Estimation of the Chandler wobble parameters by the use of the Kalman deconvolution filter

Aleksander Brzeziński[1,2] and Marcin Rajner[1]

[1)]Warsaw University of Technology, Faculty of Geodesy and Cartography

[2)]Space Research Centre, Polish Academy of Sciences, Warsaw, Poland

alek@cbk.waw.pl

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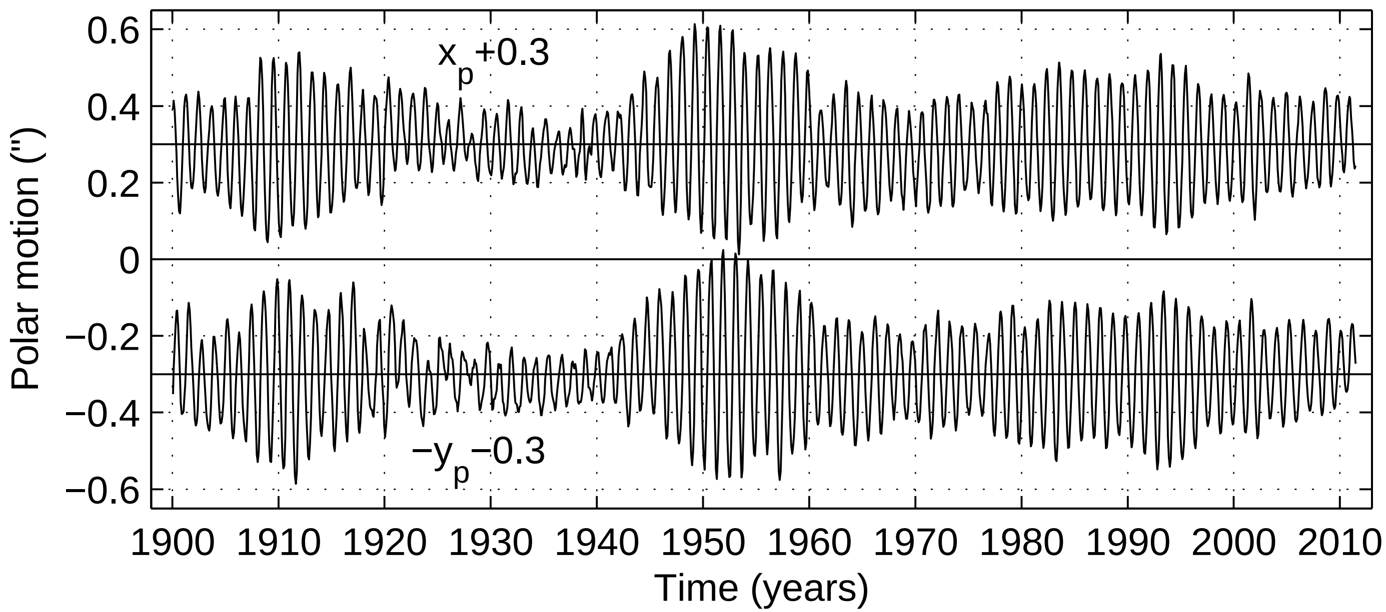
Chandler wobble: general description

- Free motion of the pole which is the largest component of polar motion
- Predicted by Euler (1765) with a period of 305 days
- Detected in astrometric observations by Chandler (1891) with a period of about 430 days
- Monitored on regular basis since the end of XIX century (see Figure 1)
- Its amplitude is changing but there is no clear decaying tendency

$\Longrightarrow$ main excitation mechanism: irregular mass redistribution on the Earth's surface



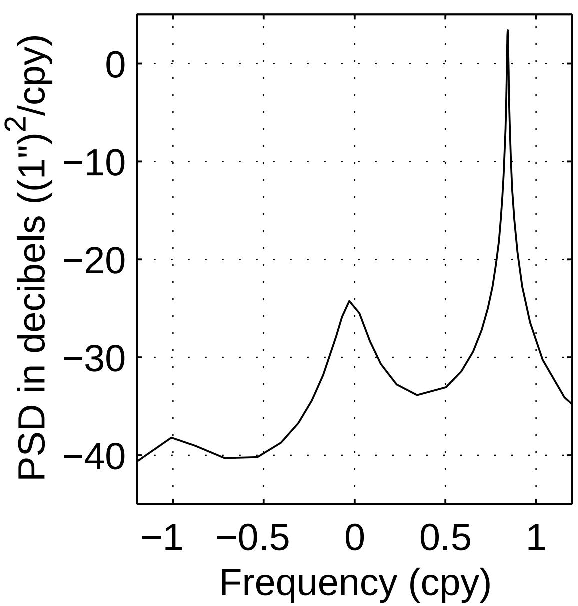


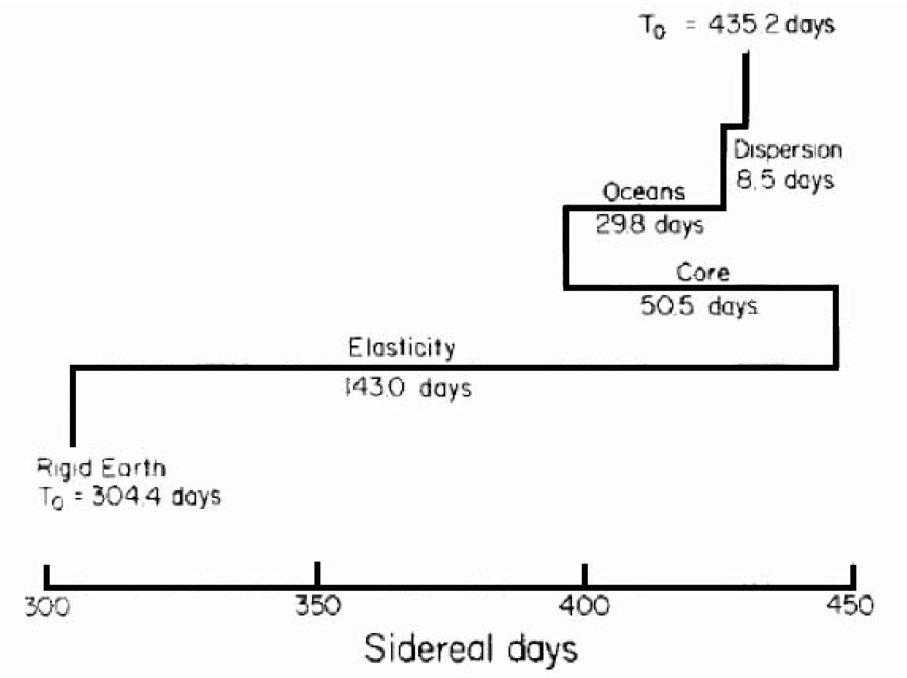
Figure 1: Polar motion series IERS C01 after removal of the annual sinusoid and the 1st order polynomial (left) and the power spectral density (PSD) function of $p = x_p - iy_p$ (right)

Chandler wobble parameters: geophysical interpretation

Parameters of the Chandler resonance, the frequency F_c (or, equivalently, the period $T_c = 1/F_c$) and the quality factor Q_c are important for studying global dynamics of the Earth because

- they define the equation describing geophysical excitation of polar motion
- are closely related to various geophysical parameters; for details see the paper

Smith and Dahlen, 1981, The period and Q of the Chandler wobble, *Geo. J. R. Astr. Soc.*, 64(223–281)



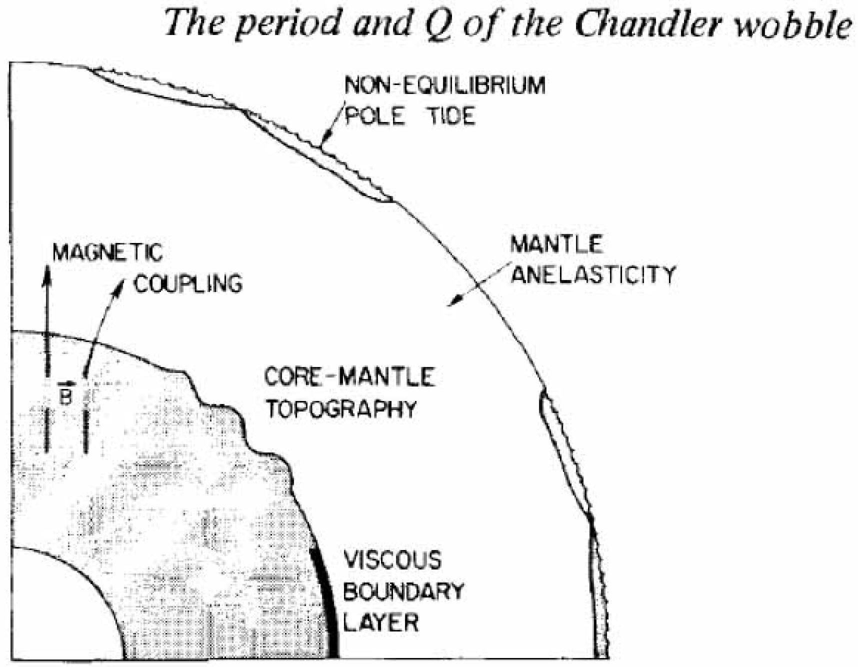


Figure 2: Geophysical properties of the Earth contributing to the Chandler wobble period T and quality factor Q (from (Smith and Dahlen, 1981))

Chandler wobble: the stochastic model of Jeffreys

Harold Jeffreys (1891-1968), the English astronomer and geophysicist, proposed in 1940 a simple stochastic model for the observed Chandler wobble. From the point of view of mathematics, this is the model of the randomly excited free oscillation with damping. This model can be applied for different purposes such as

- estimation of the free wobble parameters and their statistics
- reliable estimation of the observed polar motion power spectrum and cross-power spectrum
- filtering and prediction of the polar motion data

The idea of Jeffreys (1940) was further developed by Wilson (1979), Wilson and Vicente (1980; 1990) and many other researchers.

The algorithms which are described below and applied for estimation of the CW parameters, are based on the stochastic model of Jeffreys (1940).

THE VARIATION OF LATITUDE

Harold Jeffreys, F.R.S.

heavy pendulum is bombarded by a number of boys with pea-shooters, whose rate of firing and aim are both imperfect. Consequently the pendulum receives a series of impulses at irregular intervals and a vibration is produced. But this is not strictly periodic, and if we adopt any given period there will be considerable variations of amplitude and phase from time to time. Harmonic analysis over a long interval may fail to reveal a definite periodicity at all, because if irregular disturbances started the pendulum in one phase, new ones may very well reverse it later. Nevertheless the pendulum has a natural period, with some damping, and it is legitimate to ask how this can be found from observations of its displacement at regular intervals; par-

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If the observations are at intervals of time τ, and there is a disturbance ϵ_n, η_n in any interval, we shall have the recurrence relations

$$l_n = \alpha l_{n-1} - \beta m_{n-1} + \epsilon_n, \quad (5)$$

$$m_n = \beta l_{n-1} + \alpha m_{n-1} + \eta_n, \quad (6)$$

where

$$\alpha = e^{-k\tau} \cos \gamma\tau; \qquad \beta = e^{-k\tau} \sin \gamma\tau. \quad (7)$$

Now suppose that the disturbances are independent, ϵ_n and η_n each having standard error σ. Then the joint probability of pairs of values of l and m will follow the rule:

Chandler wobble: theoretical description

Equation of polar motion excitation

$$\dot{p} - i\sigma_c p = -i\sigma_c \chi \tag{1}$$

where

- i .... imaginary unit
- $p = x_p - iy_p$ .... observed polar motion
- $\chi = \chi_1 + i\chi_2$ .... geophysical excitation function (AAM, OAM, HAM)
- $\sigma_c = 2\pi F_c \left(1 + i/2Q_c\right)$ ... complex angular frequency of CW
- F_c , $T_c = 1/F_c$, Q_c .... CW frequency, period and quality factor

The solution of eq.(1) is

$$p(t) = p(t_o)\exp(i\sigma_c(t - t_o)) - i\sigma_c \int_{t_o}^{t} \exp\left[i\sigma_c(t - \tau)\right]\chi(\tau)d\tau \tag{2}$$

Assumption: χ is a stochastic process with power spectral density (PSD) near F_c equal S_c

Remark: if χ is a white noise, parameters F_c, Q_c and S_c are related by the following equation

$$2\pi F_c Q_c S_c = P_c \tag{3}$$

where P_c is a power, or a variance, of the Chandler wobble (Munk and MacDonald, 1960)

Chandler wobble: parameter estimation based on the Jeffreys model

Table 1: Selected estimates of the CW parameters: the frequency F_c, the quality factor Q_c and the excitation power S_c at the resonant frequency. Units are cycles per year (cpy) for F_c and mas^2/cpy for S_c.

Source	Method	F_c		Q_c		S_c	
Jeffreys (1940)	AR	0.8177	± 0.0127	46	(37–60)		
Jeffreys (1968)	AR	0.8432	± 0.0043	61	(37–193)		
Ooe (1978)	MEM–AR	0.8400	± 0.0039	96	(50–300)	51	(14–98)
Wilson & Vicente (1980)	MLM–ARIMA	0.8430	± 0.0070†	175	(48–1000)†	33	(22–52)†
Wilson & Vicente (1990)	MLM–ARIMA	0.8435	± 0.0022	179	(74–789)		
Kuehne & Wilson (1996)	PM+EXC	0.831	± 0.004	72	(30–500)		
Furuya & Chao (1996)	PM+EXC	0.8422	± 0.0035	49	(35–100)		

† These are not 1-σ but 90% confidence intervals

Legend: method of data analysis

AR autoregressive modeling

MEM–AR maximum entropy method o fitting AR spectrum

MLM–ARIMA autoregressive integrated moving-average model fitted by maximum likelihood method

PM+EXC estimate based on simultaneous use of polar motion and geophysical excitation data

Model of the excitation function:

$$\chi = \chi_o + \chi_u \tag{4}$$

where χ_o is the observed excitation function and χ_u is the residual unknown excitation. These functions are generated by the stochastic processes described by the following equations

$$\dot{\chi}_o = k_o\chi_o + w_o; \qquad \dot{\chi}_u = k_u\chi_u + w_u, \tag{5}$$

in which the coefficients k_o, k_u fulfill the stationarity condition $\Re k_o, \Re k_u \leq 0$ and $\{w_o\}, \{w_u\}$ are zero-mean, white Gaussian noises with power spectral densities q_o, q_u, respectively.

Kalman filter for polar motion excitation:
having defined the state vector $[p, \chi_o, \chi_u]^T$ we can implement Kalman filter for the linear system defined by equations (1), (4), (5), as described in details by Brzeziński (1992).

Application to real data:
by using simultaneous observations of p and χ_o as an input for the Kalman recursion we can smooth p and χ_o and estimate the unknown excitation χ_u.

CW parameters estimation:
a straightforward approach is to find F_c, Q_c which minimize the mean squared value of the estimated unknown excitation χ_u. In the following analysis we assumed $k_o = k_u = 0$, that is the random walk model for both χ_o and χ_u.

Chandler wobble: simultaneous analysis of the polar motion and excitation data

Possible extensions of the algorithm

- no observed excitation (pure deconvolution procedure)
- more then one observed excitation; two options are possible:
  - observed excitation series can be treated separately
  - can be added prior to the analysis (we adopted this option here)

Estimation of the Chandler resonance parameters F (or T) and Q, and the confidence intervals

- compute mean-squared value V of the residual excitation and find its minimum with respect to F and Q
- this procedure can be identified under certain assumptions with the maximum likelihood algorithm and the confidence intervals defined by the cross-sections $V = V_{min}(1 + \varepsilon)$

Chandler wobble: data analysis

Polar motion data

- `POLE2010` – Kalman filter combination series (Ratcliff and Gross, 2011), 1900.0 – 2011.5
- `C01` – IERS combination of the optical astrometry observations (Vondrák *et al.*, 1995) with the BIH and IERS solutions, 1900.0 – 2012.0.
- `C04` – IERS combined solution (Bizouard and Gambis, 2009), 1962.0 – 2009.6

Atmospheric excitation data

- `NCEP`: AAM series estimated from the output fields of the U.S. NCEP-NCAR reanalysis project (Kalnay et al., 1996), 1948 – 2012
- `ERA-interim`: ERA Interim reanalysis from ECMWF (Uppala et al., 2008), 1989 – 2009

Oceanic excitation data

- `ECCO`: OAM series based on the MIT global ocean circulation model (Gross et al., 2003)
  - `ECCO1` – ECCO_50yr solution, 1949 – 2002
  - `ECCO2` – c20010701 solution, 1980 – 2001
- `OMCT`: OAM series computed from the Ocean Model for Circulation and Tides (Dobslaw and Thomas, 2007), 1989 – 2009

Chandler wobble: data analysis

Hydrology excitation data

- `LSDM`: HAM series estimated from the output of the global hydrological model LSDM (Dill, 2008), 1989 – 2009

Data analysis

- For each input time series (PM, AAM, OAM, HAM) estimate parameters of the model comprising the sum of complex sinusoids with periods ± 1, $\pm 1/2$, $\pm 1/3$ years (the sign $+/-$ is for prograde/retrograde motion) and the 4-th order polynomial accounting for low-frequency variation
- Compute the residual series by removing the polynomial-harmonic model
- Estimate CW parameters for the following combinations of the data sets
  - 1948.0 – 2009.5: C01/NCEP, POLE2010/NCEP
  - 1949.0 – 2003.0: C01/NCEP/ECCO1, POLE2010/NCEP/ECCO1
  - 1980.0 – 2002.3: C01/NCEP/ECCO2, POLE2010/NCEP/ECCO2
  - 1989.0 – 2009.0: C04/ERA-interim/OMCT/LSDM

Chandler wobble parameters estimation: results

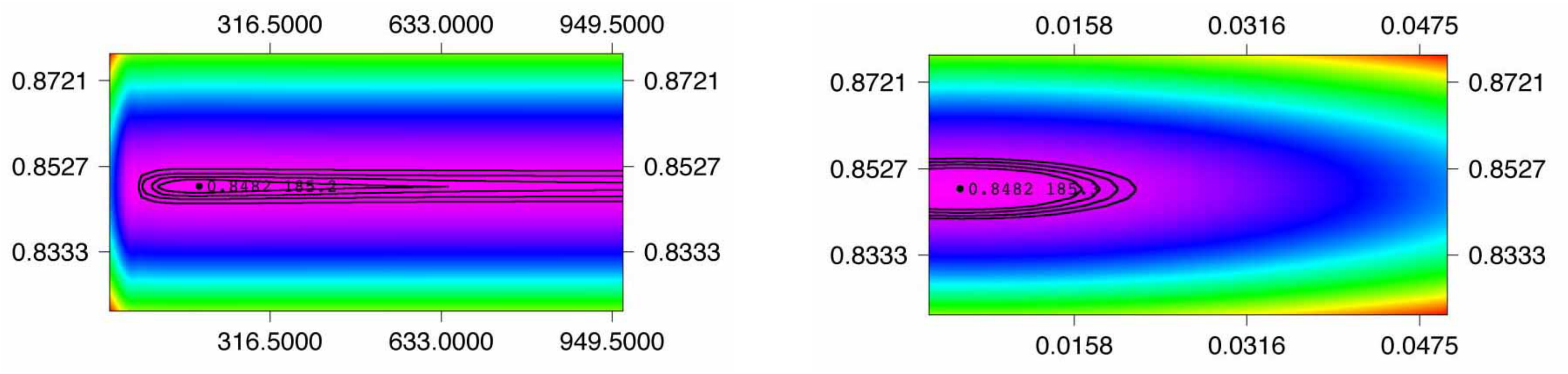

Figure 3: Chandler wobble parameters estimation from simultaneous analysis of PM and AAM data. The mean-squared value of residual excitation is shown as function of F and Q (left), and of F and 1/Q (right). Period of analysis: 1948-2010, input data PM – IERS C01, and AAM – NCEP reanalysis.

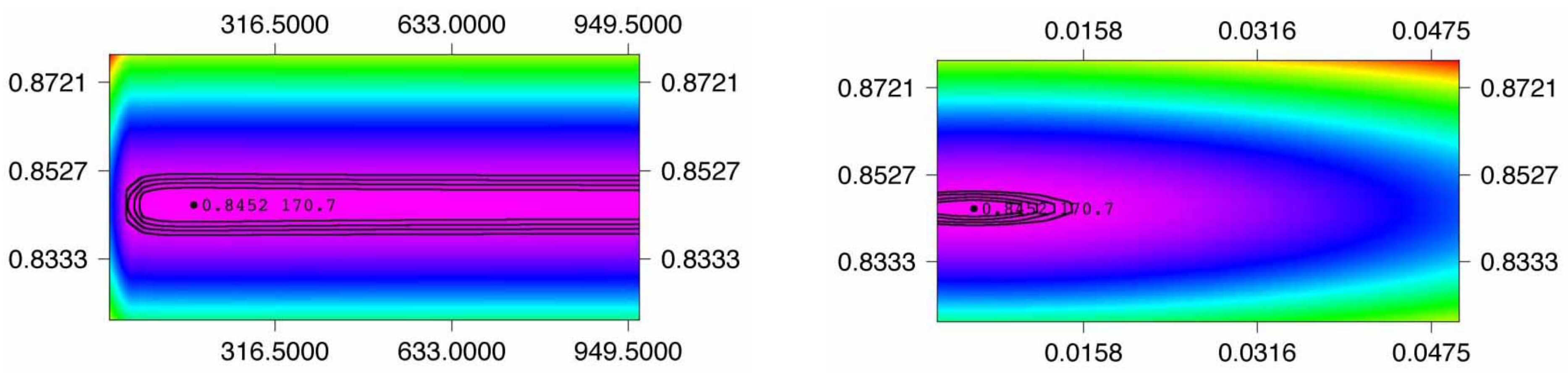

Figure 4: Same as in Fig.3 but for the PM series Pole 2010

Chandler wobble parameters estimation: results

Table 1: Chandler wobble parameters estimated from the simultaneous analysis of polar motion and AAM data: period T_c in days, quality factor Q_c and mean-squared value V_c of residuals (in mas^2).

(a) Period of analysis 1900.0–2011.5.

Data sets	F_c	T_c	Q_c	V_c
C01	0.8440	432.8	∞	1245
POLE2010	0.8435	433.0	2400	913

(b) Period of analysis 1948.0–2009.5.

Data sets	F_c	T_c	Q_c	V_c
C01	0.8449	432.3	235	1022
C01/NCEP	0.8459	431.8	111	1063
POLE2010	0.8434	433.1	128	678
POLE2010/NCEP	0.8459	431.8	90	708

(c) Period of analysis 1949.0–2002.9.

Data sets	F_c	T_c	Q_c	V_c
C01/NCEP	0.8498	429.8	230	1188
C01/NCEP/ECCO1	0.8492	430.1	380	1162
POLE2010/NCEP	0.8443	432.6	77	691
POLE2010/NCEP/ECCO1	0.8436	433.0	87	669

Chandler wobble parameters estimation: results

Table 1 (d) Period of analysis 1980.0–2002.1.

Data sets	F_c	T_c	Q_c	V_c
C01	0.8418	433.9	∞	235
C01/NCEP	0.8439	432.8	125	269
C01/NCEP/ECCO2	0.8445	432.5	83	253
POLE2010	0.8417	433.9	∞	185
POLE2010/NCEP	0.8439	432.8	127	217
POLE2010/NCEP/ECCO2	0.8445	432.5	85	202

Table 1 (e): Period of analysis 1989.0–2009.0.

Data sets	F_c	T_c	Q_c	V_c
C04	0.8423	433.6	269	186
C04/ERA-interim	0.8474	431.0	82	175
C04/ERA-interim/OMCT	0.8472	431.1	64	144
C04/ERA-interim/OMCT/LSDM	0.8491	430.2	53	163

Chandler wobble parameters estimation: summary and conclusions

- the estimates of the frequency F_c and the quality factor Q_c are statistically independent
- analysis of the polar motion data alone yields reliable estimates of the CW frequency but is less useful for constraining the quality factor
- the estimated frequency is in most cases consistent with the reference value $F_c = 0.8435 \pm 0.0022$ cpy
- the estimated quality factor is lower than the reference value $Q_c = 179\ (74 - 789)$

$\Rightarrow$ still to be done: clarify the problem of assigning the confidence intervals to to the estimated parameters