Next step in Earth interior modeling for nutation

Véronique Dehant, Marta Folgueira, Mihaela Puica, Laurence Koot, Tim Van Hoolst, Antony Trinh,

Royal Observatory of Belgium,

Complutense University of Madrid

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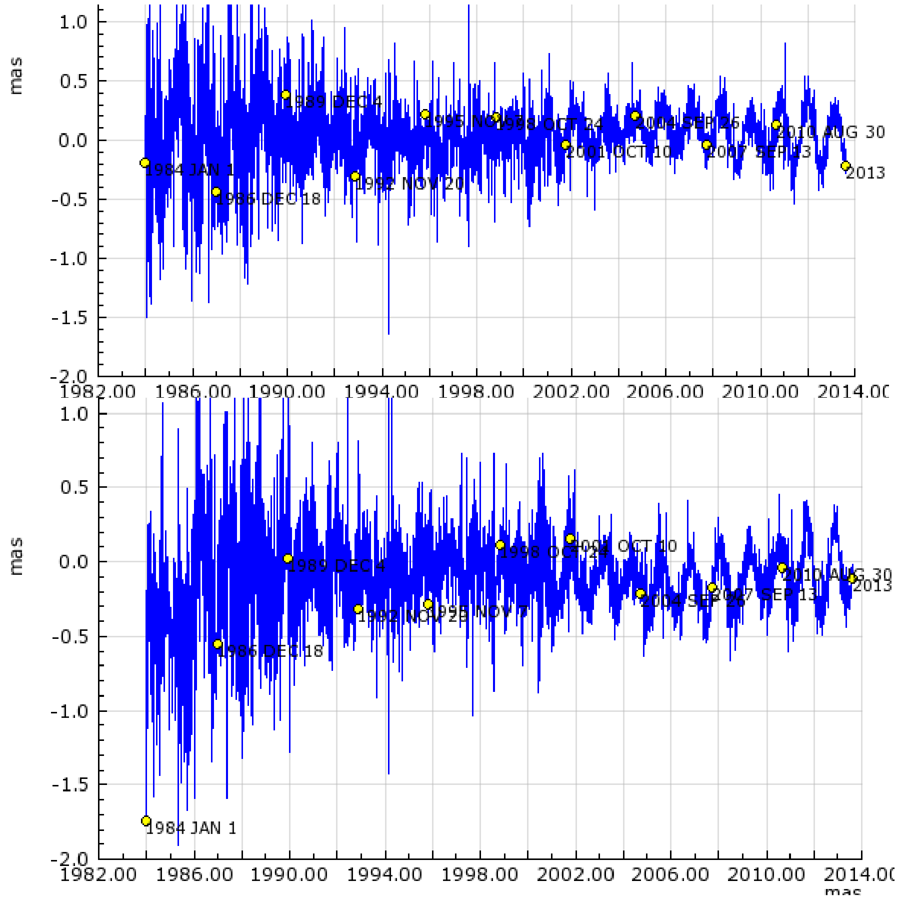
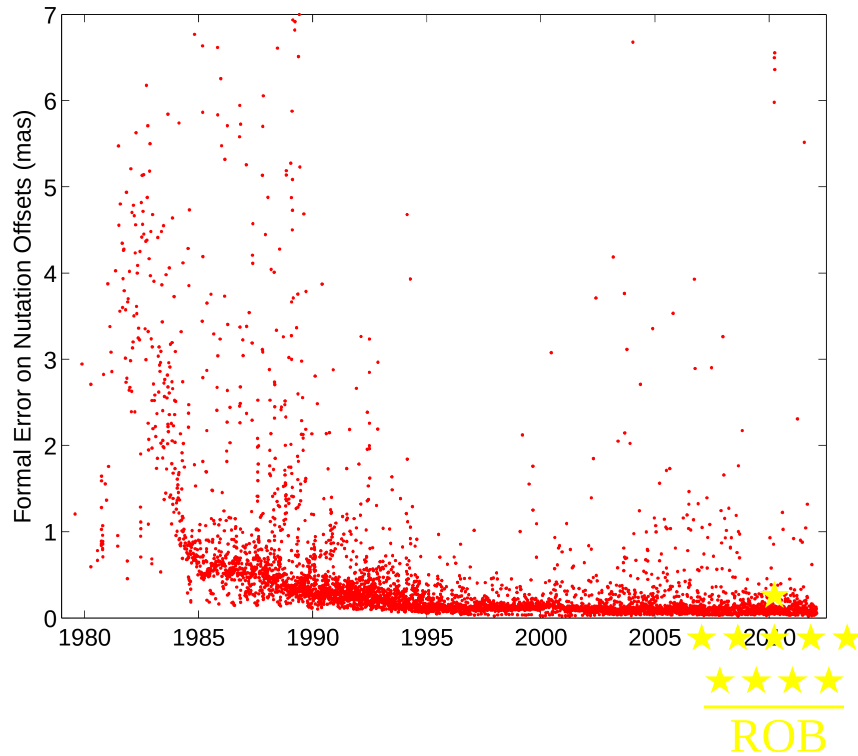
Nutation Series

- Longer time series
  - Better adjustment of long-period terms (e.g., 18.6-yr)
  - Improvement of the formal error
  - Can choose to drop data before 1995

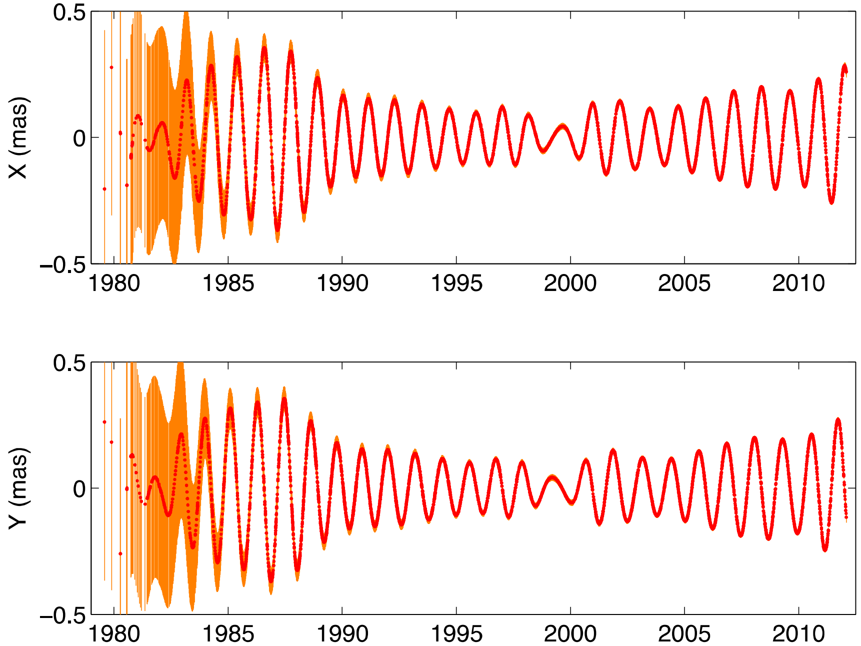




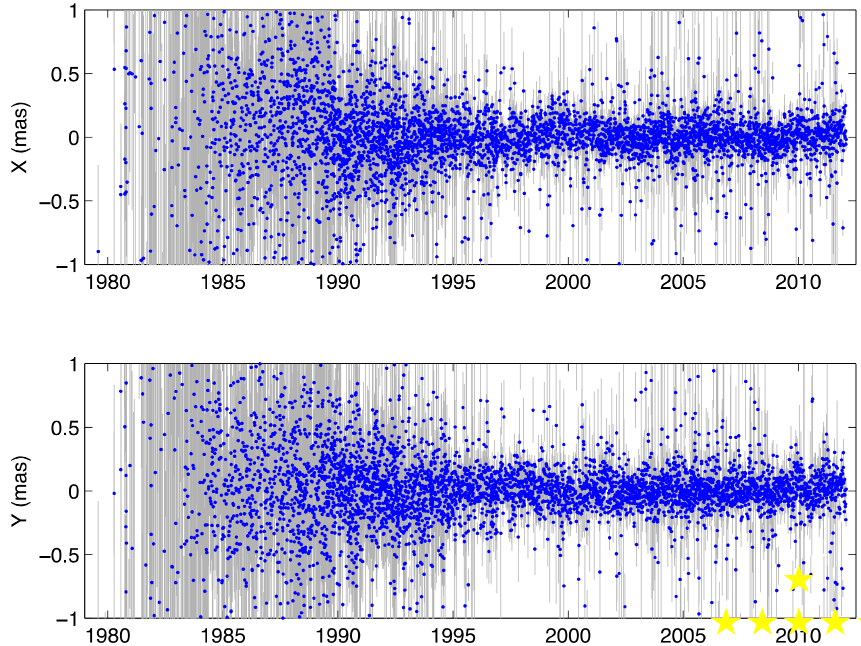
Some Challenges for the free nutations

- The amplitudes can only be observed due to the poor knowledge of their excitation! Only FCN free mode observed. (but resonance)
- Explain the variations of the FCN amplitude/phase
- Detect a signal related to the FICN (see work of Lambert et al.)

The FCN



The nutation after removal of the FCN and main tidal terms



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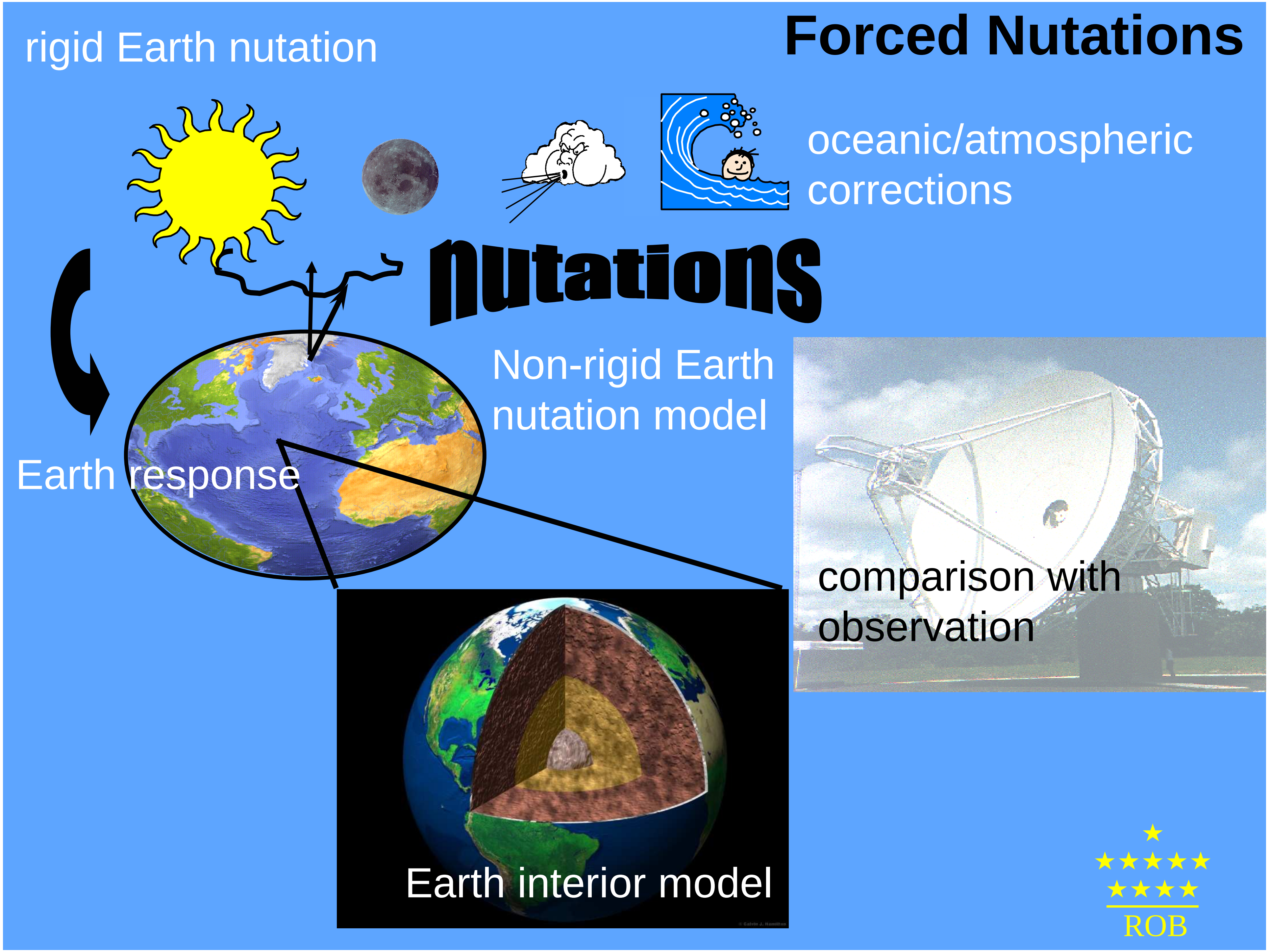
Forced Nutations
rigid Earth nutation
oceanic/atmospheric corrections
nutations
Non-rigid Earth nutation model
Earth response
comparison with observation
Earth interior model
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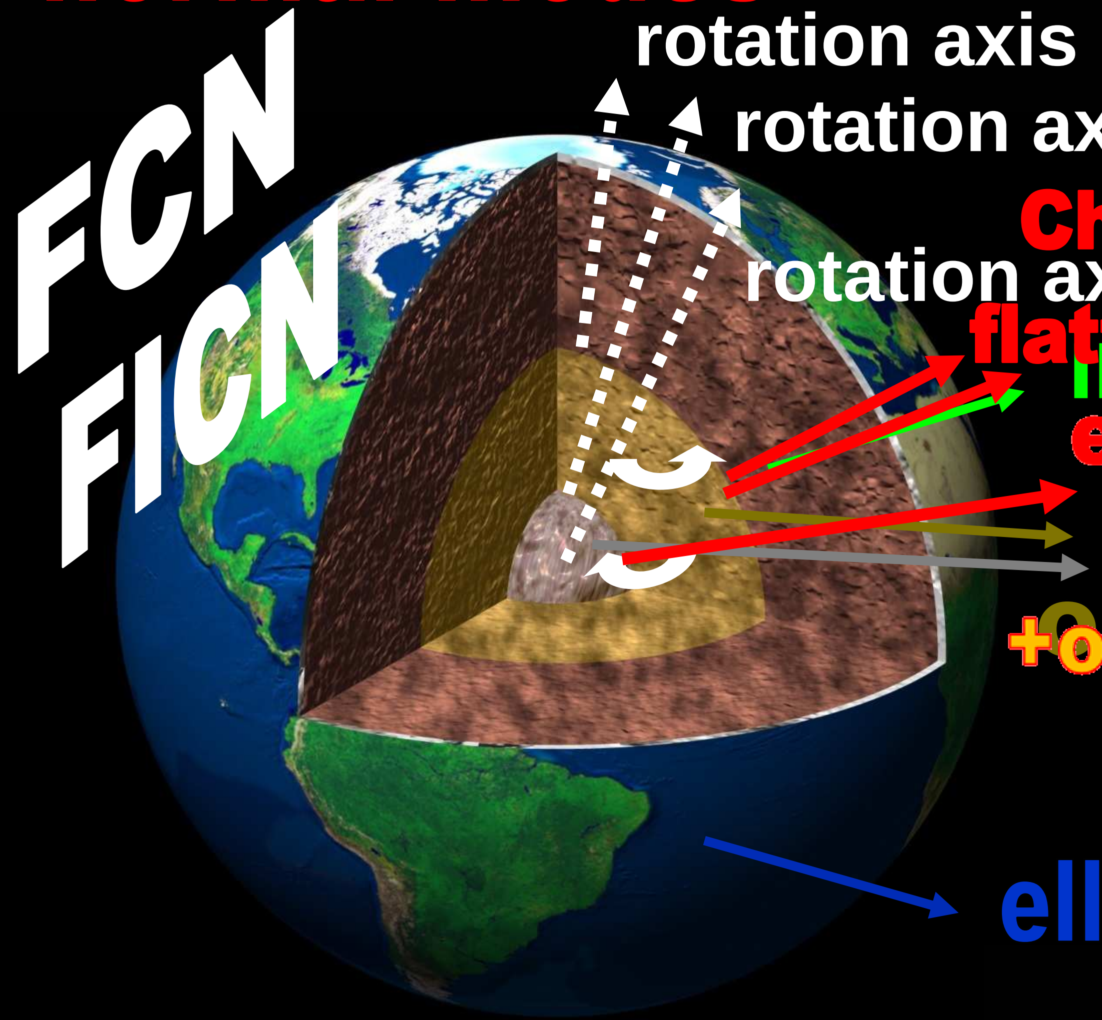
structure of the Earth's interior for its response

+ normal modes



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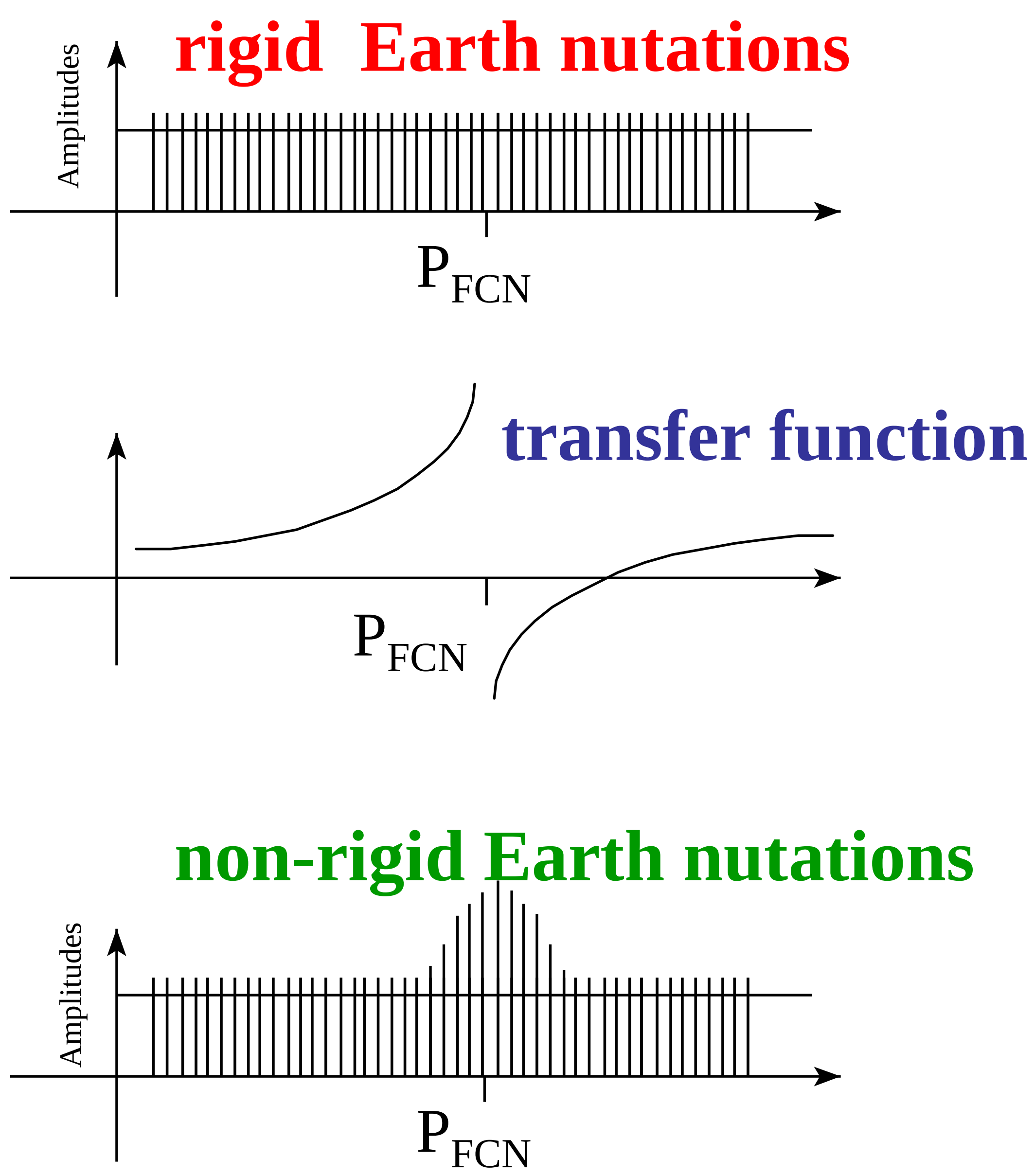

1. calculate rigid nutations (precision better than observation precision) from **celestial mechanics**
2. Calculate response of planet (transfer function in frequency domain) from **geophysics**

Earth: **amplifications** up to 30 mas

Earth rotation changes due to the core; core-mantle coupling

→ coupling mechanisms:

- ☞ topographic torque
- ☞ gravitational torque
- ☞ viscous torque
- ☞ electromagnetic torque

classically

this talk

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Electromagnetic torque + viscous torque: dissipative

- Outer core electrical conductivity: known from laboratory experiments: **$5\ 10^5$ S m^{-1}** (Stacey & Anderson 2001).
- Lowermost mantle electrical conductivity (~200 m layer at the base of the mantle): unknown but has to be lower than that of the core.

  σ_m= 10 S m^{-1}, $5\ 10^4$ S m^{-1}, $5\ 10^5$ S m^{-1}
- RMS of the radial magnetic field at the CMB: from surface magnetic field measurements: > 0.3 mT.
- Viscosity of the outer core fluid close to the CMB:
  - molecular viscosity: ~10^{-6} m^2 s^{-1} (laboratory experiments and ab initio computations).
  - eddy viscosity: < 10^{-4} m^2 s^{-1} (Buffett & Christensen 2007).

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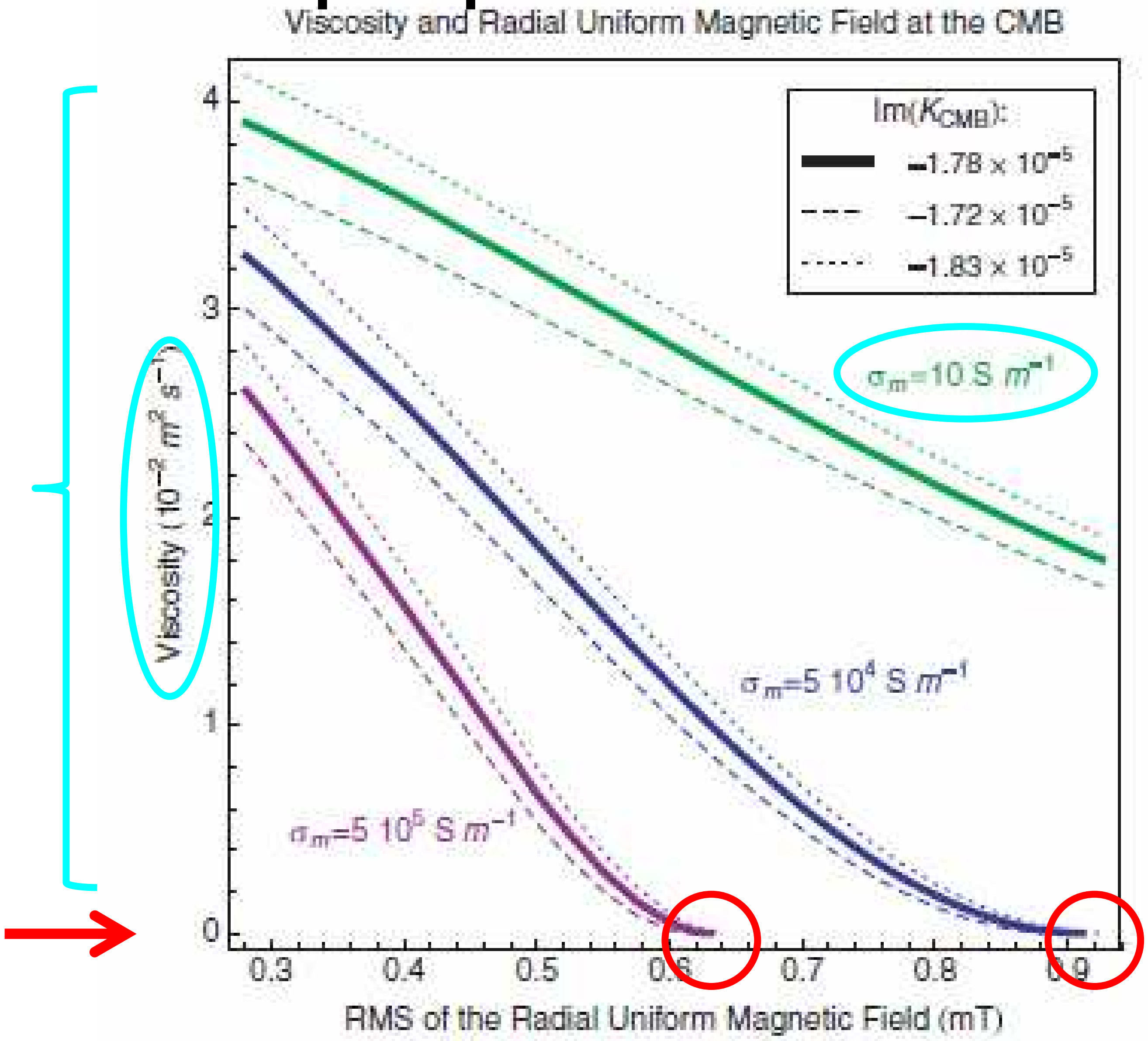
Constraints on the physical properties of the CMB



Coupling model used: Buffet et al. 2002 for EM and Mathews & Guo 2005 for viscomagnetic

From Koot et al. 2010

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Constraints on the physical properties of the CMB

- For EM coupling only: RMS of the radial magnetic field at the CMB: 0.7 mT or higher.
- Viscomagnetic coupling:
  - Allows for lower values of the magnetic field at the CMB.
  - Allows for lower values of mantle conductivity.
  - Outer core viscosity: ~10^{-2} m^2 s^{-1}.

  ➡ Very high value, unlikely to be realistic.

➡ Electromagnetic coupling (if no topography) how to explain large RMS?

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Earth rotation changes due to the core; core-mantle coupling

→ coupling mechanisms:

- ☞ topographic torque
- ☞ gravitational torque
- ☞ viscous torque
- ☞ electromagnetic torque

classically

How to explain high magnetic field?

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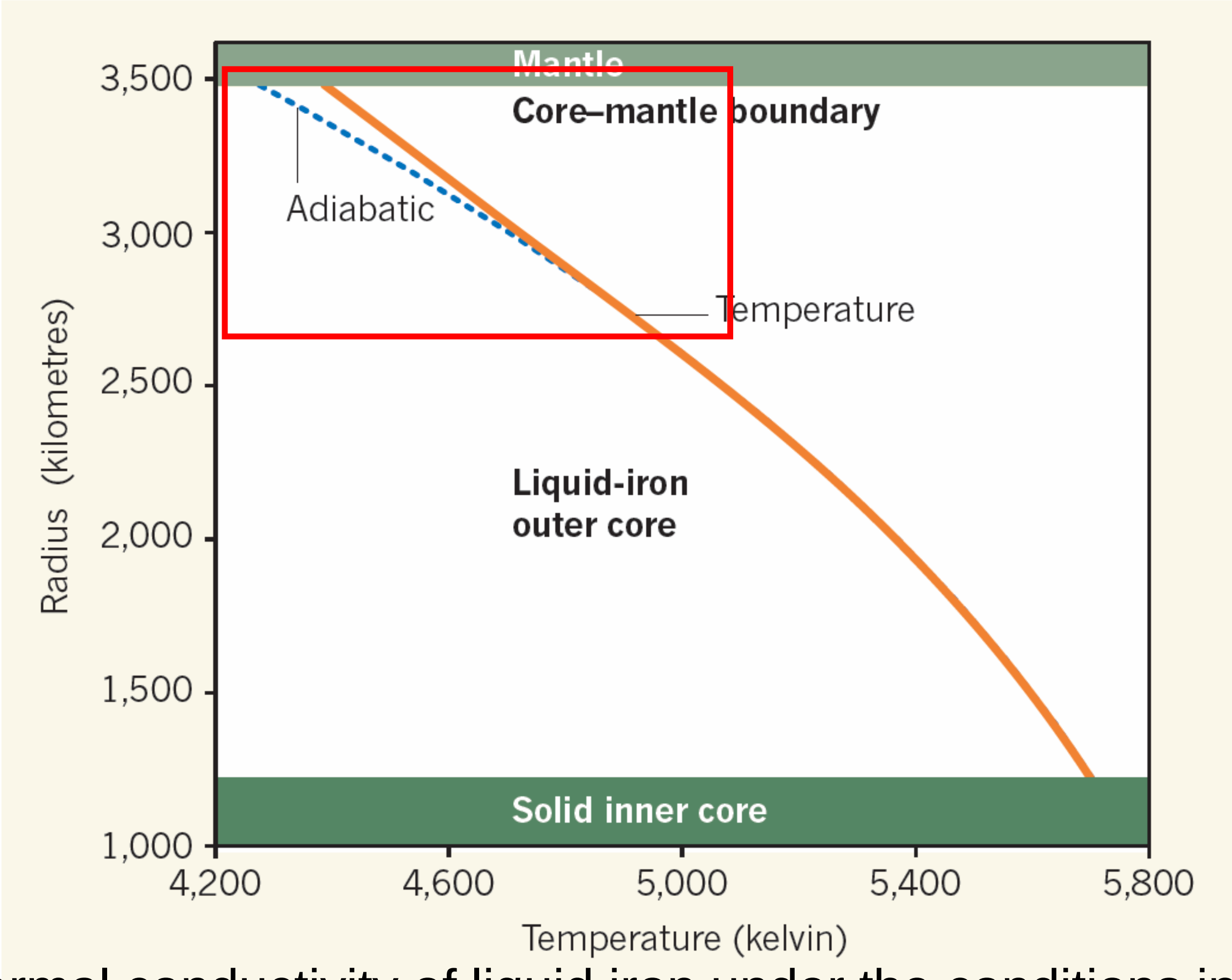
Higher thermal conductivity at the top of the core



Laboratory: thermal conductivity of liquid iron under the conditions in Earth's core is several times higher than previous estimates (Pozzo et al. *Nature* 2012)

→ increase of heat to be carried by conduction in this layer

→ less heat to drive convection in the core

→ decrease in electrical resistance

→ more generation than loss of magnetic field balance equation

Buffett, Nature, 2012

Earth rotation changes due to the core; core-mantle coupling

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classically

heat conductivity

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➡ **Topographic coupling**

how to explain CMB coupling constants?

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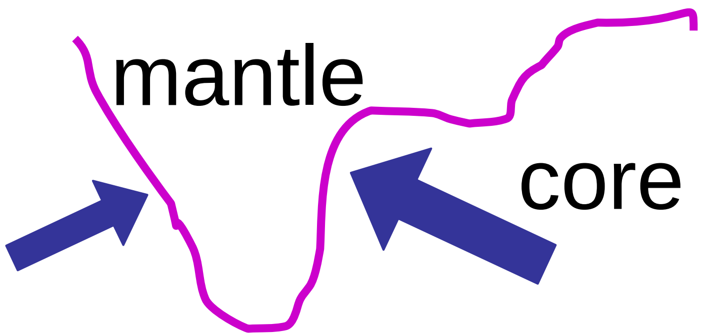
Core Angular Momentum exchange due to **topographic** torque at CMB

☞ pressure at CMB

☞ core-mantle boundary topography (<2km)

Difficult, challenging, controversial

but cannot be ruled out



e.g. Hide 1977

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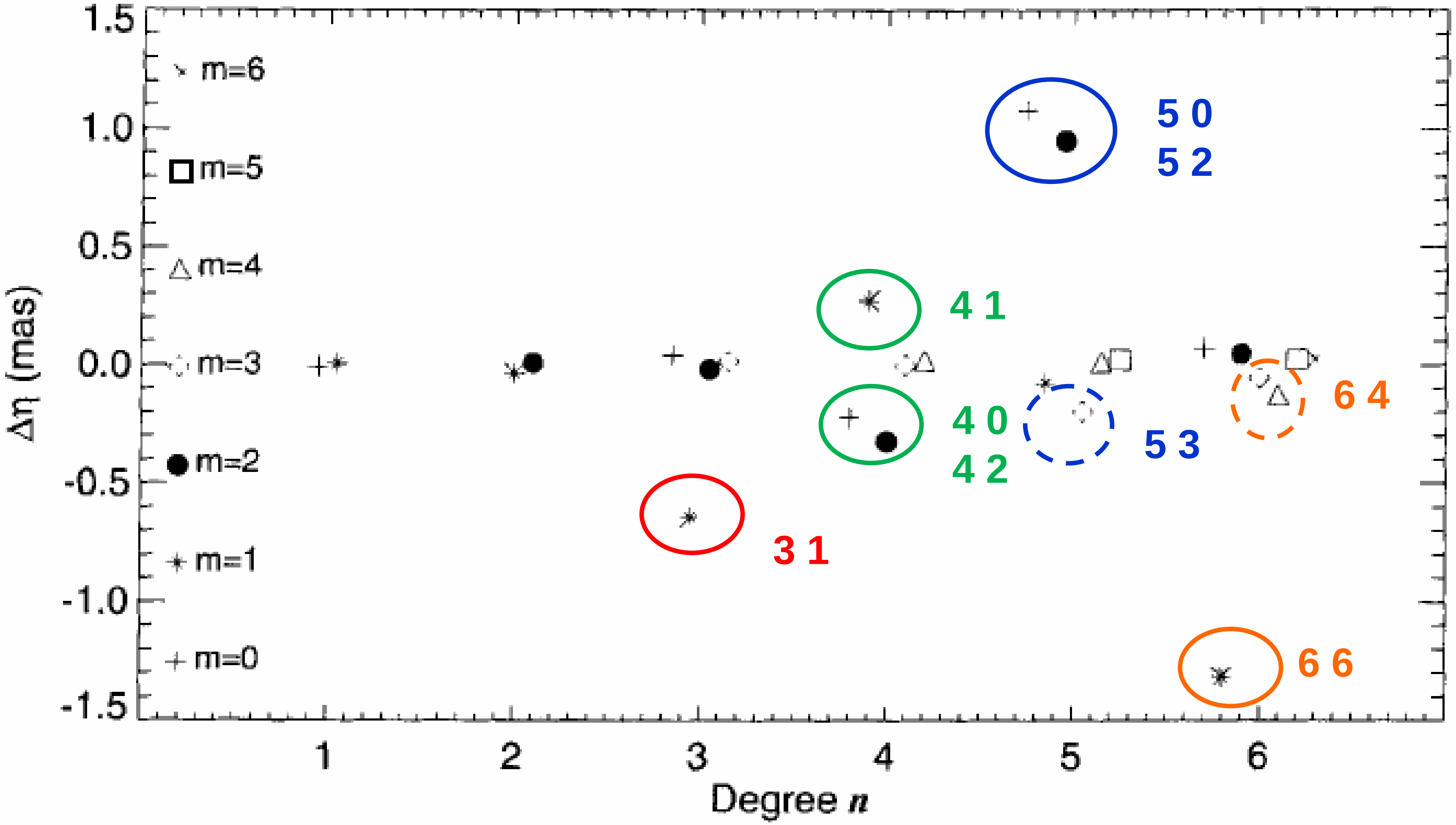

From Wu and Wahr, 1997

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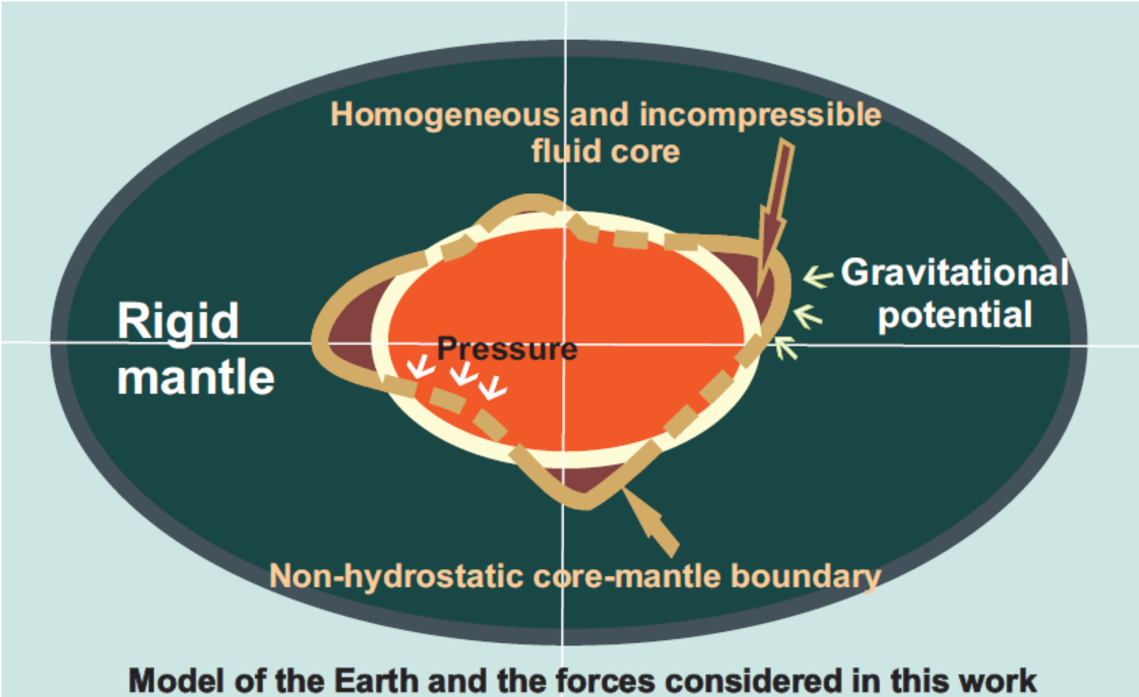
Topographic coupling



- Why only some of the topography coefficients are important?
- Related to resonance with inertial waves

  [*when perturbing a rotating fluid, the particle motion is characterized by a low-frequency oscillation called inertial wave*]
- Related to the geometry of the core and of the topography

☞ **Analytical approach**

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Research objective and strategy

- Aim at obtaining torque and associated effects on nutation
- Strategy:
  1
  – Establish the motion equations and boundary conditions in the fluid;
  – Compute analytically/numerically the solutions;
  – Obtain the dynamic pressure as a function of the physical parameters;
  – Determine the topographic torque.
- Assessment: Comparison with Wu and Wahr (1997) who used a numerical technique

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Differential equations and boundary conditions

- Linearized Navier-Stokes equation:

The oscillations of a rotating fluid where the restoring force involves Coriolis force are inertial waves

$$\nabla \cdot \vec{v} = 0 \qquad \chi = \Omega^2 \left(m_1^f \, xz + m_2^f \, yz \right)$$

$$\begin{cases} i\,\sigma_m \vec{q} + 2\vec{\hat{z}} \times \vec{q} + \nabla \Phi = 0 \\ \nabla \cdot \vec{q} = 0 \\ \vec{n} \cdot \vec{q} + \Omega^{-1} L^{-1} \vec{n} \cdot \vec{v} = 0 \end{cases}$$

Coriolis

where $\Phi = \frac{\phi}{\Omega^2 L^2}$ and $\phi = \frac{p}{\rho_f} + \chi$.

Process for obtaining the solutions and the torque

- $\vec{q}$ as a function of $\nabla\Phi$: $$\vec{q} = \frac{-i\,\sigma_m}{4-\sigma_m^2}\left[\nabla\Phi - \frac{2}{i\,\sigma_m}\vec{\hat{z}}\times\nabla\Phi - \frac{4}{\sigma_m^2}(\vec{\hat{z}}\cdot\nabla\Phi)\,\vec{\hat{z}}\right]$$

- Expression for Φ: $$\Phi = \sum_{l=1} a_l^k P_{lk}(\frac{\sigma_m}{2})\,Y_l^k(\vartheta,\lambda).$$

- Expression of $\vec{v}$ in function of χ: $$v_1 + iv_2 = -\frac{i}{\Omega}\left(\frac{\partial\chi}{\partial x} + i\frac{\partial\chi}{\partial y}\right)$$ $$v_3 = \frac{i}{\Omega}\frac{d\chi}{dz}$$

$$\vec{\Gamma}_{topo} = \vec{\Gamma}_0 + \int\int_{CMB} \vec{r}\times\vec{\hat{n}}\rho_f(\chi-\phi)\,dS = \vec{\Gamma}_0 + \vec{\Gamma}^{\chi}_{topo} + \vec{\Gamma}^{\phi}_{topo}$$

$$\Gamma_1^{\phi} = -\frac{i}{2}\sum_{l=1}^{l_{max}}\sum_{k=-l}^{l}(-1)^k\left[\sqrt{l-k}\sqrt{l+k+1}\,\epsilon_l^{-k-1} + \sqrt{l-k+1}\sqrt{l+k}\,\epsilon_l^{-k+1}\right]P_{lk}(\frac{\sigma_m}{2})\,a_l^k$$

$$\Gamma_2^{\phi} = \frac{1}{2}\sum_{l=1}^{l_{max}}\sum_{k=-l}^{l}(-1)^k\left[-\sqrt{l+k+1}\sqrt{l-k}\,\epsilon_l^{-k-1} + \sqrt{l-k+1}\sqrt{l+k}\,\epsilon_l^{-k+1}\right]P_{lk}(\frac{\sigma_m}{2})\,a_l^k$$

$$\Gamma_3^{\phi} = i\sum_{l=1}^{l_{max}}\sum_{k=-l}^{l}(-1)^k k\,\epsilon_l^{-k}\,P_{lk}(\frac{\sigma_m}{2})\,a_l^k$$

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Final expressions for nutation

Equation for obtaining the analytical expression of a_l^k in function of the topography coefficients ε_l^k

$$
\begin{aligned}
& \sin^2\theta \sum_{l,k} Y_l^k \left[k P_{lk}\left(\frac{\sigma_m}{2}\right) - \left(1 - \frac{\sigma_m^2}{4}\right) P'_{lk}\left(\frac{\sigma_m}{2}\right)\right] a_l^k \\
+ \; & \sin^2\theta \left[2\sqrt{\frac{2\pi}{15}} \left(1 + 3\sum_{n=0} \epsilon_n^m Y_n^m\right) \left(\frac{(\sigma_m^2 + \sigma_m - 2)}{2\sigma_m} Y_2^1 m_f^- + \frac{(-\sigma_m^2 + \sigma_m + 2)}{2\sigma_m} Y_2^{-1} m_f^+ \right) \right. \\
& \left. \quad + \sqrt{\frac{2\pi}{3}} \frac{(4 - \sigma_m^2)}{2\sigma_m} \Psi \left(-Y_1^1 m_f^- + Y_1^{-1} m_f^+ \right) \right] \\
+ \; & \cos^2\theta \sqrt{\frac{2\pi}{3}} \Psi \left(-\frac{(\sigma_m + 2)}{2} Y_1^1 m_f^- + \frac{(\sigma_m - 2)}{2} Y_1^{-1} m_f^+ \right) \\
+ \; & \sqrt{\frac{2\pi}{15}} \sum_{n=1} m \epsilon_n^m Y_n^m \left(\frac{(\sigma_m + 2)}{2} Y_2^1 m_f^- + \frac{(\sigma_m - 2)}{2} Y_2^{-1} m_f^+ \right) \\
= \; & 0
\end{aligned}
$$

functions of the frequency

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Particular case of flattening only

$$\begin{pmatrix} \alpha_0^0[\dots]_0^0 & \gamma_0^0[\dots]_2^0 & 0 & 0 \\ \beta_2^0[\dots]_0^0 & \alpha_2^0[\dots]_2^0 & \gamma_2^0[\dots]_4^0 & 0 \\ 0 & \beta_4^0[\dots]_2^0 & \alpha_4^0[\dots]_4^0 & \gamma_4^0[\dots]_6^0 \\ 0 & 0 & \beta_6^0[\dots]_4^0 & \alpha_6^0[\dots]_6^0 \end{pmatrix} \cdot \begin{pmatrix} a_0^0 \\ a_2^0 \\ a_4^0 \\ a_6^0 \end{pmatrix} = \begin{pmatrix} f_0^0(\epsilon_0^0, \epsilon_2^0) \\ f_2^0(\epsilon_0^0, \epsilon_2^0) \\ f_4^0(\epsilon_0^0, \epsilon_2^0) \\ f_6^0(\epsilon_2^0) \end{pmatrix}$$

functions of the frequency

General case

$$\begin{pmatrix} \alpha_{l_i}^k[\dots]_{l_i}^k & 0 & \dots & & \\ \beta_{l_i+2}^k[\dots]_{l_i+2}^k & 0 & \dots & & \\ \vdots & \vdots & \ddots & & \\ \dots & \gamma_{l-2}^k[\dots]_l^k & \dots & & \\ \dots & \alpha_l^k[\dots]_l^k & \dots & \dots & \dots \\ \dots & \beta_{l+2}^k[\dots]_l^k & \dots & \dots & 0 \\ \vdots & \vdots & \dots & \dots & \gamma_{l_f-2}^k[\dots]_{l_f}^k \\ 0 & 0 & & \dots & \alpha_{l_f}^k[\dots]_{l_f}^k \end{pmatrix}$$

$$\delta_l^k = \alpha_l^k[\dots]_l^k \cdot \delta_{l-2}^k - \beta_l^k[\dots]_{l-2}^k \cdot \gamma_{l-4}^k[\dots]_{l-2}^k \cdot \delta_{l-4}^k$$

$$\delta_0^k = \alpha_0^k[\dots]_0^k \qquad \delta_2^k = det(\begin{pmatrix} \alpha_0^k[\dots]_0^k & \gamma_0^k[\dots]_0^k & 0 \\ \beta_2^k[\dots]_2^k & \alpha_2^k[\dots]_2^k & \gamma_2^k[\dots]_4^k \\ 0 & \gamma_4^k[\dots]_2^k & \alpha_4^k[\dots]_4^k \end{pmatrix})$$

$$
\begin{pmatrix}
\alpha_0^0[\dots]_0^0 & \gamma_0^0[\dots]_2^0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta_2^0[\dots]_0^0 & \alpha_2^0[\dots]_2^0 & \gamma_2^0[\dots]_4^0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_4^0[\dots]_2^0 & \alpha_4^0[\dots]_4^0 & \gamma_4^0[\dots]_6^0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_6^0[\dots]_4^0 & \alpha_6^0[\dots]_6^0 & \gamma_6^0[\dots]_8^0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_8^0[\dots]_6^0 & \alpha_8^0[\dots]_8^0 & \gamma_8^0[\dots]_{10}^0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_{10}^0[\dots]_8^0 & \alpha_{10}^0[\dots]_{10}^0 & \gamma_{10}^2[\dots]_{12}^0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \beta_{12}^0[\dots]_{10}^0 & \alpha_{12}^0[\dots]_{12}^0 & \gamma_{12}^2[\dots]_{14}^0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_{14}^0[\dots]_{12}^0 & \alpha_{14}^0[\dots]_{14}^0 & \gamma_{14}^2[\dots]_{16}^0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{16}^0[\dots]_{14}^0 & \alpha_{16}^0[\dots]_{18}^0
\end{pmatrix}
\cdot
\begin{pmatrix}
a_0^0 \\ a_2^0 \\ a_4^0 \\ a_6^0 \\ a_8^0 \\ a_{10}^0 \\ a_{12}^0 \\ a_{14}^0 \\ a_{16}^0
\end{pmatrix}
$$

functions of the frequency

$$
= \begin{pmatrix}
f_0^0(\epsilon_0^0, \epsilon_2^0, \epsilon_4^0, \epsilon_2^1, \epsilon_4^1, \epsilon_2^{-1}, \epsilon_4^{-1}) \\
f_2^0(\epsilon_0^0, \epsilon_2^0, \epsilon_4^0, \epsilon_6^0, \epsilon_2^1, \epsilon_4^1, \epsilon_6^1, \epsilon_2^{-1}, \epsilon_4^{-1}, \epsilon_6^{-1}) \\
f_4^0(\epsilon_0^0, \epsilon_2^0, \epsilon_4^0, \epsilon_6^0, \epsilon_8^0, \epsilon_2^1, \epsilon_4^1, \epsilon_6^1, \epsilon_8^1, \epsilon_2^{-1}, \epsilon_4^{-1}, \epsilon_6^{-1}, \epsilon_8^{-1}) \\
f_6^0(\epsilon_2^0, \epsilon_4^0, \epsilon_6^0, \epsilon_8^0, \epsilon_{10}^0, \epsilon_2^1, \epsilon_4^1, \epsilon_6^1, \epsilon_8^1, \epsilon_{10}^1, \epsilon_2^{-1}, \epsilon_4^{-1}, \epsilon_6^{-1}, \epsilon_8^{-1}, \epsilon_{10}^{-1}) \\
f_8^0(\epsilon_4^0, \epsilon_6^0, \epsilon_8^0, \epsilon_{10}^0, \epsilon_4^1, \epsilon_6^1, \epsilon_8^1, \epsilon_{10}^1, \epsilon_4^{-1}, \epsilon_6^{-1}, \epsilon_8^{-1}, \epsilon_{10}^{-1}) \\
f_{10}^0(\epsilon_6^0, \epsilon_8^0, \epsilon_{10}^0, \epsilon_6^1, \epsilon_8^1, \epsilon_{10}^1, \epsilon_6^{-1}, \epsilon_8^{-1}, \epsilon_{10}^{-1}) \\
f_{12}^0(\epsilon_8^0, \epsilon_{10}^0, \epsilon_8^1, \epsilon_{10}^1, \epsilon_8^{-1}, \epsilon_{10}^{-1}) \\
f_{14}^0(\epsilon_{10}^0, \epsilon_{10}^1, \epsilon_{10}^{-1}) \\
0
\end{pmatrix}
$$

frequency values for getting determinant =0
→ resonances

Final expressions for LOD

Equation for obtaining the analytical expression of a_l^k in function of the topography coefficients ε_l^k

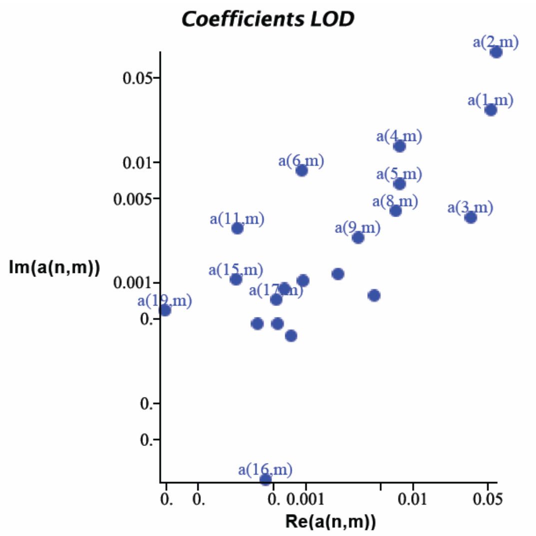
$$\sum_{l,k} Y_l^k \left[k P_{lk}\left(\frac{\sigma_m}{2}\right) - \left(1-\frac{\sigma_m^2}{4}\right) P'_{lk}\left(\frac{\sigma_m}{2}\right) \right] a_l^k - 2\left(1-\frac{\sigma_m^2}{4}\right) \sum_{n=1} m\epsilon_n^m Y_n^m m_3 = 0$$

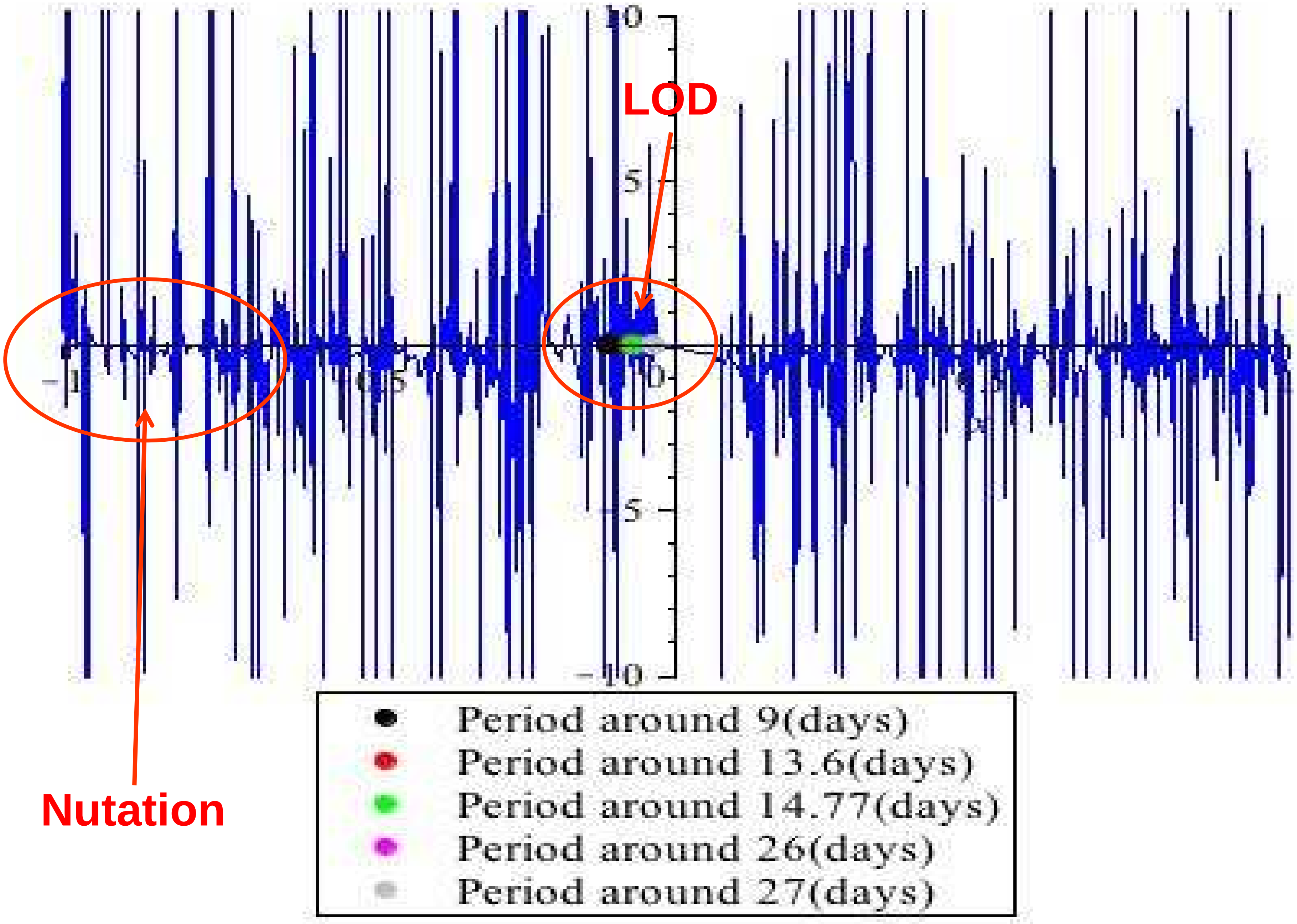
functions of the frequency

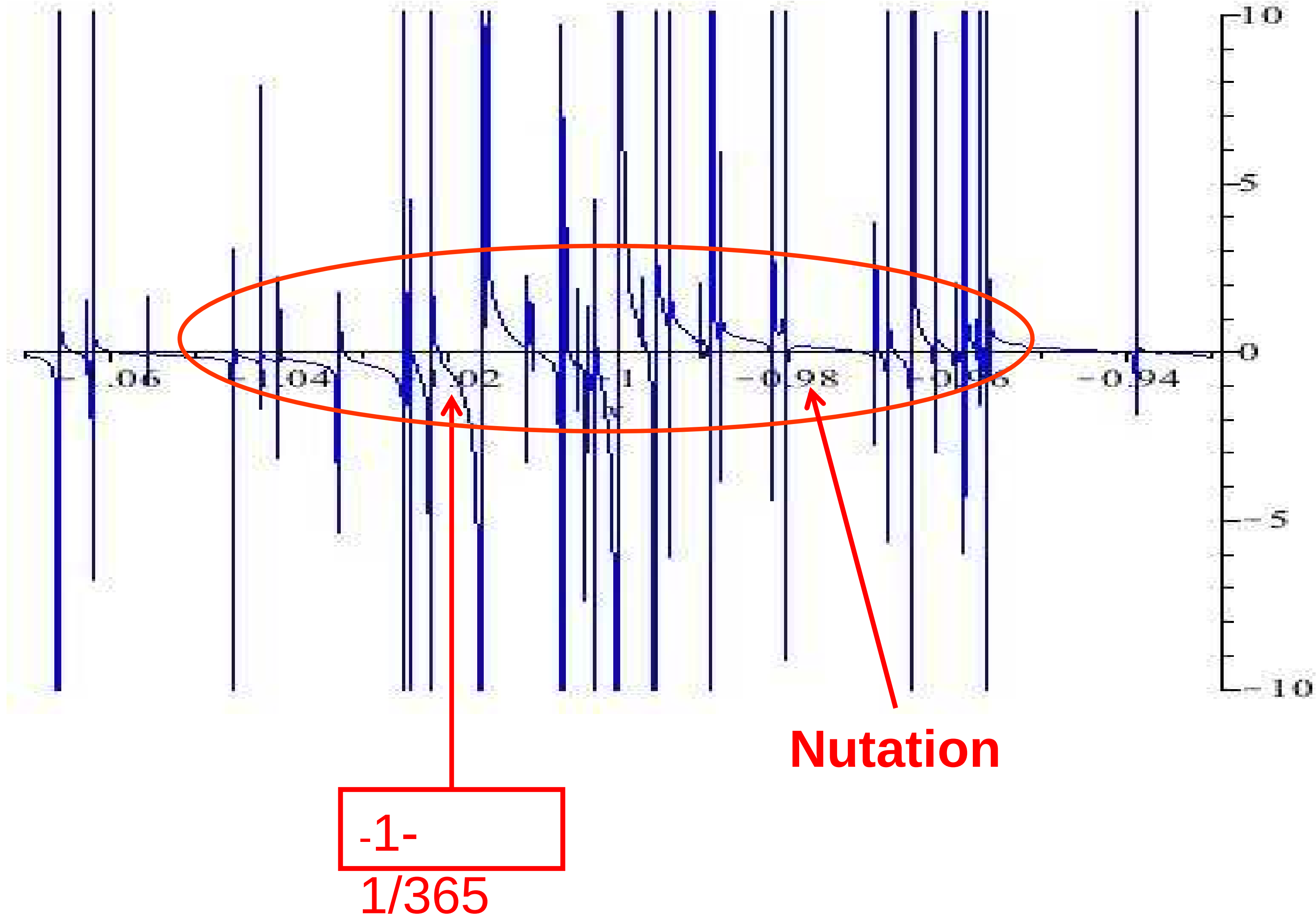
This is very similar but simpler than for nutation.

Kaula-kind law for topography

$$a_n^m = \frac{2\left(1-\frac{\sigma_m^2}{4}\right) m\epsilon_n^m m_3}{m P_n^m\left(\frac{\sigma_m}{2}\right) - \left(1-\frac{\sigma_m^2}{4}\right) P_n^{m\prime}\left(\frac{\sigma_m}{2}\right)}$$




LOD
Nutation
10
5
−5
−10
−1
−0.5
0
Period around 9(days)
Period around 13.6(days)
Period around 14.77(days)
Period around 26(days)
Period around 27(days)



10
5
0
−5
−10
−1.06
−1.04
−1.02
−1
−0.98
−0.96
−0.94
Nutation
-1-
1/365

Earth rotation changes due to the core; core-mantle coupling

+ Core stratification

→ coupling mechanisms:

- ☞ topographic torque
- ☞ gravitational torque
- ☞ viscous torque
- ☞ electromagnetic torque

adopted model

heat conductivity

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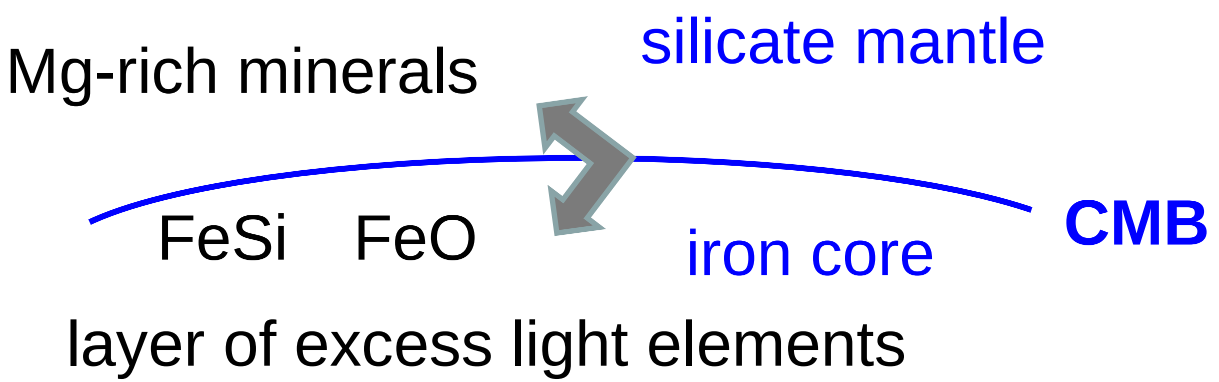
Buffett EPSL (2011)

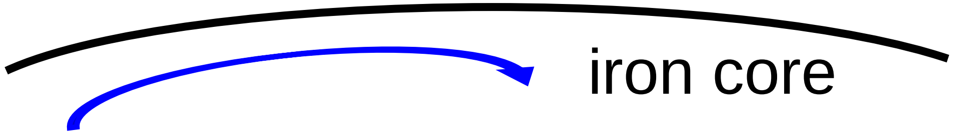
Topography, stratification, and magnetism

- chemical interactions between the core and the mantle



Buffett EPSL (2011)

Stratification and magnetism



motion almost parallel to constant density surfaces

little change in density and the resulting buoyancy forces are weak

Earth rotation changes due to the core; core-mantle coupling

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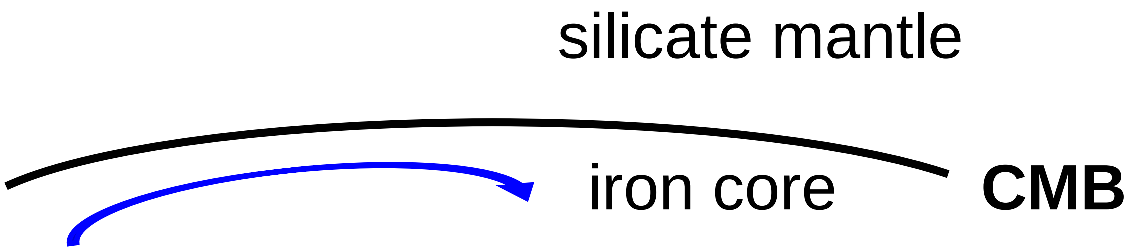

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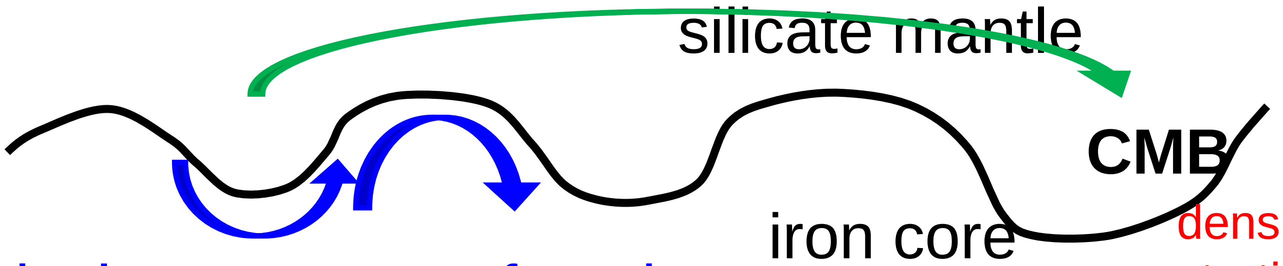

vertical component of motion

stratification

density field in a stratified fluid is disturbed and a buoyancy force arises

Required strength of the radial magnetic field can be lowered.

Further from the boundary the stratified fluid is swept past the mantle with the underlying tidal flow

Earth rotation changes due to the core; inner core-outer core coupling

→ coupling mechanisms:

- ☞ ~~topographic torque~~
- ☞ gravitational torque
- ☞ viscous torque
- ☞ electromagnetic torque

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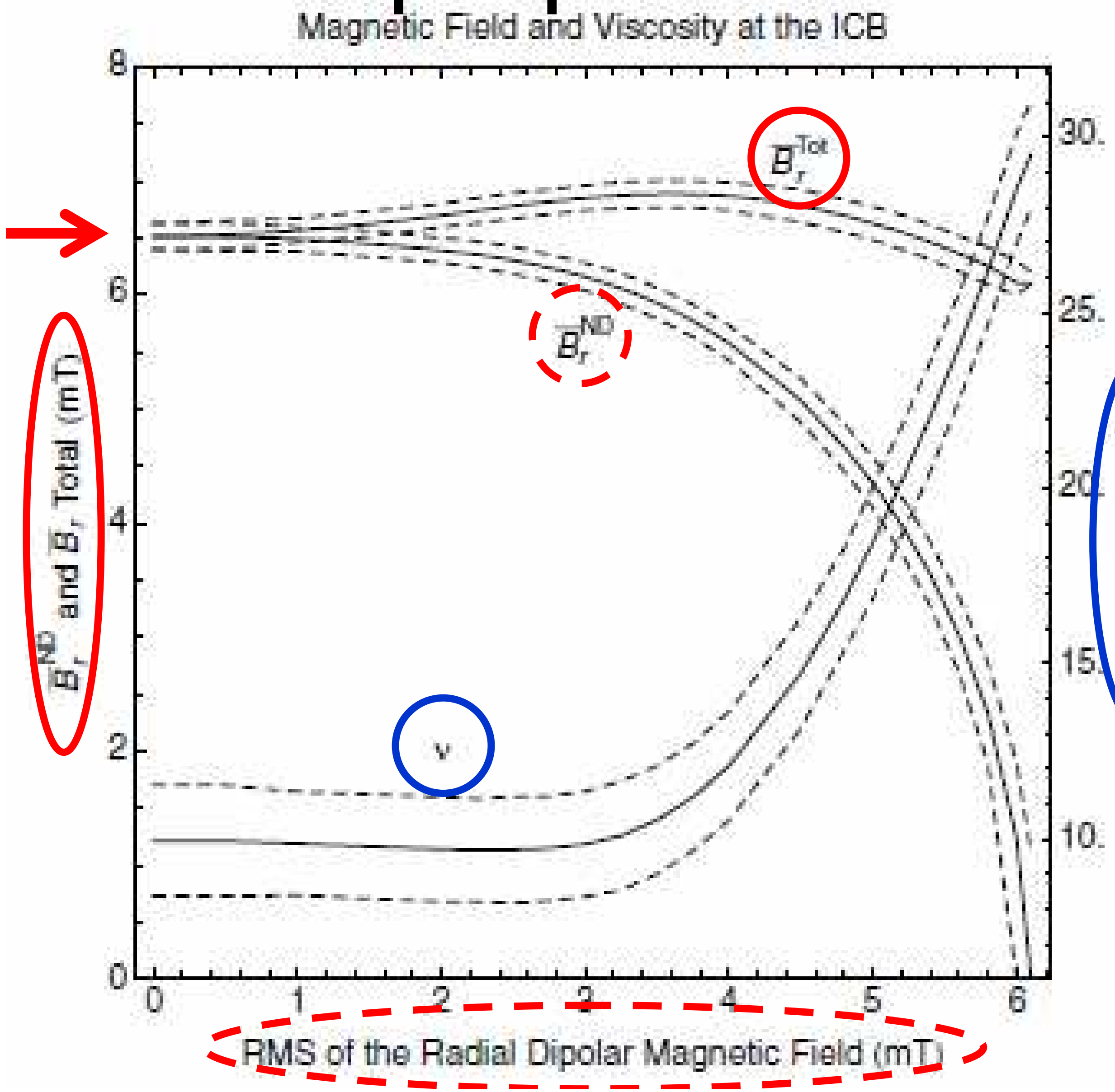
Constraints on the physical properties of the ICB



Visco-magnetic coupling at the ICB

- Electrical conductivity of the outer and inner cores: known from laboratory experiments.
- Unknown parameters:

-Magnetic field at the ICB

-Viscosity of the outer core at the ICB

From Koot et al. 2010

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Constraints on the physical properties of the ICB

- No solution for a purely EM coupling.
- Outer core viscosity: ~ 10 $m^2\ s^{-1}$: **unrealistic!**
- RMS of the mag. field at the ICB: 6-7 mT.

Another mechanism is required to explain the observed damping of the FICN mode !

Inner core viscous deformation?

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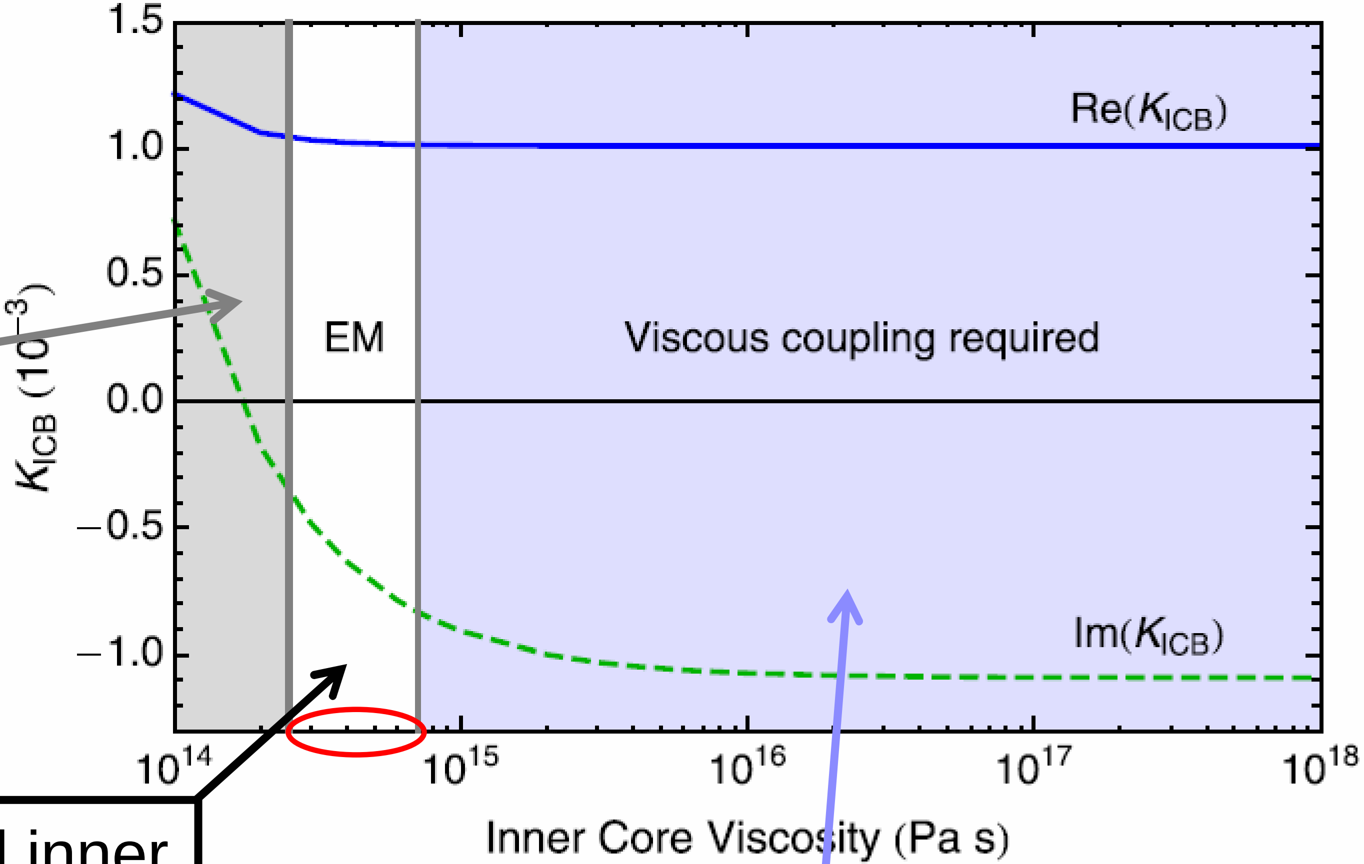
Real and **imaginary** parts of the coupling constant K_{ICB} which satisfies the observational constraints



too much energy dissipated by inner core deformation

EM coupling and inner core viscosity can explain observation

inner core viscosity contribution too small, EM coupling alone at ICB cannot explain observation

Koot & Dumberry EPSL (2011)

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Inner core viscous deformation?

Koot & Dumberry EPSL (2011)

– **For Inner core viscosity: ~2-7 10^{14} Pa s.**

– **RMS of the mag. field at the ICB: 4.5 - 6.5 mT**

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Modelling the Earth's rotation

- Interior → **Realistic** Model → Rotation
- Current model IAU2000 (Mathews et al. 2002)
  - **interior** properties summarised in a set of **parameters**
  - poorly known parameters are **estimated**, **improving** knowledge of the Earth's **interior** (Koot et al. 2010, 2011…)
  - other parameters are **computed** for a **spherical** Earth
- Former model IAU1980 (Wahr 1981)
  - **full** consideration of the **polar flattening**
  - disregarded **non-hydrostaticity** which affects the **FCN period** (Gwinn et al. 1986), eventually **discarded**
  - since then **refined** (e.g. Huang et al. 2011), now working on **non-hydrostaticity** and associated **triaxiality**

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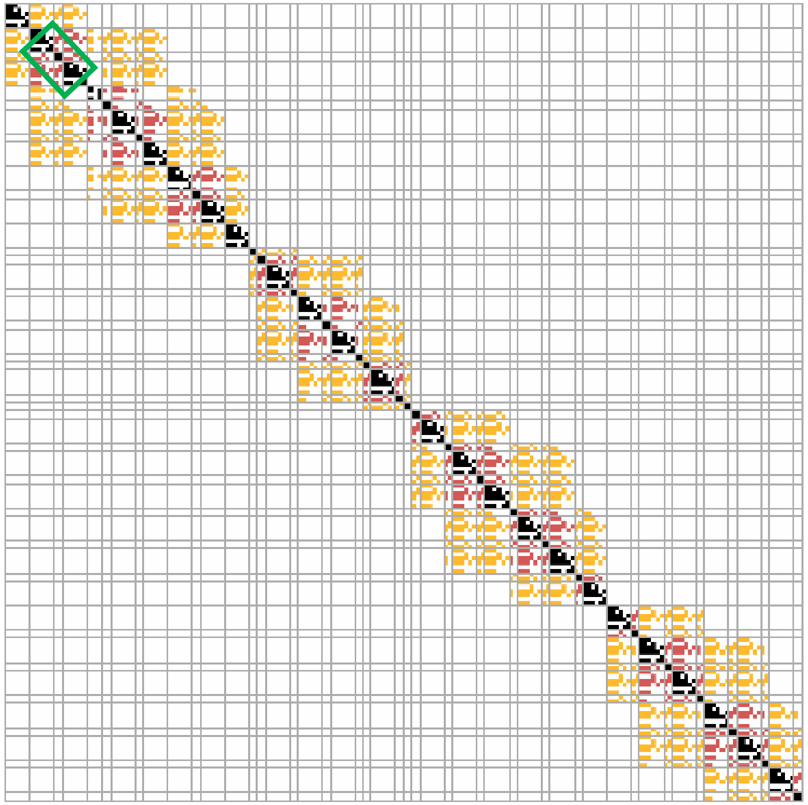
Non-hydrostaticity & Triaxiality



- **Spectral** analysis of the equations of **continuum mechanics**
- Rotation perturbations modelled as infinitesimal **toroidal degree-1** displacement
- ODE submatrix:
  - **spherical**, non-rotating
  - **biaxial**, rotating
  - **triaxial**, rotating, convecting

Work of Antony Trinh

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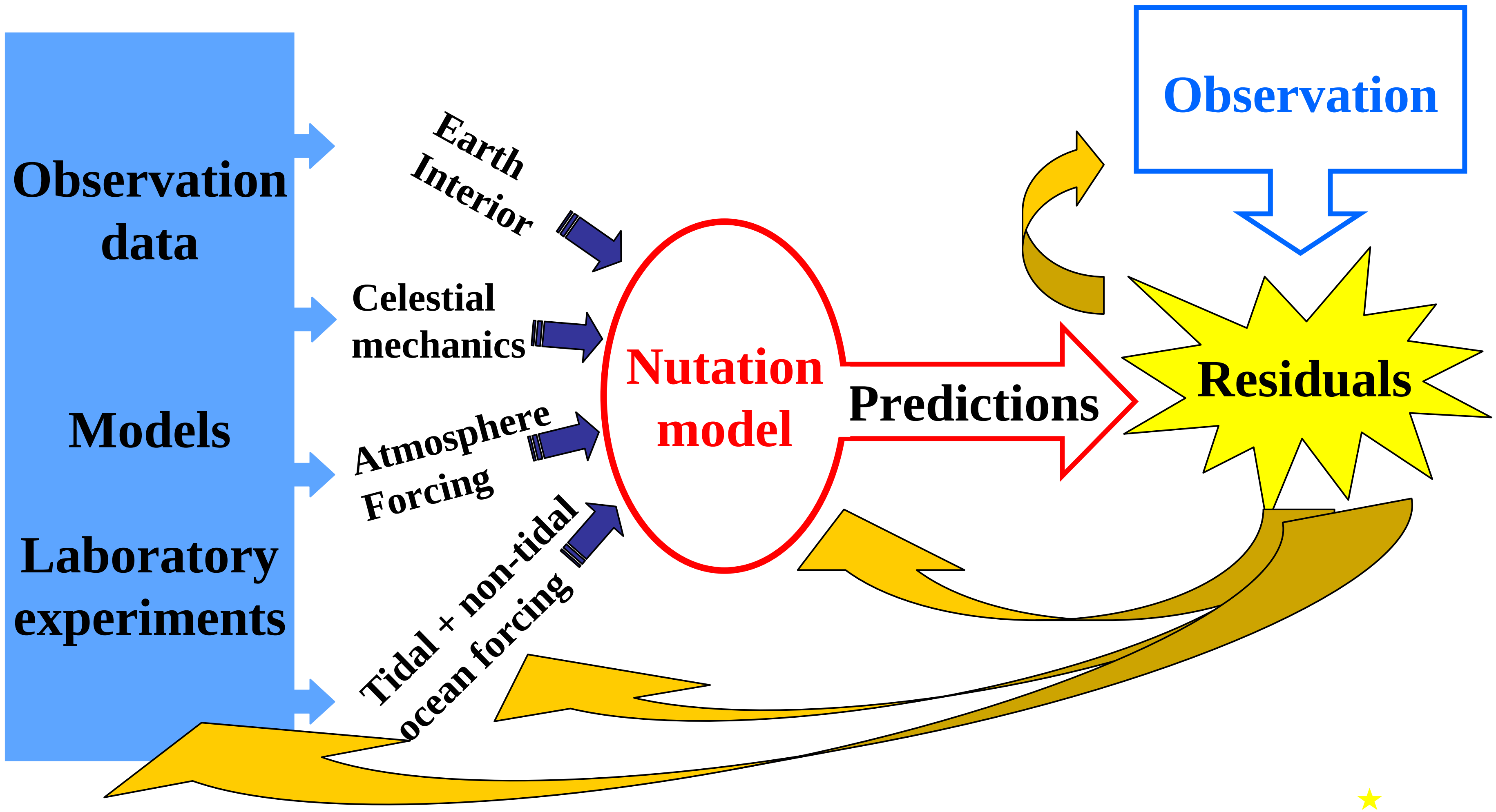

Better understanding of the Earth interior!