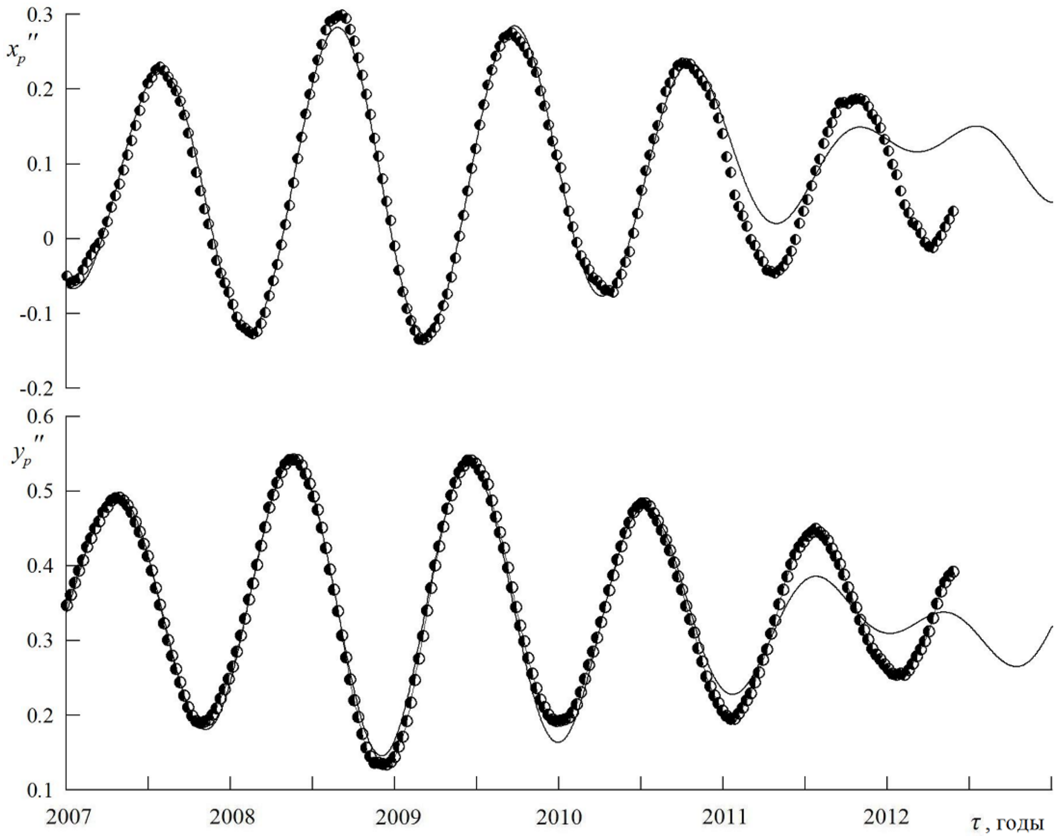
$$\left[1 + \varepsilon_r \sum_{i=1}^{N} C_i \cos(2\pi \vartheta_i \tau + \gamma_i) \right] \dot{l.o.d.}(\varphi, t) = -\frac{D_0}{r_0} M_r^{S,L} + \varepsilon_r \left[\sum_{i=1}^{N} \frac{C_i}{2\pi \vartheta_i} \sin(2\pi \vartheta_i \tau + \gamma_i) l.o.d.(\varphi, t) - \frac{D_0}{r_0} \Delta M_r^{\;S,L}(\dot{\Omega}, \dot{I}) \right]$$

$$\frac{d[\mathrm{UT1} - \mathrm{TAI}](\varphi, t)}{dt} = -D_0^{\,-1} l.o.d.(\varphi, t)$$



$\Delta M_q^{\;S,L}(\dot{\Omega}, \dot{I})$ - additional terms of specific lunar-solar gravitational-tidal moment in the spatial Earth-Moon system problem

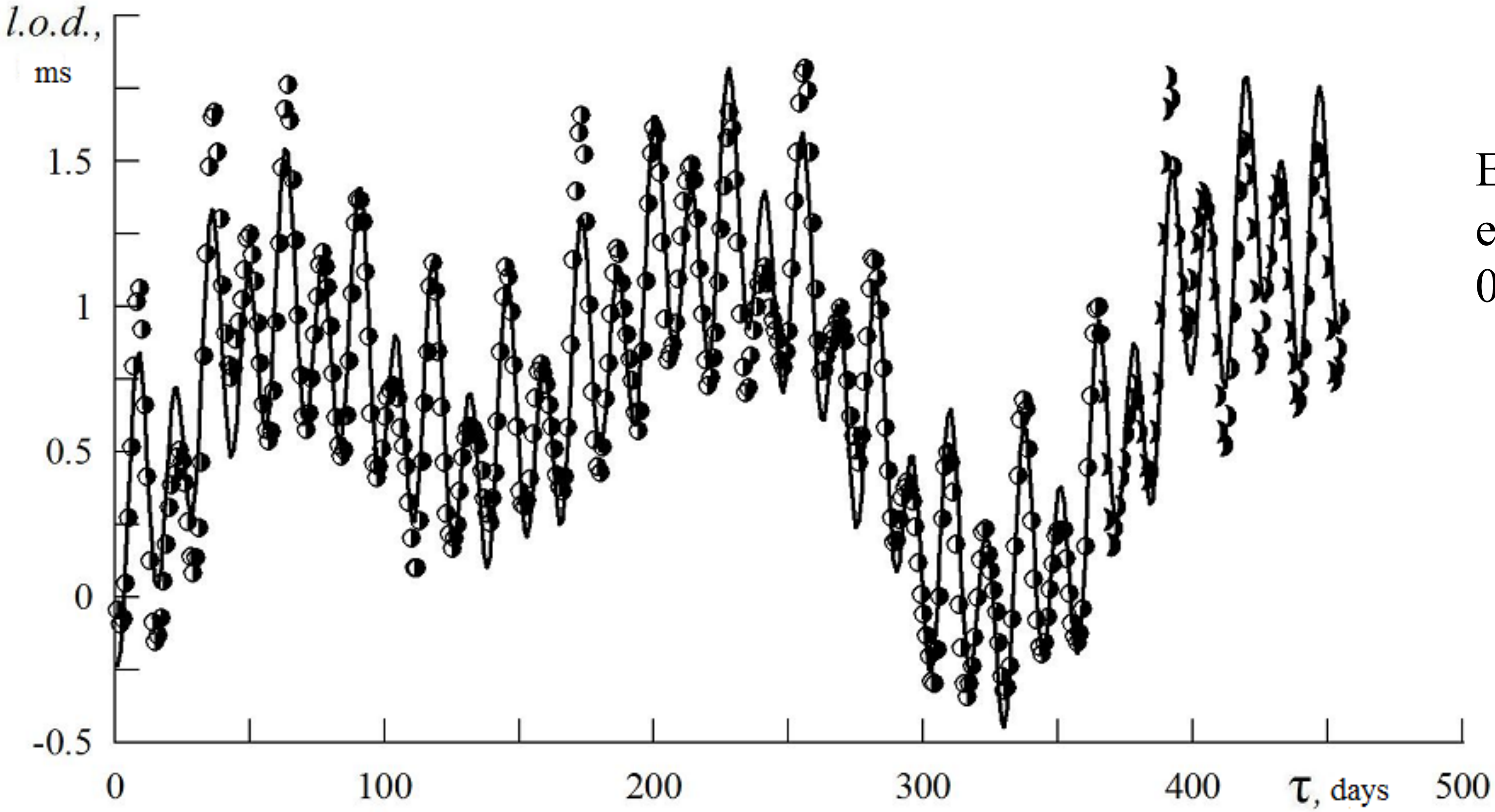
Simulation: pole coordinates



Estimation error: 0.003’’

Interpolation (2007-2010) and forecast (September 2010-2012) of x_p, y_p. Solid curve – theoretical model, dots are IERS observation data

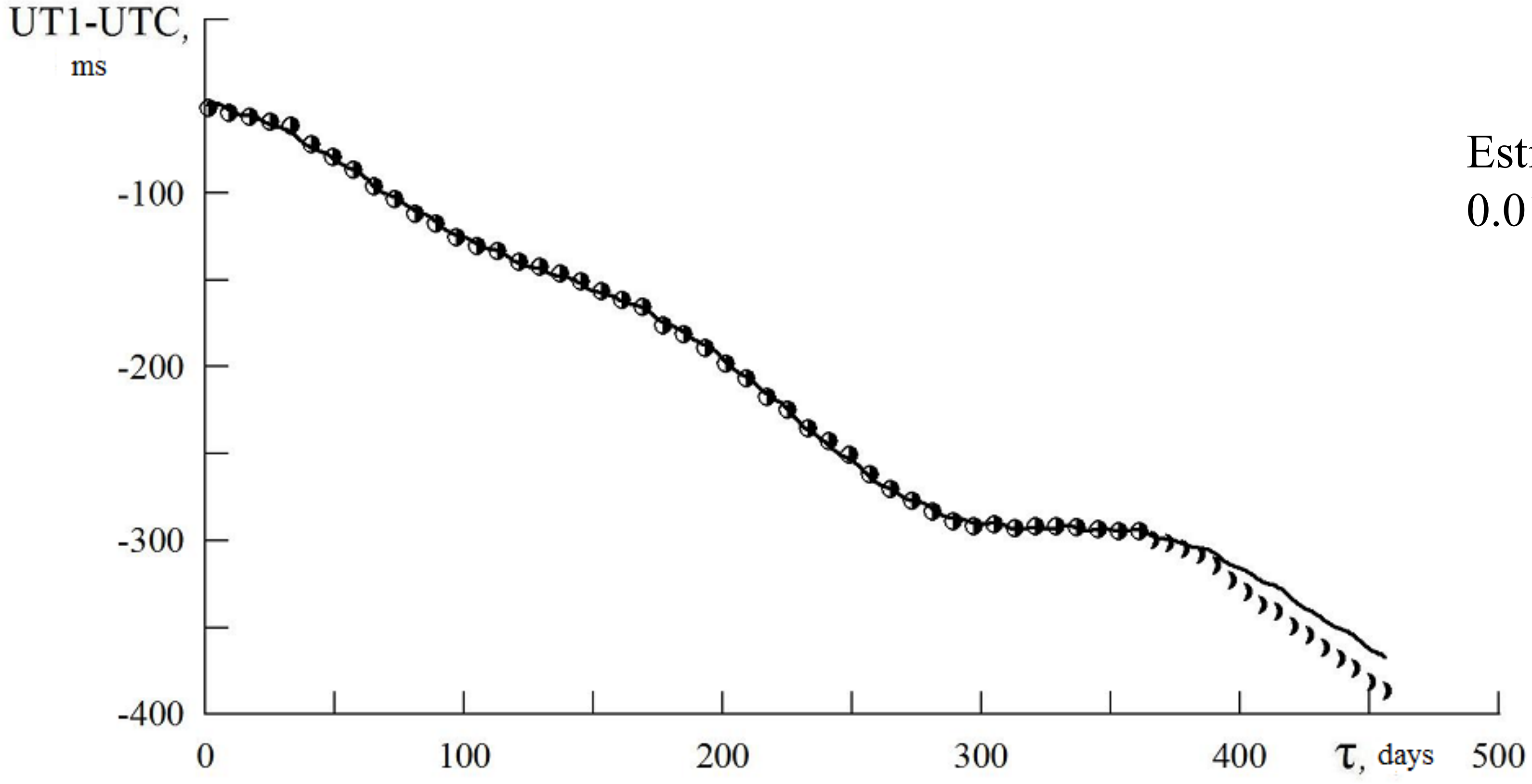
Simulation: l.o.d.



Estimation error: 0,3 ms

Interpolation (Septemper 2010 - Septemper 2011) and forecast (September 2011 - January 2012) of l.o.d. Solid curve – theoretical model, dots are IERS observation data

Simulation: UT1-UTC



Estimation error:
0.01 ms

UT1-UTC interpolation (Septemper 2010 - Septemper 2011) and forecast (September 2011 - January 2012) of UT1-UTC. Solid curve – theoretical model, dots are IERS observation data.

Differential equation of EOP

- For the coordinates x_p, y_p, $l.o.d.(t)$, UT1-TAI

$$\dot{x}_p + N_x y_p + \sigma_x x_p = \kappa_q r_0^{\,2} + M_p^{\,S,L} + \varepsilon_p \left[2 r_0 \delta r(t) k_q + r_0^{\,2} \sum_{i=1}^{N} A_i \cos(2\pi \vartheta_i \tau + \alpha_i) + \Delta M_p^{\,S,L}(\dot{\Omega}, \dot{I}) \right]$$

$$\dot{y}_p - N_y x_p + \sigma_y y_p = -\kappa_p r_0^{\,2} + M_q^{\,S,L} + \varepsilon_q \left[-2 r_0 \delta r(t) k_p + r_0^{\,2} \sum_{i=1}^{N} B_i \cos(2\pi \vartheta_i \tau + \beta_i) + \Delta M_q^{\,S,L}(\dot{\Omega}, \dot{I}) \right]$$

$$\left[1 + \varepsilon_r \sum_{i=1}^{N} C_i \cos(2\pi \vartheta_i \tau + \gamma_i) \right] l.\dot{o}.d.(\varphi, t) = -\frac{D_0}{r_0} M_r^{S,L} + \varepsilon_r \left[\sum_{i=1}^{N} \frac{C_i}{2\pi \vartheta_i} \sin(2\pi \vartheta_i \tau + \gamma_i) l.o.d.(\varphi, t) - \frac{D_0}{r_0} \Delta M_r^{\,S,L}(\dot{\Omega}, \dot{I}) \right]$$

$$\frac{d[\mathrm{UT1-TAI}](\varphi, t)}{dt} = -D_0^{\,-1} l.o.d.(\varphi, t)$$

$\Delta M_q^{\,S,L}(\dot{\Omega}, \dot{I})$ - additional terms of specific lunar-solar gravitational-tidal moment in the spatial Earth-Moon system problem

Refined model of the pole's motion and l.o.d.

- Solutions for the refined model

$$x_p = \overline{x}_p + \Delta x_p,$$

$$y_p = \overline{y}_p + \Delta y_p,$$

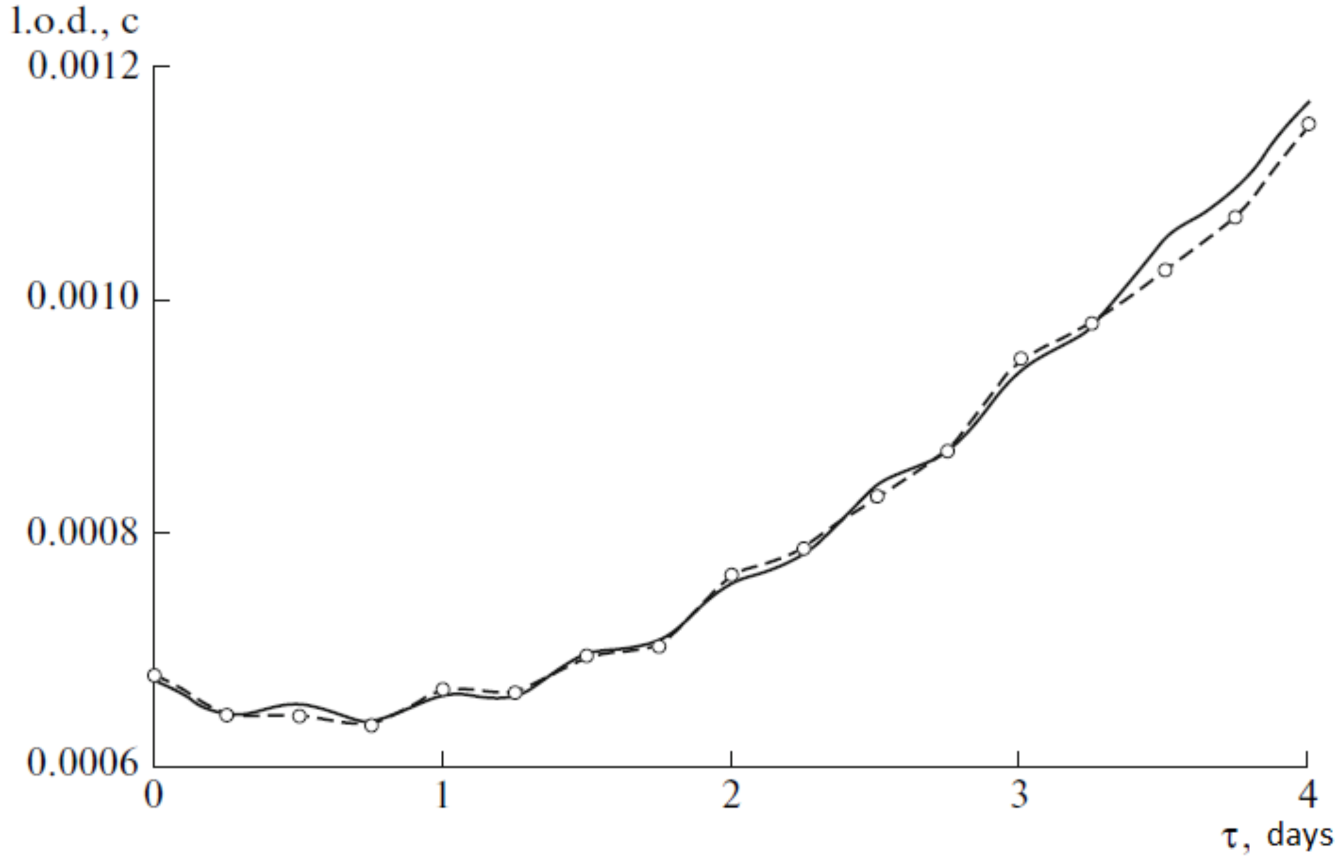
$$l.o.d.(\varphi,\tau) = \overline{l.o.d.(\tau)} + \Delta_\varphi^S l.o.d.(\tau) + \Delta_\varphi^L l.o.d.(\tau)$$

$\overline{x}_p, \overline{y}_p, \overline{l.o.d.}$ - Solutions for the basic model

$\Delta x_p, \Delta y_p$ - Additional terms, taking into account lunar perturbations

$\Delta_\varphi{}^S l.o.d.(\tau), \Delta_\varphi{}^L l.o.d.(\tau)$ -Additional terms, as combination diurnal and half diurnal with solar-lunar gravity-tidal perturbations

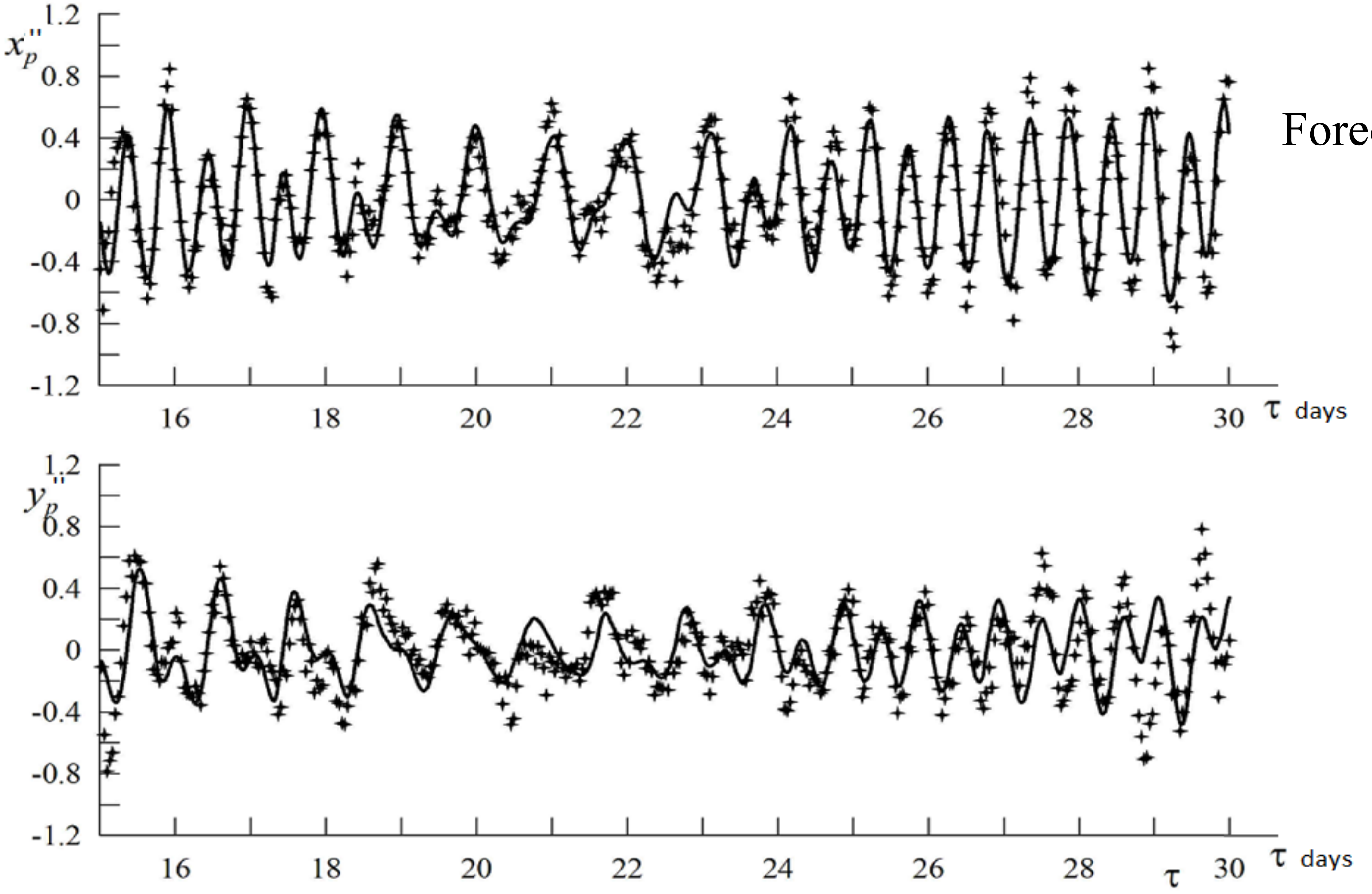
Near-diurnal l.o.d.



Forecast error:
0.0005 c

Interpolation (2.5 days) and forecast (1.5 days) diurnal oscillations of the l.o.d. (φ,τ). Solid curve – theoretical model, dash-dots – observations data

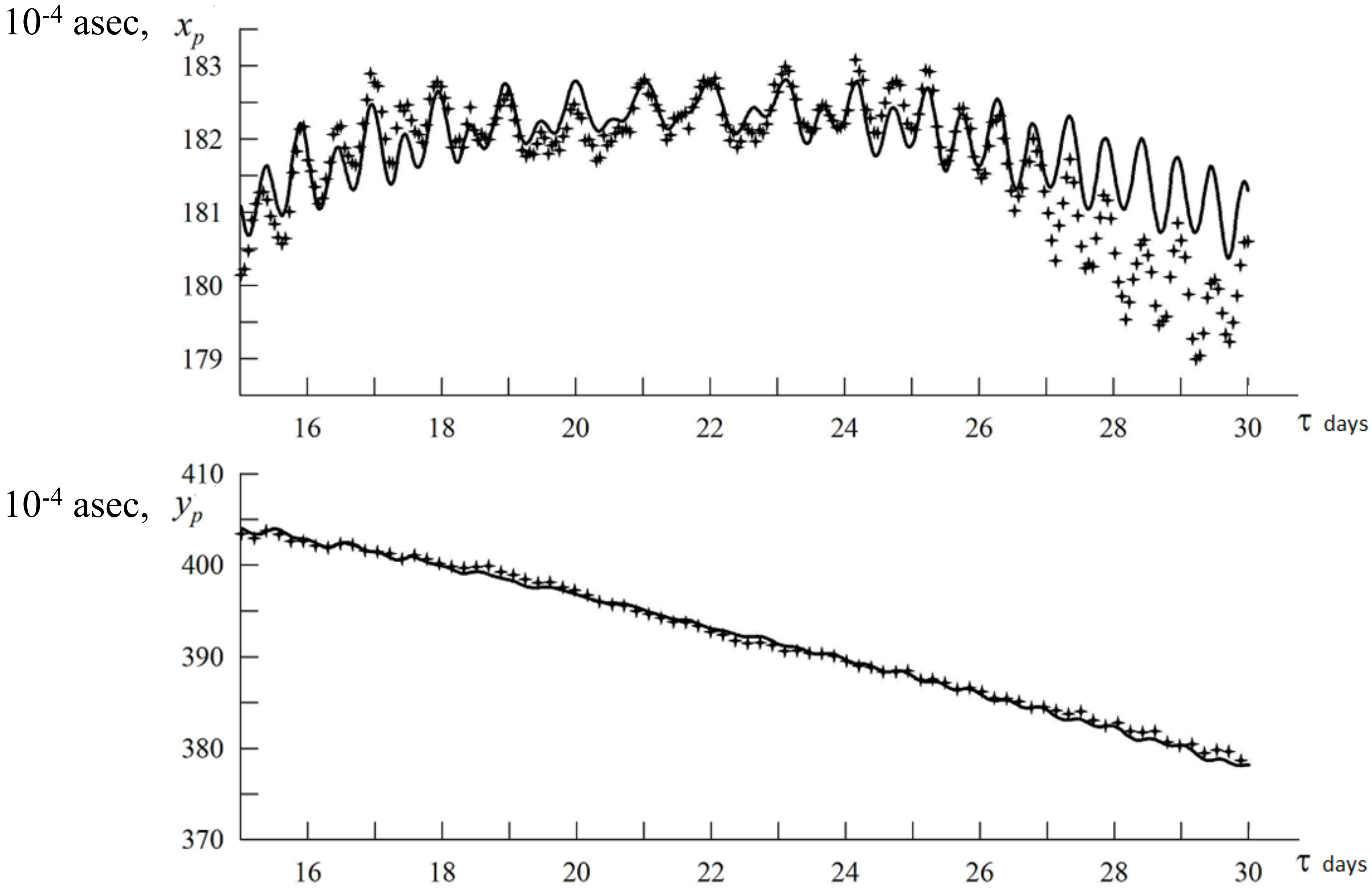
Near-diurnal polar motion



Forecast error: 0.02”

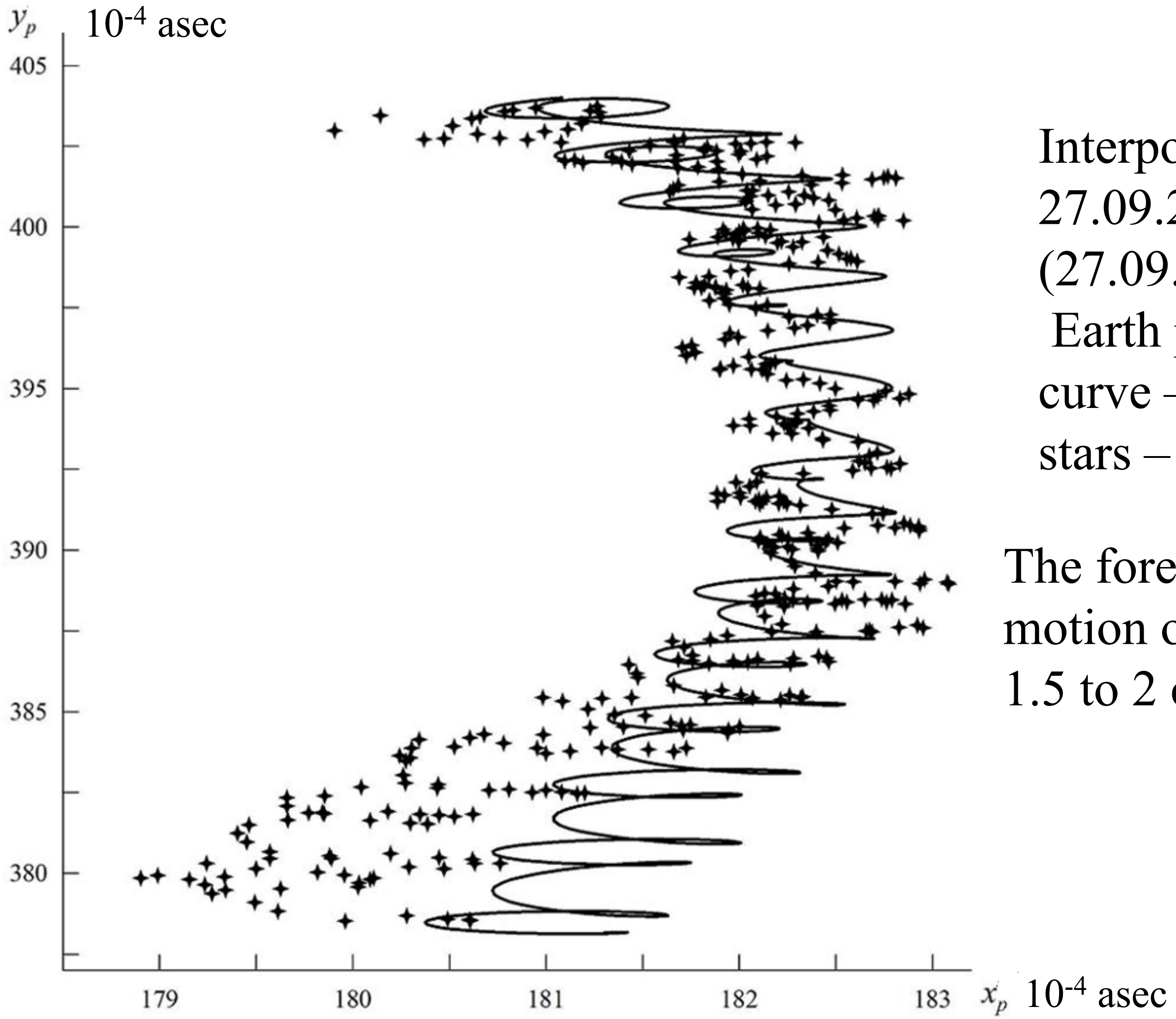
Interpolation of the subdiurnal oscillations for x_p and y_p of the Earth’s pole from 15.09.2011 to 27.09.2011 and forecast for 3 days (solid curve) in comparison with VLBI data (stars).

Polar motion simulation



Interpolation (15.09.2011 - 27.09.2011) and forecast (27.09.2011 – 29.09.2011) Earth pole coordinates. Solid curve – theoretical model, stars – observations data

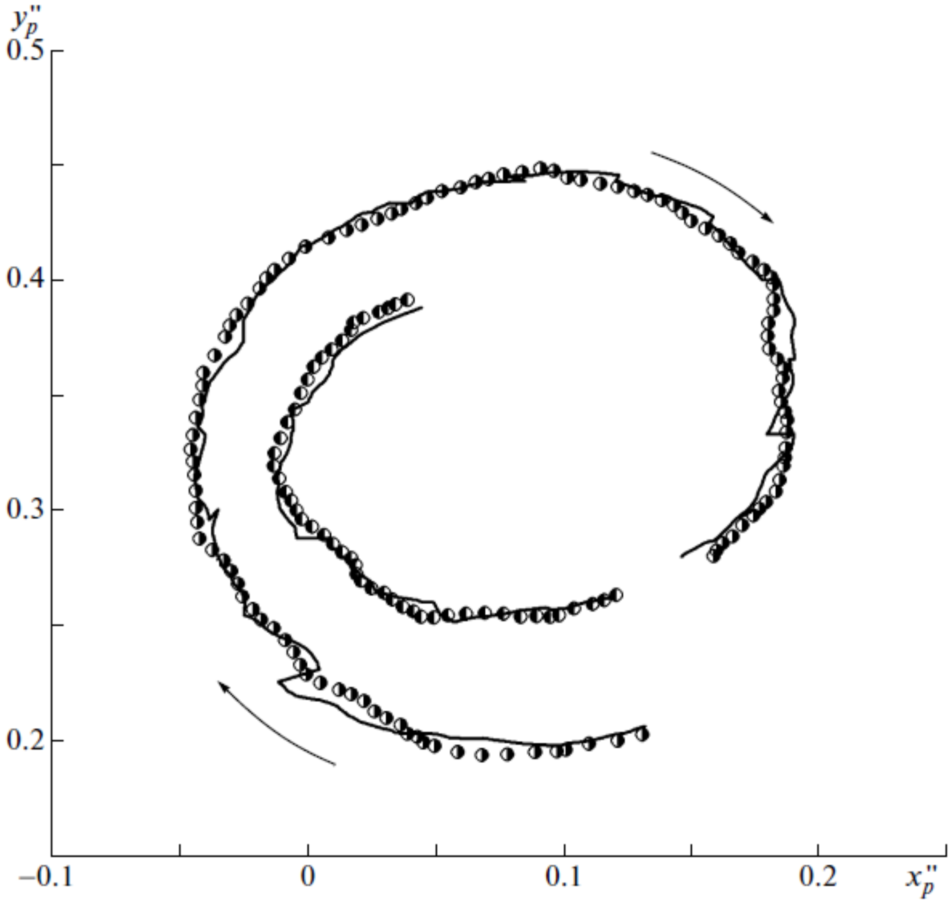
Polar trajectory simulation



Interpolation (15.09.2011 - 27.09.2011) and forecast (27.09.2011 – 29.09.2011) Earth pole trajectory. Solid curve – theoretical model, stars – observations data

The forecast error of the intradays motion on the Earth's pole is from 1.5 to 2 cm.

Earth’s pole trajectory



Rms deviation

$$\sigma_x = 4.99\times10^{-5}$$

$$\sigma_y = 1.36\times10^{-5}$$

Twenty-day forecasts for the trajectory of the Earth’s pole corresponding to the time interval from January 1, 2011 to December 6, 2011 and forecasts for the interval from January 1, 2012 to May 28, 2012 (solid curve); the points show the IERS data.

Thank you for listening!