On the changes of IAU 2000 nutation theory stemming from IAU 2006 precession theory

A. Escapa[1], J. Getino[2], J. M. Ferrándiz[1], & T. Baenas[1]

[1]Dept. Applied Mathematics. University of Alicante. Spain
[2]Dept. Applied Mathematics. University of Valladolid. Spain

Alberto.Escapa@ua.es

Contents

Contents

Precession–nutation motion

- Precession–nutation motion is a basic ingredient to establish the transformation that relates the celestial and terrestrial reference systems
- It provides the time evolution of the celestial pole with respect to the reference celestial system
- From a dynamical point of view precession–nutation is a single entity, although it is conventionally separated into long–term (precession) and non long–term (nutation) parts
- This motion is realized by International Astronomical Union (IAU) model for precession–nutation plus additional corrections
- The model describes the evolution of the Celestial Intermediate Pole due to the external torques exerted by the Moon, the Sun, and the planets

Current standards

- Last IAU models for precession–nutation have consisted of two separated parts comprising precession and nutation
  - IAU 1976 precession model (Lieske et al. 1977) + IAU 1980 nutation model (Wahr 1981)
  - IAU 2000 nutation model (Mathews et al. 2002) + IAU 1976 precession model (updated with corrections to precession rates)
  - IAU 2000 nutation model (Mathews et al. 2002) + IAU 2006 precession model (Capitaine et al. 2003)
- To ensure compatibility between precession and nutation models, this approach requires the introduction of some corrections in the nutation or the precessional part
- It is the case of current IAU precession–nutation model, where the nutation part (IAU 2000) must be adjusted to the precession part (IAU 2006), although there is no explicit mention to this issue in IAU resolutions

IAU 2000 nutation corrections I

- From the perspective of the nutation model, the main changes of IAU 2006 precession model (Capitaine et al. 2003, IAU 2006 Resolution B1) are
  - A change in the value of the obliquity

$$\epsilon_A = 84381.40600 - 46.836769\, t + \cdots \text{ (IAU 2006)}$$
$$\epsilon_A = 84381.448 - 46.8150\, t + \cdots \text{ (IAU 1976)}$$

  - The introduction of a time rate change of Earth J_2 parameter

$$\dot{J}_2 = -3 \times 10^{-9}\, century^{-1}$$

- The induced corrections in the IAU nutational part were given in Capitaine et al. (2005). They consist in the global rescaling

$$\Delta\psi_{\mathsf{IAU2006}} = \left(\frac{\sin \epsilon_{0\ \mathsf{IAU2000}}}{\sin \epsilon_{0\ \mathsf{IAU2006}}} + t\, \dot{J}_2/J_2 \right) \Delta\psi_{\mathsf{IAU2000}}$$

$$\Delta\epsilon_{\mathsf{IAU2006}} = \left(1 + t\, \dot{J}_2/J_2 \right) \Delta\epsilon_{\mathsf{IAU2000}}$$

IAU 2000 nutation corrections II

- Numerically, the adjustments greater than 1 microarcsecond (μas) are
  - Due to the change of the obliquity value (periodic)

$$d\psi = -8.1 \sin \Omega - 0.6 \sin(2F - 2D + 2\Omega)$$

  - Due to J_2 time rate (secular mixed)

$$\begin{aligned} d\epsilon &= -25.6\ t \cos \Omega - 1.6\ t \cos(2F - 2D + 2\Omega) \\ d\psi &= +47.8\ t \sin \Omega + 3.7\ t \sin(2F - 2D + 2\Omega) + \\ &\quad +0.6\ \ t \sin(2F + 2\Omega) - 0.6\ \ t \sin(2\Omega) \end{aligned}$$

- However, they are considered in some relevant sources
  - IERS Conventions (2010), sec. 5.6.3
  - The Explanatory Supplement to the Astronomical Almanac, p. 211
  - SOFA routines
- They give raise to the so called IAU 2000A$_{\mathrm{R06}}$ nutation model

Objective

- In this work, we aim at providing an alternative independent derivation (analytical) of the adjustments of nutation series induced by obliquity value changes and the J_2 time rate
- That is to say, our objective is to check the validity and scope of the adjustment nutation formulas

$$\Delta\psi_{\mathsf{IAU2006}} = \left(\frac{\sin\epsilon_{0\ \mathsf{IAU2000}}}{\sin\epsilon_{0\ \mathsf{IAU2006}}} + t\,\dot{J}_2/J_2\right)\Delta\psi_{\mathsf{IAU2000}}$$

$$\Delta\epsilon_{\mathsf{IAU2006}} = \left(1 + t\,\dot{J}_2/J_2\right)\Delta\epsilon_{\mathsf{IAU2000}}$$

Contents

Analytical solution

- The effects to be discussed are at the μas level, hence at this stage we consider a rigid symmetrical Earth model and a first order theory
- The equations of motion are constructed and solved by means of the Hamiltonian formalism of the rigid Earth (Kinoshita 1977)
- However, we have performed from the beginning the computations of the nutations (Escapa 2013), since we have introduced some variations in Kinoshita's scheme
  - A different unperturbed problem $\mathcal{H}_0 = \mathcal{T}$
  - Inclusion of the time rate of J_2
  - Full inclusion of the time rate of the orbital coefficients (due to the secular variation of sun eccentricity)
- Our preliminary results show that at the μas level the adjustments can be modeled through the motion of the angular momentum axis

Contents

Corrections due to a change in obliquity I

- The nutations (periodic) can be written as

$$\Delta\psi = \frac{k}{\sin\epsilon_0}\sum_{i\neq 0}\frac{1}{\bar{n}_i}B_i'\left(\epsilon_0\right)\sin\Theta_i,\ \Delta\epsilon = -\frac{k}{1}\sum_{i\neq 0}\frac{m_{i5}}{\bar{n}_i}\frac{B_i\left(\epsilon_0\right)}{\sin\epsilon_0}\cos\Theta_i,$$

$$B_i\left(\epsilon\right) = -\frac{1}{6}A_i^{(0)}\left(3\cos^2\epsilon - 1\right) + \frac{1}{2}A_i^{(1)}\sin 2\epsilon - \frac{1}{4}A_i^{(2)}\sin^2\epsilon$$

  where k is proportional to the dynamical ellipticity, $\bar{n}_i \simeq \dot{\Theta}_i$, and $A_i^{(j)}$ depend on the orbital motion of the external bodies
- When changing the value ϵ_0 these formulas lead to a global rescaling
- But that change also affects in a different way to each term in longitude and obliquity, providing new corrections
- We also find some new corrections to the secular mixed terms

Corrections due to a change in obliquity II

Numerically, the adjustments in μas are (cutoff 0.5 μas)

- Periodic terms
  - Global rescaling part (IERS correction)

$$d\psi = -8.1\sin\Omega - 0.6\sin(2F - 2D + 2\Omega)$$

  - New part (same order of magnitude as the considered one)

$$\begin{aligned} d\psi &= -7.5\sin\Omega + 0.5\sin(2F - 2D + 2\Omega) \\ d\epsilon &= +0.8\cos\Omega \end{aligned}$$

  - Total correction

$$\begin{aligned} d\psi &= -15.6\sin\Omega \\ d\epsilon &= +\ 0.8\cos\Omega \end{aligned}$$

- Secular mixed terms

$$d\psi = -8.1\ t\sin\Omega$$

Corrections due to the J_2 rate I

- The J_2 rate induces a time dependence in k

$$k = k_0 \left(1 + t \frac{\dot{H}_d}{H_d}\right) \simeq k_0 \left(1 + t \ \frac{\dot{J}_2}{J_2}\right)$$

- The nutations are given by

$$\Delta\psi = \frac{k_0}{\sin\epsilon_0} \left(1 + t \ \frac{\dot{J}_2}{J_2}\right) \sum_{i \neq 0} \frac{1}{\bar{n}_i} B_i' \left(\epsilon_0\right) \sin\Theta_i$$

$$\Delta\epsilon = -k_0 \left(1 + t \ \frac{\dot{J}_2}{J_2}\right) \sum_{i \neq 0} \frac{m_{i5}}{\bar{n}_i} \frac{B_i \left(\epsilon_0\right)}{\sin\epsilon_0} \cos\Theta_i$$

- They are the same as those reported in Capitaine et al. (2005)
- Strictly speaking they are only valid for the first order part of the nutations, since some the second order parts are proportional to k^2 and the rescaling factor would be different

Corrections due to the J_2 rate II

- We have also found new out of phase terms (periodic)

$$\Delta\psi = \frac{k_0}{\sin\epsilon_0}\frac{\dot{J}_2}{J_2}\sum_{i\neq 0}\frac{B_i'(\epsilon_0)}{\bar{n}_i^2}\cos\Theta_i$$

$$\Delta\epsilon = k_0\frac{\dot{J}_2}{J_2}\sum_{i\neq 0}\frac{m_{i5}}{\bar{n}_i^2}\frac{B_i(\epsilon_0)}{\sin\epsilon_0}\sin\Theta_i$$

- The same kind of terms also appear when taking fully into account the time rate of the orbital coefficients $A_i^{(j)}$ due to the secular variation of sun eccentricity

Corrections due to the J_2 rate III

Numerically, the adjustments in μas are (cutoff 0.5 μas)

- Secular mixed terms (IERS correction)

$$
\begin{aligned}
d\epsilon &= -25.6\ t\cos\Omega - 1.5\ t\cos(2F-2D+2\Omega) \\
d\psi &= +48.0\ t\sin\Omega + 3.5\ t\sin(2F-2D+2\Omega) + \\
&\quad +0.6\ \ t\sin(2F+2\Omega) - 0.6\ \ t\sin(2\Omega)
\end{aligned}
$$

- Out of phase terms
  - Due to J_2 rate

$$
\begin{aligned}
d\psi &= -1.4\cos\Omega \\
d\epsilon &= -0.8\sin\Omega
\end{aligned}
$$

  - Due to $A_i^{(j)}$ rate

$$
d\psi = -0.5\cos l'
$$

  - Total correction

$$
\begin{aligned}
d\psi &= -1.4\cos\Omega - 0.5\cos l' \\
d\epsilon &= -0.8\sin\Omega
\end{aligned}
$$

Summary I

- Current IAU model for precession–nutation consists of two separated parts comprising precession and nutation
- To ensure compatibility, this approach requires the introduction of some corrections in the nutation or the precessional part
- We have derived analytical formulas that adjust IAU 2000A nutation model (Mathews et al. 2002) to IAU 2006 precession model (Capitaine et al. 2003) taking into account
  - The change in the value of the obliquity ϵ_A
  - The introduction of a time rate change of Earth J_2 parameter
- Our preliminary results show that at the μas level the adjustments can be modeled through the motion of the angular momentum axis (at the first order)

Summary II

- Effect of the change in the obliquity value
  - It seems that the global rescale must be supplemented with additional terms of similar magnitude that affect both longitude and obliquity, providing the total correction

$$d\psi = -15.6\sin\Omega,\ d\epsilon = +0.8\cos\Omega$$

  - There are also new secular mixed terms

$$d\psi \quad = \quad -8.1\ t\sin\Omega$$

- Effect of the inclusion of time rate of J_2
  - We have recovered by an independent way the current IERS Conventions (2010) corrections for the secular mixed terms
  - There are also new out of phase periodic terms

$$d\psi = -1.4\cos\Omega - 0.5\cos l',\ d\epsilon = -0.8\sin\Omega$$

- These corrections should be considered in the actual standards and models

Acknowledgements

This work has been partially supported by the Universidad de Alicante under project GRE11-08, and Spanish grants I+D+I AYA2010-22039-C02-01, and AYA2010-22039-C02-02.