Systèmes de Référence Temps-Espace

Nutation determination by means of GNSS

- Comparison with VLBI

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Introduction

Space geodetic techniques cannot be used for a direct determination of the celestial pole offsets due to their correlations with the orbital elements of the satellite the computation of which is affected by the deficiencies in the modeling of the satellite orbits.

However, as shown by Rothacher et al. (1999), GPS can be used to estimate the nutation rates, similarly to what is done on a regular basis for LOD estimation; these authors computed a series of nutation rates covering 3.5 years, which was used for the estimation of the corrections to the IERS 1996 nutation model. However, nutation rate is not part of the IGS product and no other series of nutation rates have been provided since that time.

The purpose of this study, done in the framework of Kunliang Yao's PhD (2013), is to investigate the potential of GNSS observations for nutation estimation with the high precision currently achieved by this technique. We aim at developping the best use of GPS observations, independently of VLBI, for determining the nutation of the Earth's axis with the best possible accuracy.

Orbital elements and Earth orientation parameters

Geometical relationships

The changes in the observed orbital elements of an artificial satellite corresponding to celestial pole offsets (i.e. corrections to the coordinates, *X*, *Y* of the CIP in the GCRS), and correction to the Earth Rotation Angle (ERA) at date *t* can be expressed (at the 1st order of the offsets and with Keplerian approximation) by the following relationships (Capitaine & Wallace, 2007):

$$\Delta i = -\,dX \sin\Omega + dY \cos\Omega$$

$$\Delta\Omega \tan i = -\,dX \cos\Omega - dY \sin\Omega \cos\Omega\, \Delta i - \tan i\; d(ERA)$$

$$\Delta u_0 \sin I = -\,dX \sin\Omega + dY \cos\Omega\,,$$

Ω , i , are the right ascension of the ascending node and inclination of the orbital plane and u_0 is the argument of the latitude of the satellite at the osculating epoch; they are affected by systematic errors due to imperfect modeling.

Nutation estimation from satellite observations

- **absolute determination of the celestial pole offsets or ERA are not possible**
- **the rates in *X*, *Y* and ERA can be estimated** provided that the orbital perturbations are modeled with sufficient accuracy over the time interval of the estimation

- - - - - - - - - - - **IAU definitions** (see http://syrte.obspm.fr/iauWGnfa/NFA_Glossary.html) - - - - - - - - -

GCRS: *Geocentric celestial reference system*; ITRS: *International Terrestrial Reference System*
CIP: *Celestial Intermediate Pole*; TIRS: *Terrestrial Intermediate reference system;* CIO *Celestial intermediate origin;*
TIO: *Terrestrial intermediate origin;* ERA: *angle from the CIO to the TIO*

Main characteristics of the study

- The method used is largely based on the GNSS observations analysis strategy of the CNES-GRGS GINS multi-technique software for orbit determination and Earth dynamics studies, but with the following specificities which take advantage of the GNSS potential for estimation of the EOPs (cf. previous slide):

  (i) determination of the time derivatives of the GCRS CIP coordinates (X, Y), and ERA, with high temporal resolution, along with x_P, y_P (pole coordinates),

  (ii) computation of the satellite orbit in a reference system that minimizes the influence of the *a priori* values for precession-nutation (i.e. *X, Y*) and UT1-UTC (i.e. ERA), that GPS cannot determine directly.

- The computations are carried out by means of a new software, developed by K. Yao in Matlab environment and implementing the specificities (i) and (ii).

- The observations used are 3 years of GPS measurements from 1 January 2009, obtained from a dense and globally distributed reference station network.

The new GPS analysis software

Motivation *(cf. specificities (i) and (ii) of the previous slide)*

- To obtain long time series of time derivatives of the GCRS CIP coordinates (*X, Y*) and the ERA
- To minimize the effect (on the computations) of the *a priori* values for (*X, Y*) and ERA

Advantages of Matlab environment

- Codes easy to understand; vectorial computation are easily programmed
- Many scientific algorithms functions available
- Science programming library in C/FORTRAN can be re-used (i.e. SOFA library)
- Built-in tools available for the visualization and the data analysis, graphical interface
- etc.

Calculations

- largely based on the GPS analysis method of GINS (see Table 1 on next slide)
- GPS observations analyzed day by day
- ***Calculation part 1***: Determination of the station clock biases using the GPS ionosphere-free combination, P_c, of the pseudo-range observations
- ***Calculation part 2*** (11 successive steps; see Figure 2): Estimation of the parameters based an iterative least squares adjustment using the GPS double-differenced ionosphere-free phase observations, DDL_c

The main features of the GPS analysis software

Main features	Analysis with GINS	Analysis with the new software
Inertial reference system	GCRS	TIRS'$_0$ (*see the figure on the next slide*)
EOP estimation	(xp, yp, xpdot, ypdot), LOD	(xp, yp), ERAdot, (Xdot, Ydot)
Observations	GPS iono-free range and phase	GPS iono-free range & double-differenced phase
Ambiguities	Resolved by GINS/DYNAMO tools chains	Estimated
Solar system ephemerides	DE405	INPOP10
Earth's potential	EIGEN-GI04S	EGM2008
Station positions	ITRF08	
Antenna models	igs08.atx	
Solar radiation pressure	JPL SRP model	
Orbit integration	8-order Cowell's method	
Parameters estimation	iterative least squares adjustment	

Table 1: *The main features of the new analysis software as compared to the GRGS-CNES GINS software*

The inertial system for orbit computation

The reference system TIRS'$_0$ in which the satellite orbit computation is performed is:

- based on the ***Terrestrial Intermediate Reference System (TIRS)***: cf. Figure below,
- defined at date t_0 = 00h of the beginning epoch of the arc under analysis.

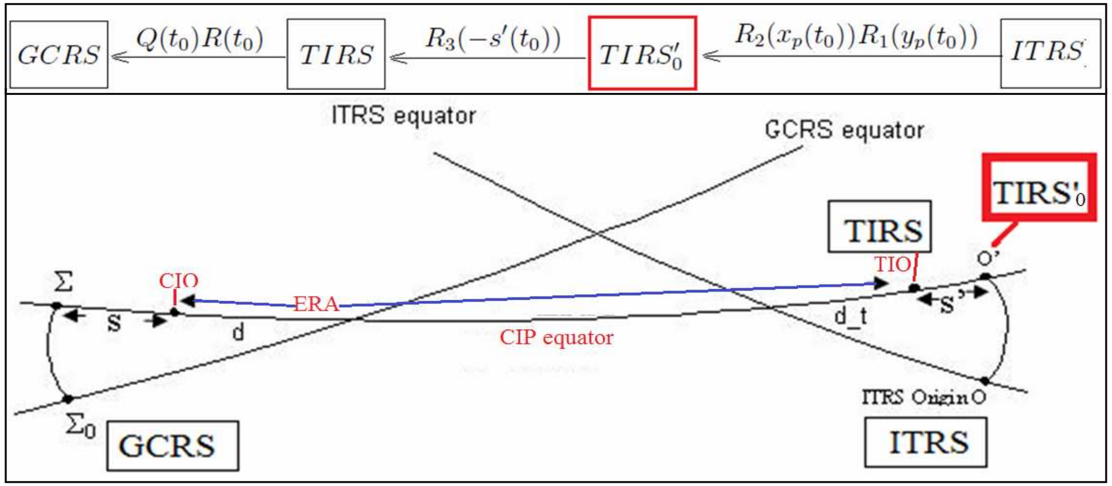

Fig. 1: *Schematic representation of the TIRS'$_0$ reference system used for the orbit integration*

TIRS'$_0$ is realized by the CIP axis and the origin O' on the CIP equator

(Note that the distance of O' from the TIO is *s*', i.e. ~0)

Formula for the coordinate transformation to the Terrestrial intermediate system TIRS'₀

Matrix transformation between GCRS and ITRS'$_0$

$$[GCRS] = \bar{Q}(X_0 + dX_0, Y_0 + dY_0)\, R_3(-\theta_0 - d\theta_0)[TIRS'_0]$$
$$\approx \bar{Q}(X_0, Y_0) R_3(-\theta_0)[TIRS'_0] \qquad \boxed{\theta_0 = -s_0 + ERA_0 + s'_0}$$

$$[TIRS'_0] \approx R_3^{-1}(-\theta_0)\bar{Q}^{-1}(X_0, Y_0)[GCRS] \quad |dX_0| < 1 \text{ mas} \quad |dY_0| < 1 \text{ mas} \quad |d\theta_0| < 1 \text{ mas}$$

This depends on the X, Y and ERA values at epoch t_0 (including the offsets w.r.t. the *a priori* values)

Matrix transformation between ITRS and ITRS'$_0$

$$[TIRS'_0] \approx \underline{H(X_0, Y_0, \theta_0, \delta X, \delta Y) R_3(-\delta\theta)} \;\; \underline{R_2(xp) R_1(yp)} \;\; [ITRS]$$
$$[ITRS] \approx \underline{R_1^{-1}(yp) R_2^{-1}(xp)} \;\; \underline{R_3^{-1}(-\delta\theta) H(X_0, Y_0, \theta_0, -\delta X, -\delta Y)} \;\; [TIRS'_0]$$

$$H(X_0, Y_0, \theta_0, \delta X, \delta Y) = \begin{bmatrix} 1 & (X_0\delta Y - Y_0\delta X)/2 & cos(\theta_0)\delta X + sin(\theta_0)\delta Y \\ -(X_0\delta Y - Y_0\delta X)/2 & 1 & -sin(\theta_0)\delta X + cos(\theta_0)\delta Y \\ -cos(\theta_0)\delta X + sin(\theta_0)\delta Y & sin(\theta_0)\delta X - cos(\theta_0)\delta Y & 1 \end{bmatrix}$$

This depends on the pole coordinates x_p, y_p and on the variations δX, δY and δERA values from epoch t_0

This is nearly not sensitive to errors in the a priori values X_0, Y_0, and ERA_0

Estimation of the parameters

Observations: $DDLc = D_b^{j,m} - D_a^{j,m} - D_b^{i,m} + D_a^{i,m} + \lambda_c N_{c,ba,ji,m} + \sigma_{L,ba,ji,m}$ (*m*: epoch; *a*, *b*: stations; *i*, *j*: satellites)

GPS double-differenced ionosphere-free phase observations

$D_{geo} = |\vec{X}_e(Te_{GPS}) - M(Tr_{GPS})\vec{X}_r(Tr_{GPS})|$ (D_{geo}: geometric distance station-satellite; *M*: matrix transformation from ITRS to the inertial system)

Steps:

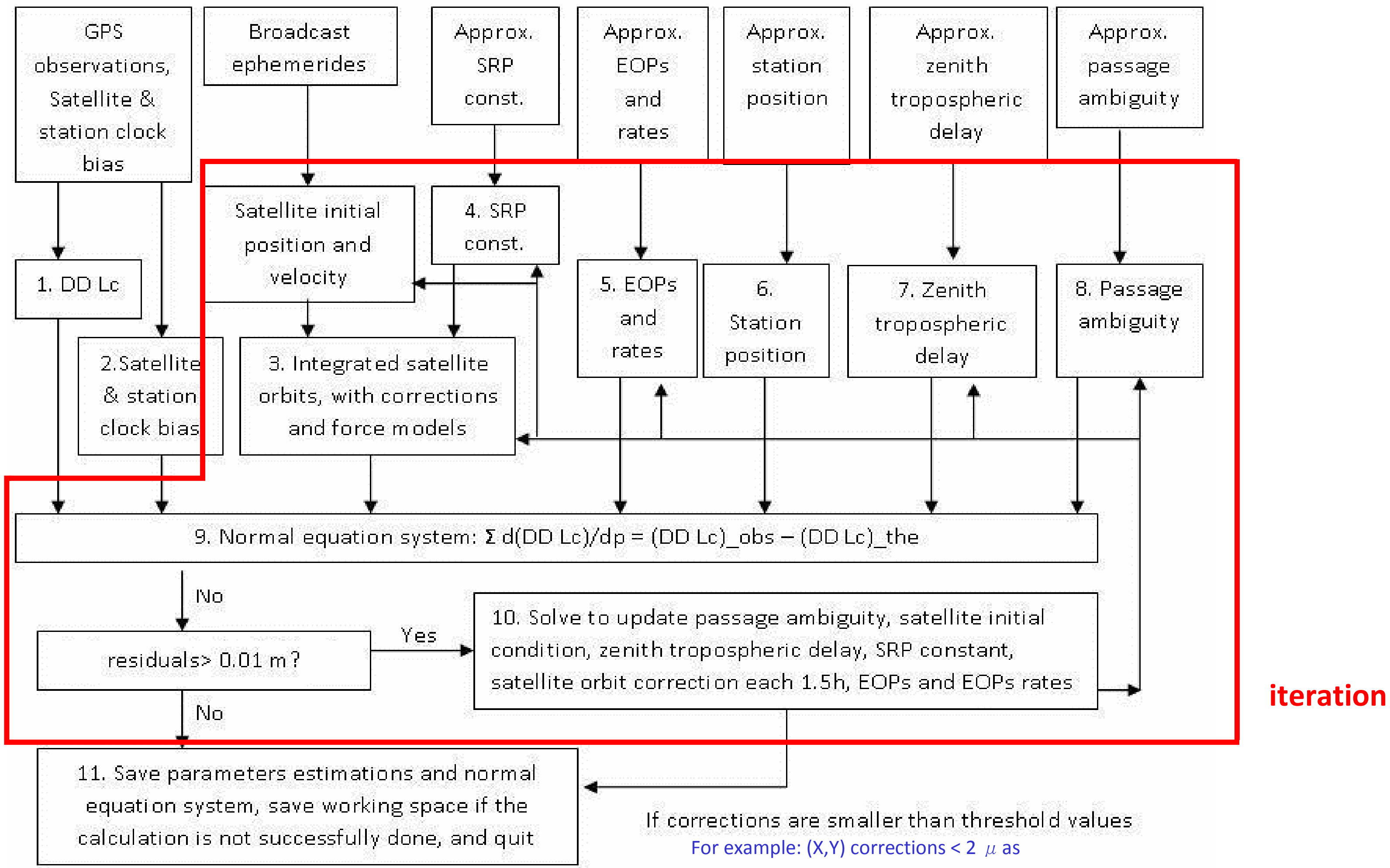

Fig. 2: *The successive steps of the new GPS analysis software for the estimation of the parameters*

IAU 2000 semi-analytical model for the time derivative of the X, Y quantities

sin. coef.	cos. coef.	Argument					period
(mas/d)	(mas/d)	l	l'	F	D	Ω	(days)
-51.16	-41648.80	0	0	2	0	2	13.66
-22.33	-8249.95	1	0	2	0	2	9.13
-6.97	-7100.50	0	0	2	0	1	13.63
3.90	-1559.86	1	0	-2	-2	-2	9.56
-3.54	-1414.41	1	0	2	0	1	9.12
-5.57	-1358.41	0	0	2	2	2	7.10
-4.77	-1311.22	2	0	2	0	2	6.86
2.54	1072.81	0	0	0	2	0	14.77
1.34	530.49	2	0	0	0	0	13.78
1.21	475.51	0	0	2	0	0	13.61

Table 2: *Semi-analytical development of the time derivative of X corresponding to IAU 2000 nutation: largest terms*

Test of the efficiency of the GPS analysis software for nutation rate estimation

- **Data set**: GPS data over 200 days from 01/01/2009 with ~115 IGS "core" stations, and ~32 satellites
- **A priori values**: IERS C04 polar motion and LOD; nutation rates equal to zero
- **Results**: estimated *X*, *Y* rates in very good agreement with the IAU 2000A nutation model
- **Estimation errors**: mostly within 0.1 μ as/s = 8.64 mas/day; will be reduced if better a priori values

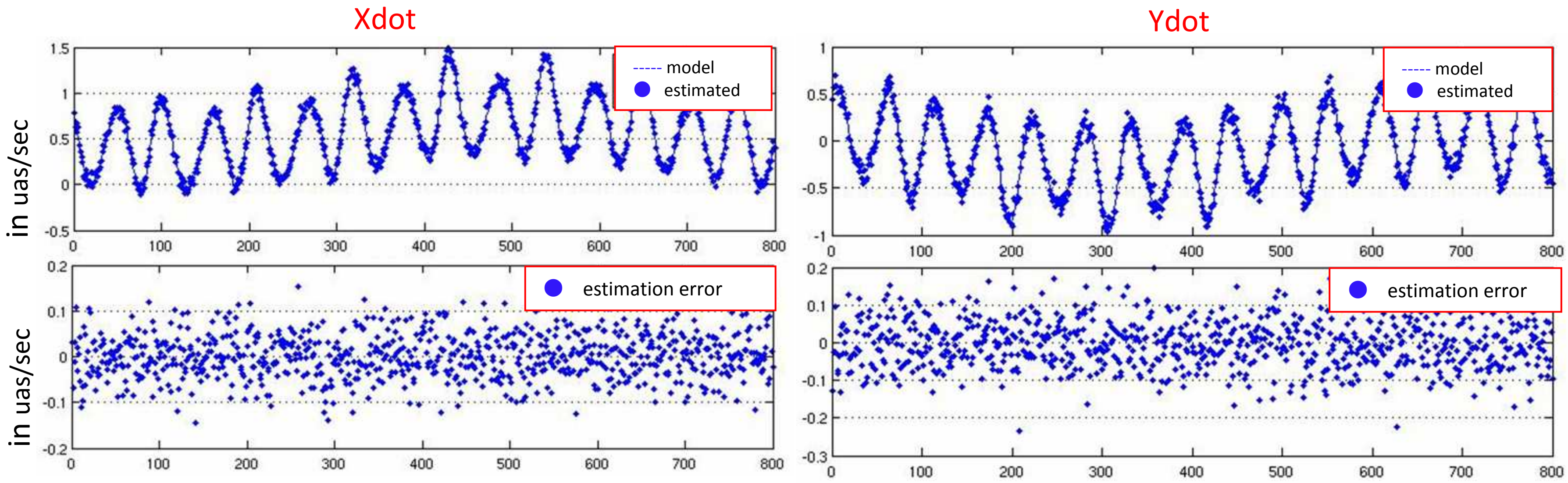

Fig. 3: *The X, Y rates estimated from GPS observations using the new analysis software and the IAU 2000 nutation*
1 point = 6 hours; interval 200 days

Computation of the time series of the parameters

Validation of the computations:

- Evaluation of the observations fitting residuals (RMS ~ 8 mm)
- Comparison of the zenith tropospheric delay with the IGS product
- Evaluation of the 3D orbit difference in the ITRF w.r.t. the IGS final orbit (typically < 5 cm)
- Comparison of the polar coordinates to those obtained by GINS (typically < 50 μas))

Data set:

- GPS data from 2009 to 2011 (3 years; 315 360 observation epochs)
- About 115 IGS “core” stations, and almost 32 satellites have been used

A priori values

- Polar motion and LOD : IERS C04
- Xdot, Ydot : IAU2006/2000A model

Results

- 3-year time series of corrections (every 6 hours) δXdot, δYdot: i.e. nutation rates
- 3-year time series of corrections to ERAdot (i.e. δLOD)
- 3-year time series of polar motion corrections (i.e. dx_p, dy_p)
- correlations bettween EOPs included between 0.02 and 0.23 (Xdot or Ydot and ERAdot)

Time series of nutation rate corrections

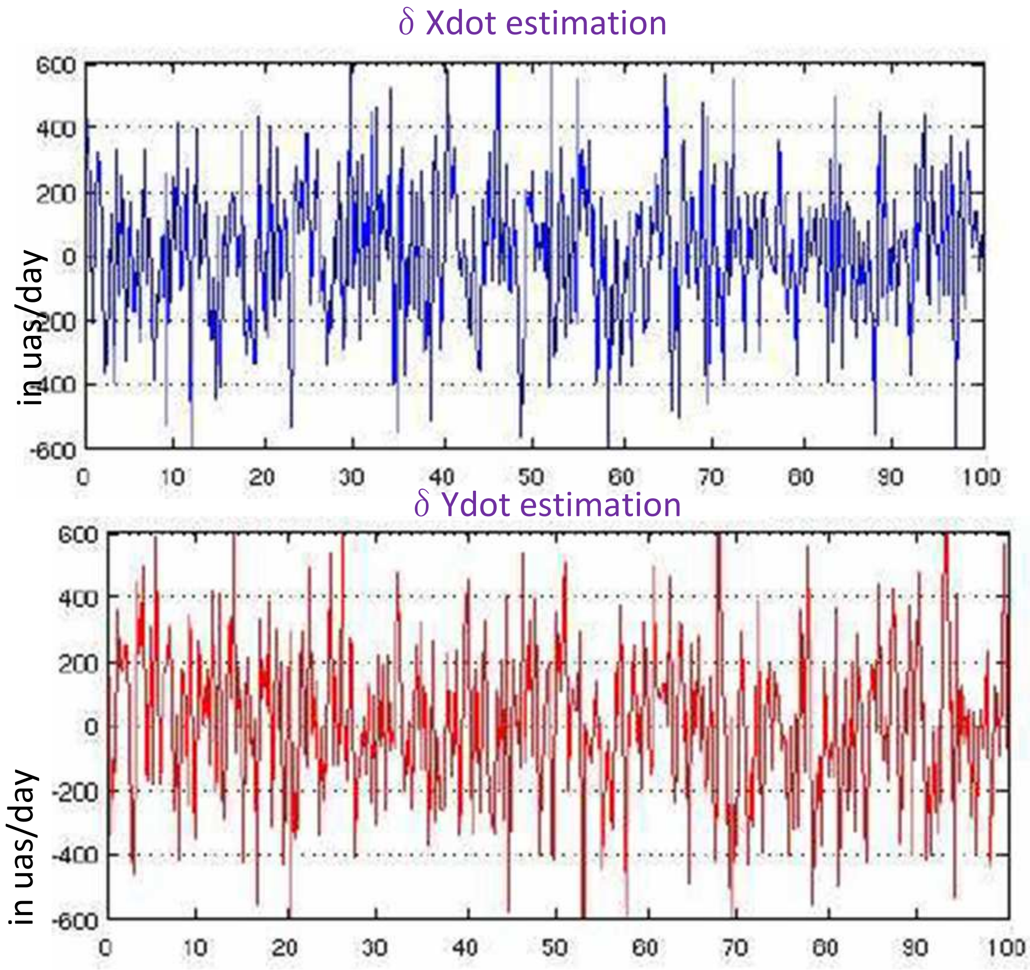

Fig. 4: *Example of nutation rates corrections (dXdot, dYdot) during the 100 days interval starting on 01/01/2009*

Spectral analysis of the time series for nutation rate corrections

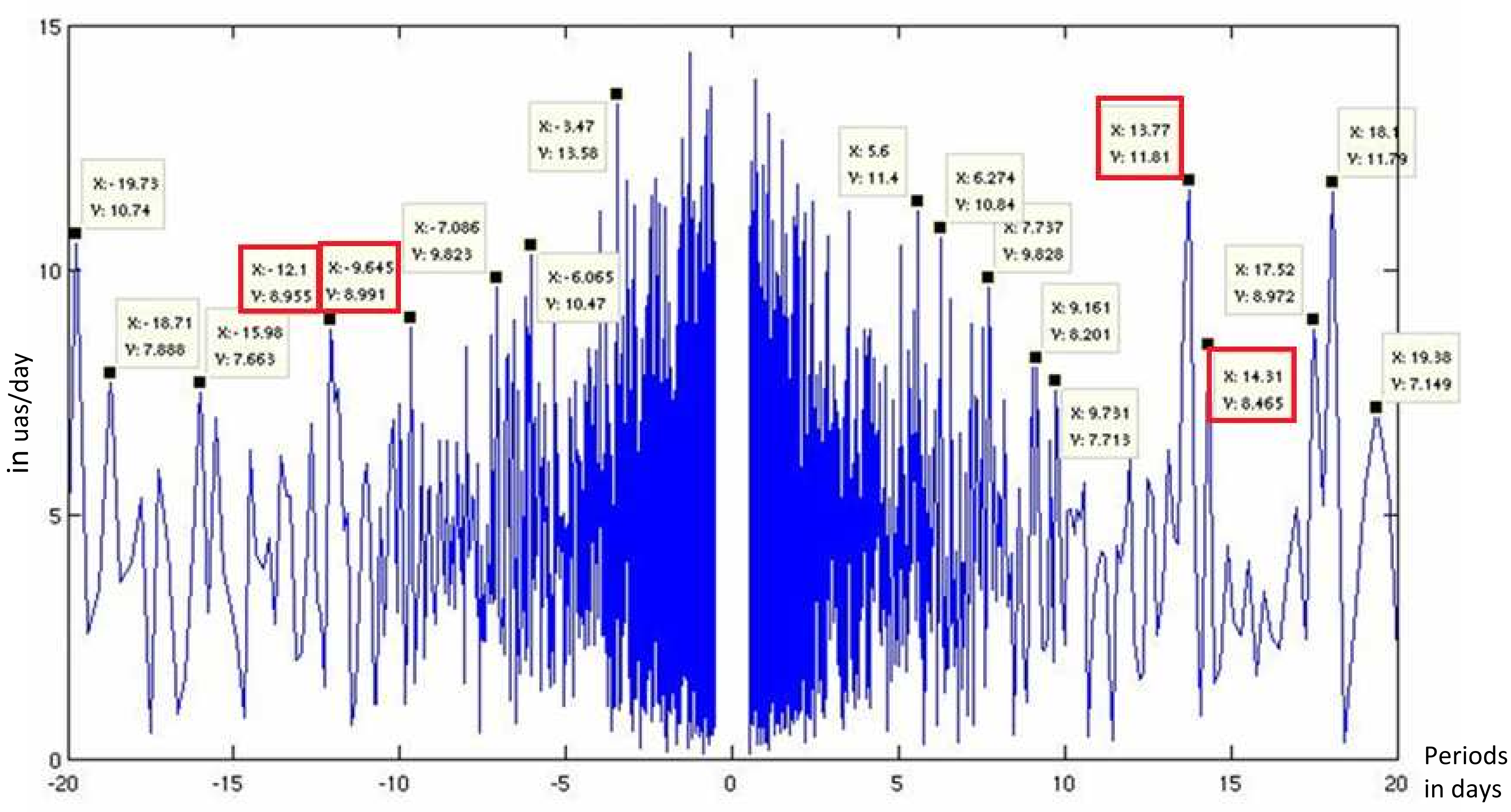

Fig. 5: *Spectrum of 3-year time series for dXdot + idYdot*

Nutation estimation from the Xdot and Ydot time series

- The 3-year (2009-2012) time series (δ Xdot, δ Ydot) are used as pseudo observations
- weighted LS harmonic adjustment of **28 circular nutation terms** based on the equations enclosed in blue

$$
\begin{aligned}
\delta X + j\delta Y &= \sum_i a_i e^{j(ARG_i)} \\
&= \sum_i (a_{real,i} + j a_{imag,i})(\cos(ARG_i) + j\sin(ARG_i)) \\
\delta X(t) &= \sum_i [a_{real,i}\cos(ARG_i) - a_{imag,i}\sin(ARG_i)] \\
\delta Y(t) &= \sum_i [a_{real,i}\sin(ARG_i) + a_{imag,i}\cos(ARG_i)] \\
\delta \dot{X}(t) &= \sum_i [-a_{real,i}\sin(ARG_i) - a_{imag,i}\cos(ARG_i)]A\dot{R}G_i \\
\delta \dot{Y}(t) &= \sum_i [a_{real,i}\cos(ARG_i) - a_{imag,i}\sin(ARG_i)]A\dot{R}G_i
\end{aligned}
$$

Amplitudes to estimate

GPS-estimated corrections to the IAU 2000A nutation

Table 3: *List of the corrections to the largest terms with periods shorter than 16 days*

period (days)	l	l'	F	D	Ω	$a_{real,i}$ μas	sigma μas	$a_{imag,i}$ μas	sigma μas
-15.39	0	1	0	-2	0	6	9	-15	9
15.39	0	-1	0	2	0	-5	9	-7	9
-14.77	0	0	0	-2	0	6	8	0	8
14.77	0	0	0	2	0	5	8	0	8
-14.19	0	1	-2	0	-2	6	8	-7	8
14.19	0	-1	2	0	2	4	8	-2	8
-13.66	0	0	-2	0	-2	1	8	3	8
13.66	0	0	2	0	2	-14	8	-8	8
-12.66	0	0	-4	2	-2	2	7	19	7
12.66	0	0	4	-2	2	-1	7	-3	7
-10.08	1	0	0	-4	0	-2	6	8	6
10.08	-1	0	0	4	0	-6	6	-5	6
-9.56	1	0	-2	-2	-2	-10	5	-2	5
9.56	-1	0	2	2	2	-3	5	-1	5
-9.13	-1	0	-2	0	-2	-4	5	-6	5
9.13	1	0	2	0	2	-6	5	3	5
-8.75	-3	0	-2	2	-2	3	5	-9	5
8.75	3	0	2	-2	2	-1	5	0	5
-7.10	0	0	-2	-2	-2	7	4	-2	4
7.10	0	0	2	2	2	-1	4	1	4
-6.86	-2	0	-2	0	-2	-3	4	1	4
6.86	2	0	2	0	2	2	4	-1	4
-5.64	-1	0	-2	-2	-2	4	3	-5	3
5.64	1	0	2	2	2	3	3	2	3
-4.68	-2	0	-2	-2	-2	3	3	-2	3
4.68	2	0	2	2	2	-1	3	3	3
-4.08	-1	0	-2	-4	-2	-1	2	0	2
4.08	1	0	2	4	2	-2	2	2	2

Comparison between the GPS and VLBI nutation estimations

period	l	l'	F	D	Ω	based on GPS, in μas		based on VLBI, in μas	
(days)						$a_{real,i}$	$a_{imag,i}$	$a_{real,i}$	$a_{imag,i}$
-14.77	0	0	0	-2	0	6 ± 8	0 ± 8	-1.2 ± 1.6	3.2 ± 1.6
14.77	0	0	0	2	0	5 ± 8	0 ± 8	2.8 ± 1.6	-1.2 ± 1.6
-13.66	0	0	-2	0	-2	1 ± 8	3 ± 8	-14.9 ± 1.6	-12.3 ± 1.6
13.66	0	0	2	0	2	-14 ± 8	-8 ± 8	-7.6 ± 1.6	11.5 ± 1.6
-9.56	1	0	-2	-2	-2	-10 ± 5	-2 ± 5	-2.5 ± 1.6	-1.8 ± 1.6
9.56	-1	0	2	2	2	-3 ± 5	-1 ± 5	-0.9 ± 1.6	-1.2 ± 1.6
-9.13	-1	0	-2	0	-2	-4 ± 5	-6 ± 5	-5.3 ± 1.8	2.8 ± 1.8
9.13	1	0	2	0	2	-6 ± 5	3 ± 5	-4.9 ± 1.8	2.5 ± 1.8
-7.10	0	0	-2	-2	-2	7 ± 4	-2 ± 4	0.5 ± 1.6	2.8 ± 1.6
7.10	0	0	2	2	2	-1 ± 4	1 ± 4	7.0 ± 1.6	-0.2 ± 1.6
-6.86	-2	0	-2	0	-2	-3 ± 4	1 ± 4	1.6 ± 1.9	-4.2 ± 1.9
6.86	2	0	2	0	2	2 ± 4	-1 ± 4	5.1 ± 1.9	-1.2 ± 1.9

Corrections to IAU 2000

Table 4: Nutation estimation results based on GPS and VLBI

GPS estimations from time series (2009–2012) of the corrections to the IAU 2006/2000 X and Y rates.

VLBI estimations from time series (1990-2012) of corrections to IAU 2006/2000 X, Y

Conclusions and perspectives

- GPS observations covering a 3-yr interval have been analyzed for estimating corrections to the short periodic terms of nutation.
- The computations have been carried out by means of a new GPS analysis software, developed by K. Yao in the framework of his PhD (2013) in Matlab environment.
- In this analysis, the satellite orbit has been integrated in an appropriate inertial system which reduces the influence of the error in the *a priori* values of the precession-nutation and ERA.
- This analysis has provided a 3-year time series of nutation rates corrections (d*X*dot, d*Y*dot) to IAU2006/2000A *a priori* values along with corrections to ERAdot and pole coordinates.
- The time series have been used to estimate corrections to the 28 largest nutation terms with period less than 20 days with an accuracy of about 10 μ as.
- This study proves that the GNSS technique **alone** (i.e. without any use of VLBI estimates) has the potential to determine the short-period terms of nutation.
- The corrections obtained by GPS for the short periodic terms of nutation has been compared with those obtained by VLBI. The two set of corrections are of the same order of magnitude, but they are not perfectly consistent – this discrerpancy requires further research.