Amplitude-frequency analysis of the Earth orientation parameters and the variation of the second zonal harmonic of the geopotential

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Abstract. Modeling temporal variations of the geopotential caused by the oscillatory-rotational processes of the Earth's motion is of a significant interest for some satellite navigation applications and geophysics problems. Observed variations of the Earth orientation parameters (EOP), the variations of the Earth's gravitational field and oscillations in the large-scale geophysical events appear to be in a considerable correlation.

Based on the celestial mechanics' methods namely the spatial version of the problem of the Earth-Moon system in the gravitational field of the Sun a mathematical model of the rotational-oscillatory motion of the elastic Earth is developed. A comparison between the real and theoretically obtained Earth's pole motion trajectory as well as between real and derived variations of the length of the day demonstrate the adequacy of the derived model to the International Earth Rotation and Reference System Service (IERS) observations data.

Based on the amplitude-frequency analysis of the oscillation processes in the motion of the Earth's pole and on the observations' data of the SLR a modeling of the second zonal harmonic of the geopotential has been carried out. Its functional dependence on the amplitude and phase of the Earth's pole oscillatory process is shown by the means of periodic functions of time.

Differential equations of the oscillations of the Earth's pole on diurnal time scales can be derived from the dynamic Euler–Liouville equations for a space analogue of the problem called a "deformable Earth–Moon" in the gravitational field of the Sun:

$$\frac{d}{dt}\left[(1+\chi_A+\kappa_p r_0)p\right]+N_p\frac{B}{A}(1+\chi_B)q=\kappa_q r^2+M_p^{SL}(\Omega,I,\pi),$$

$$\frac{d}{dt}\left[(1+\chi_B+\kappa_q r_0)q\right]-N_q\frac{A}{B}(1+\chi_A)p=-\kappa_p r^2+M_q^{SL}(\Omega,I,\pi),$$

$$N=\sqrt{N_pN_q(1+\chi_A)(1+\chi_B)}\cong(0.84\div0.85)\omega_0,$$

$$A=A^*+\delta A,\quad B=B^*+\delta B,\quad C=C^*+\delta C,\quad \chi_A=\frac{\delta A}{A^*},\quad \chi_B=\frac{\delta B}{B^*},\quad \kappa_p=\frac{\delta J_{pr}}{A^*},\quad \kappa_q=\frac{\delta J_{qr}}{B^*}$$

Here, $\omega=(p,q,r)T$ is the angular velocity vector in a coordinate system related to Earth; N is the Chandler frequency; ω_0 is the mean motion of the Earth in its orbit around the Sun; A^*, B^*, and C^* are the effective central principal moments of inertia taking into account the deformations of the "frozen" Earth's shape; δJ_{ij} $(i, j = p, q, r)$ are the small variations of the inertia tensor containing different harmonic components (zonal, tesseral, sectoral) due to the perturbing effect of gravitational tides caused by the Sun and the Moon, and to other factors;

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χ_A, χ_B, κ_p, κ_q correspond to tidal humps and cusps, respectively; $\boldsymbol{M}(\Omega, I, \pi)$ are specific lunar and solar gravitational tidal torques depending on the Euler variables (the nutation θ, precession ψ, and proper rotation φ) and the mean motions of the Earth and the Moon; Ω is the longitude of the ascending node of the lunar orbit; π is the longitude of perigee of the lunar orbit; and I is the inclination between the Moon's orbital plane and the ecliptic.

Let adduce the results of simulation of variations δc_{20} based on amplitude-frequency analysis of the Earth Pole oscillations and SLR data. The equations for amplitude and phase variables of the Earth Pole modulation motion have following form assumed $x_p=c_x+a\cos\psi$, $y_p=c_y+a\sin\psi$ (the values c_x, c_y are determined by choice of ITRF system): $a=\mu_t\cos(\psi-\nu_t)$, $\psi=N-a^{-1}\mu_t\sin(\psi-\nu_t)$, where $\mu_t=\mu(t)$, $\nu_t=\nu(t)$, are the functions of time which are expanded into a sum of periodical components; the values $\mu_t\cos\nu_t$, $\mu_t\sin\nu_t$ are perturbations which lead to the observed oscillations of the Earth Pole and which have the dimensions of specific moment of force. The solution of system is represented as follows:

$$a(\nu_i,\xi_i,\alpha_i)=a_0+\sum_i\xi_i\cos(2\pi\nu_i\tau+\alpha_i),$$

$$\psi(\vartheta_i,\eta_i,\beta_i)=\psi_0+2\pi N\tau+\sum_j\eta_j\sin(2\pi\vartheta_j\tau+\beta_j),$$

$$a_0,\psi_0=const,$$

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The expression for δc_{20} based on the analysis of non-averaging along φ equations and observation and measurement data is assumed in the form:

$$\delta c_{20}=\lambda_t^C\cos\nu_t+\lambda_t^S\sin\nu_t+f(\psi(\nu_i,\xi_i,\alpha_i),\psi(\vartheta_i,\eta_i,\beta_i)),$$

$$f(a,\psi)=\varepsilon a\int\frac{\mu_t\sin(\psi-\nu_t)}{a}dt$$

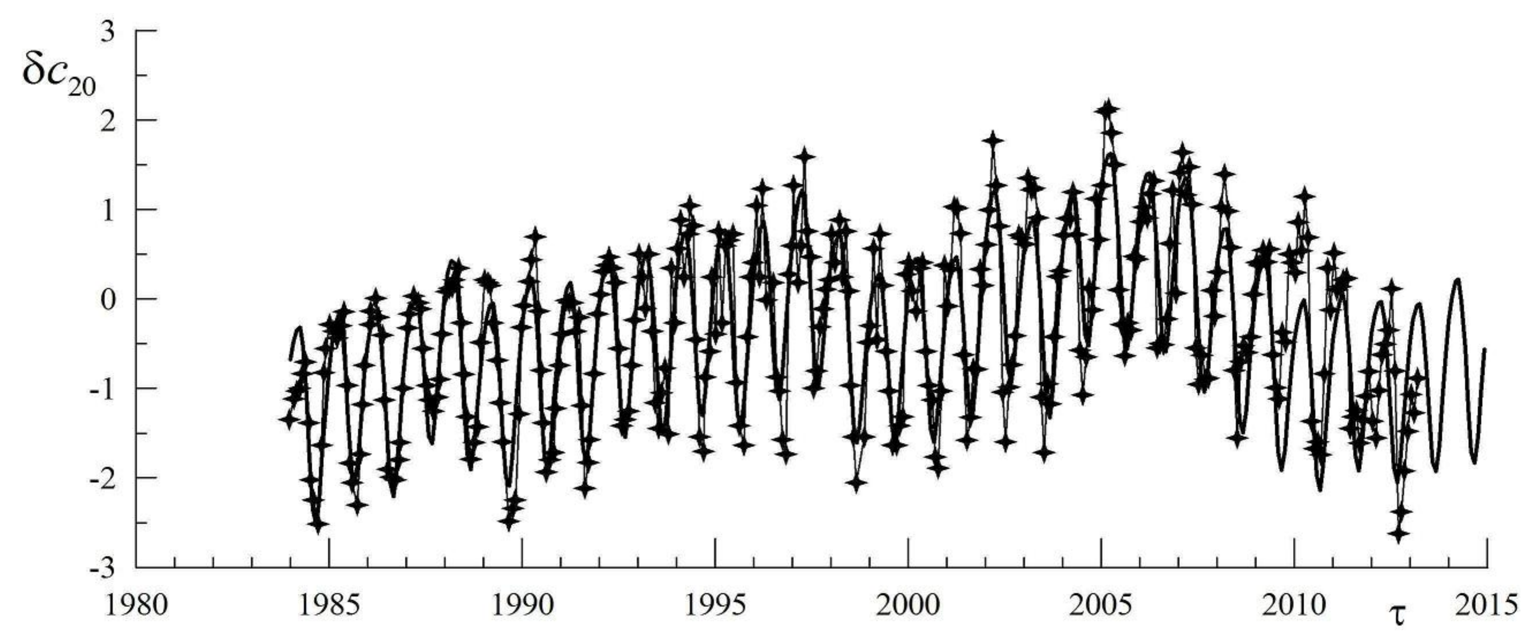

Figure *a*

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Here f is the function of amplitude and phase of the Pole oscillation; dimensioned coefficient ε is refined from SLR observations and has the order 10^{-3} (the amplitude a of the Earth Pole oscillations is expressed in angular milliseconds).
The interpolation of expression δc_{20} along SLR measurements data from 1984 till 2008 including is presented on fig. a and extrapolation for six years with the forecast for two years is given there. The contrast line on the figure is obtained theoretical curve, the connected by line stars are the measurements data. The component with oscillations corresponded to expansion of function $f(a,\psi)$ for harmonic components is released during interpolation. The coefficients of these oscillations (amplitudes and phases) are considered as unknown and must be determined based on the least square method on the interval of interpolation of SLR observations and measurements data.
The observed curve is shown on fig. b with comparison of function $f(a,\psi)$ that built directly from observations of the Earth Pole oscillations. For these the amplitude and the phase of the Earth Pole oscillations are extracted from IERS observations and measurements data preliminarily.

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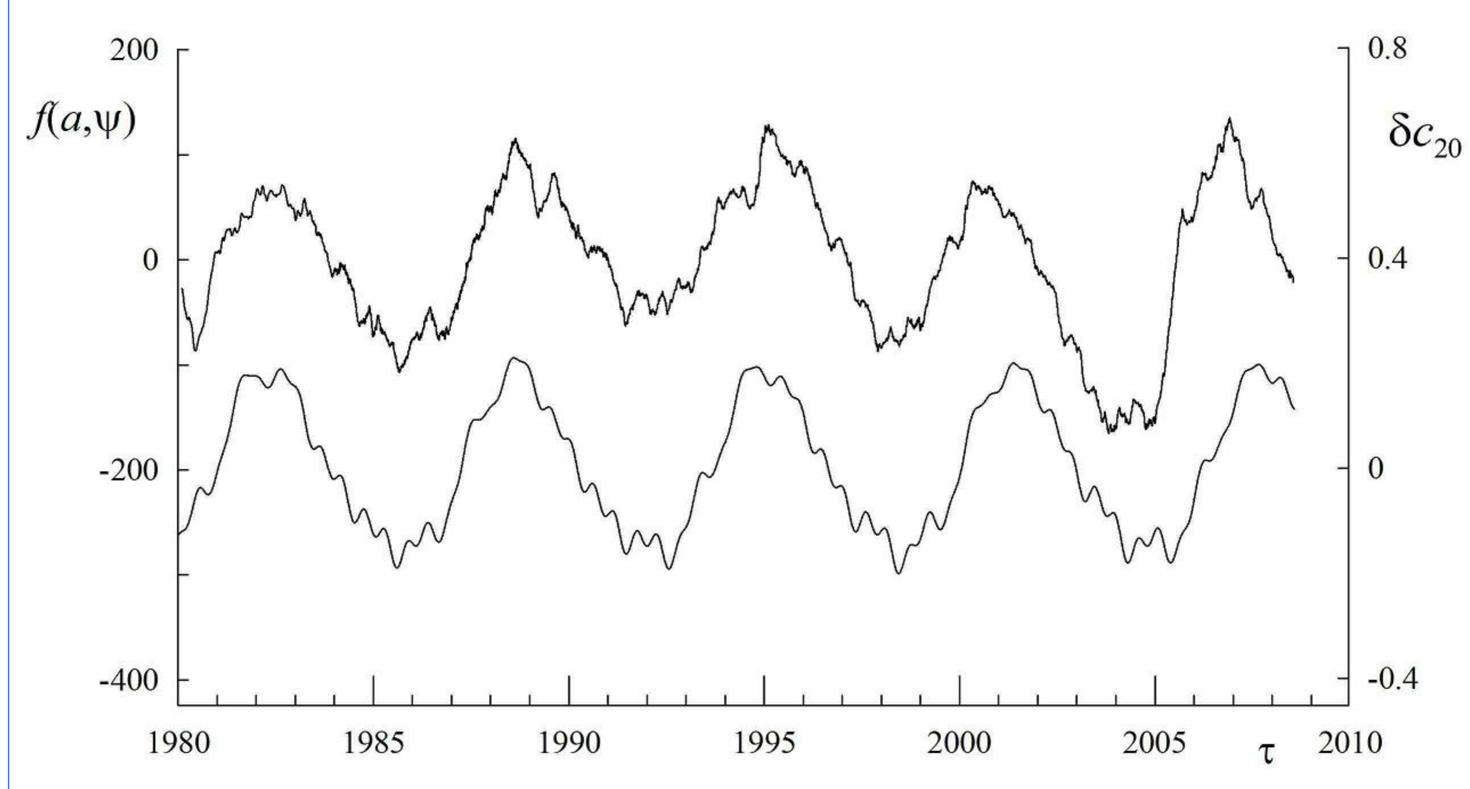

Figure *b*

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