Comparison of

geodetic & modeled excitation functions

by Allan variance

Christian BIZOUARD

Observatoire de Paris /SYRTE

Interpretation of polar motion and length of day variation

Sub-secular time scale : linear differential equations

« Geodetic » excitation ≡ geophysical / astronomical excitation

Angular momentum balance

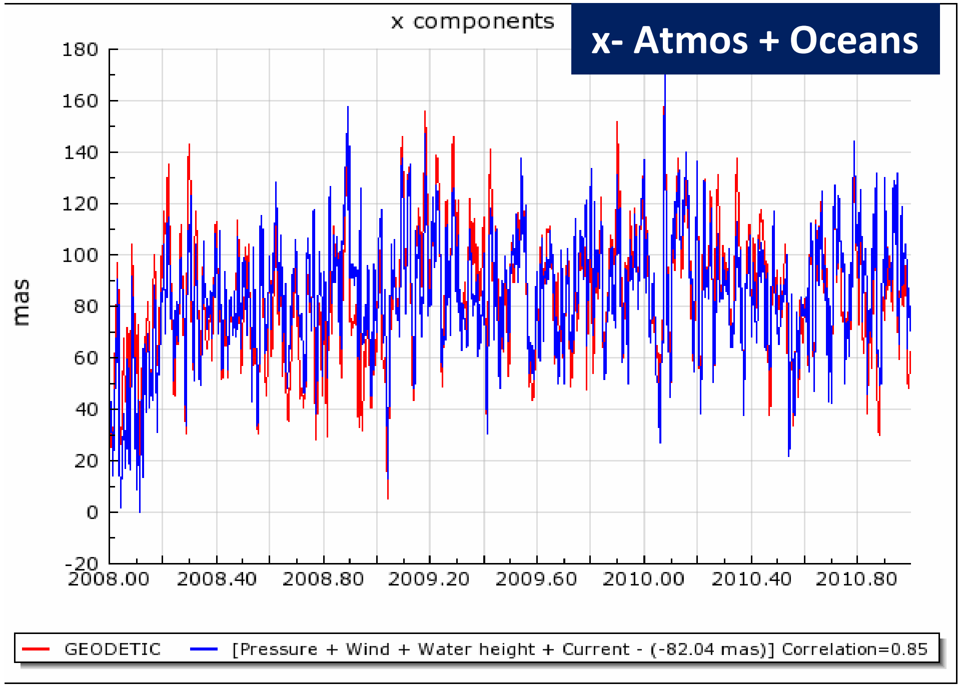
- System: {non rigid Earth including fluid layers}
- Takes into account rheological property (centrifugal deformation) and internal structure (fluid core)

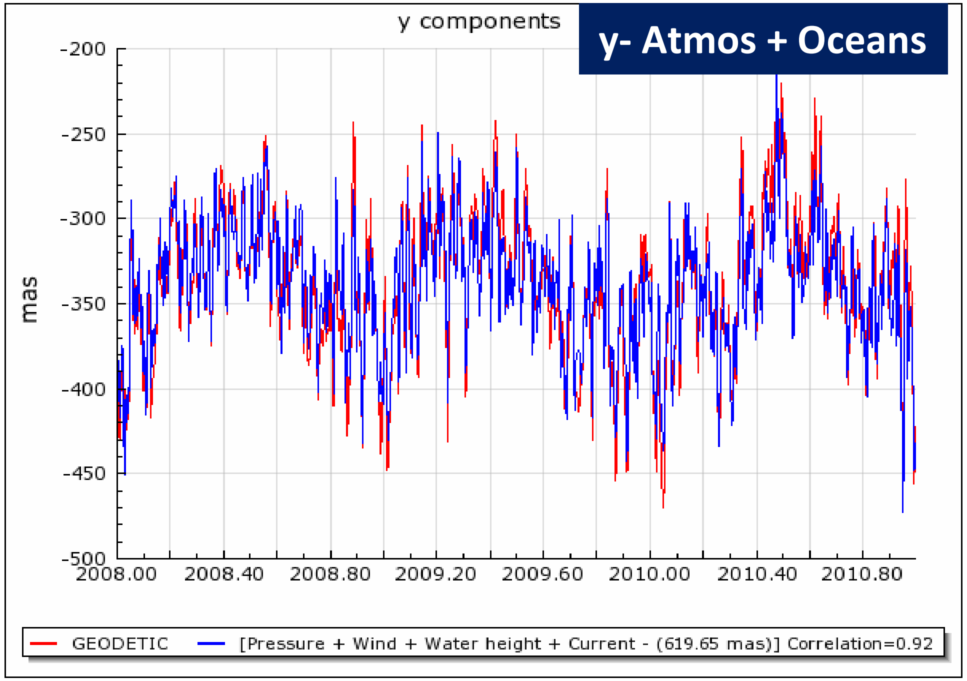
Space geodesy: Earth Rotation Parameters

Observations & models:

- ❑Meteorological, oceanic, hydrological, …
- ❑luni-solar tide

Common comparison of Geodetic and modelled excitations of the Earth rotation





- Statistical coefficients like correlation and explained variance do not make any difference between stationnary signal (e.g. stable seasonal signal) and unstable signal
- We would like to grasp the stochastic content of the signal and its stability in function of the time scale.

Statistical comparison by Allan variance

Allan variance analysis: it permits to quantify the stability of a time series at a given time scale. Also, as well as a spectral density, its slope (in log-log scale) allows to characterise the noise at play

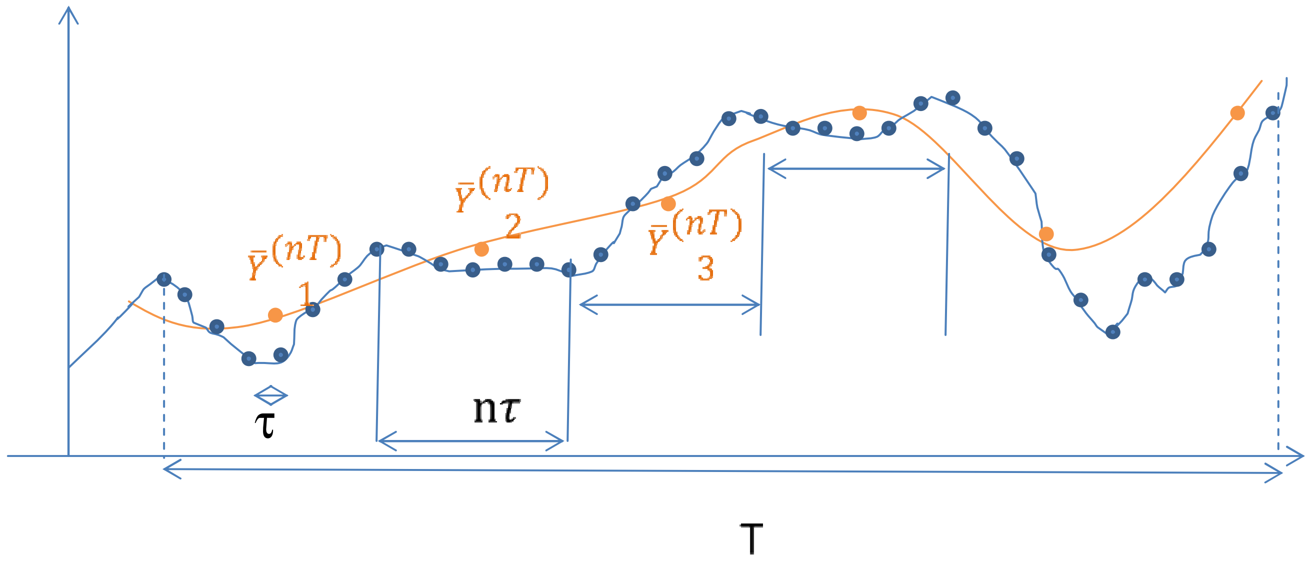
1. Widely used for time scale comparison

1. sometimes used for analysing EOP and station coordinate time series

1. Now, we shall look at geodetic excitation & modelled excitation (global fluid angular momentum)

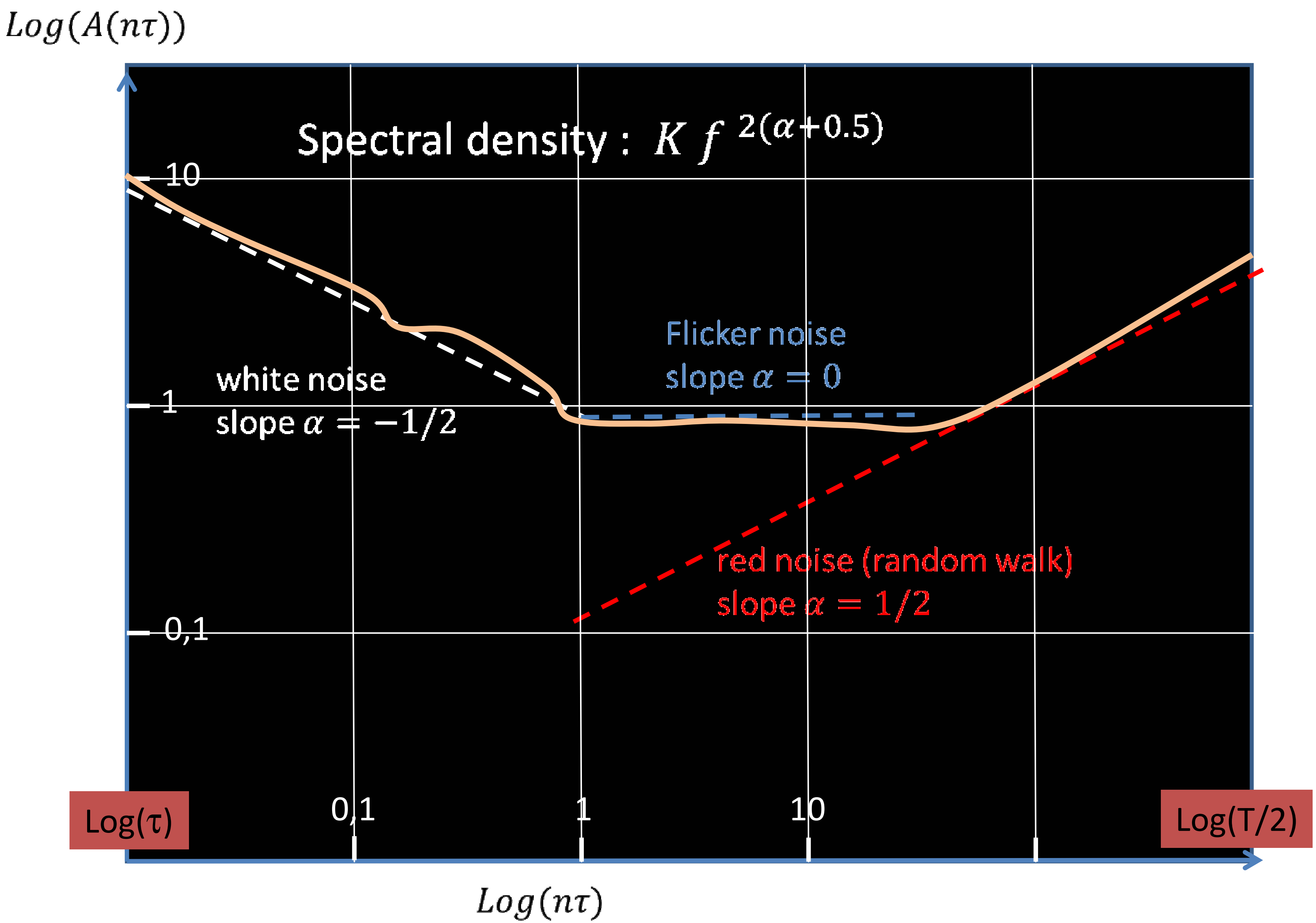
What is Allan variance or deviation?

- Equally sampled series y_i with time interval τ over period T



- Consecutive intervals of period $n\tau$ (n+1 points) and average $\bar{y}_j^{(n\tau)} = \frac{1}{n+1}\sum_{i=1}^{n+1} y_i$
- Allan variance: $\mathrm{AV}(n\tau) = \frac{1}{2}\mathrm{var}(\bar{y}_{j+1}^{(n\tau)} - \bar{y}_j^{(n\tau)})$
- The smaller is $\mathrm{AV}(n\tau)$, the more stable is the time series y_i at time scale $n\tau$.
- AV($n\tau$) related to autocovariance of the averaged series $\bar{y}$: AV(nτ)= $R_{\bar{y}\bar{y}}(0) - R_{\bar{y}\bar{y}}(n\tau)$
- **Allan deviation** : $\mathrm{A}(n\tau) = \sqrt{\mathrm{AV}(n\tau)}$: A($n\tau$) or AV($n\tau$) is an index of the signal stability at time scale $n\tau$

Allan deviation A($n\tau$) in log scales and corresponding noise



Preparation of the time series

- Geodetic excitation (G) derived from EOP C04 : p = x – i y and LOD:

$$\chi = p + i \frac{1}{\tilde{\sigma}_c} \dot{p} \qquad \qquad \chi_3 = \frac{\Delta LOD}{LOD}$$

- Angular momentum functions of the coupled Atmospheric (A) / Oceanic (O) / Hydrological models (H)

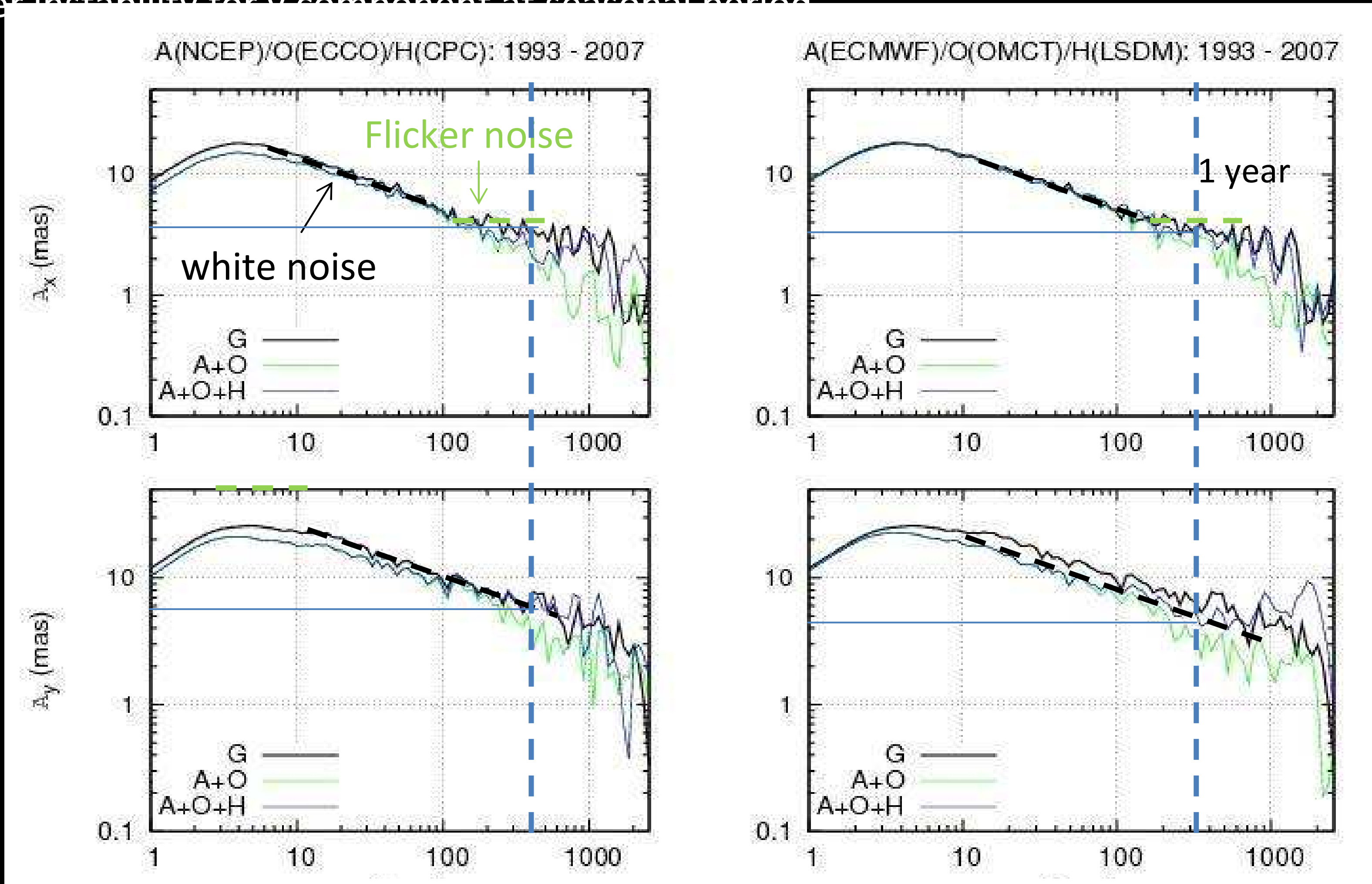
χ_A	χ_O	χ_H
NCEP	ECCO	CPC
ECMWF	OMCT	LSDM

- Removal of the harmonic seasonal terms and bias/secular trend (by least-square fit) before computing Allan deviation

- Geodetic excitation has a lower quality before 1993 (the advent of GPS)

Equatorial component analysis over 2000-2007

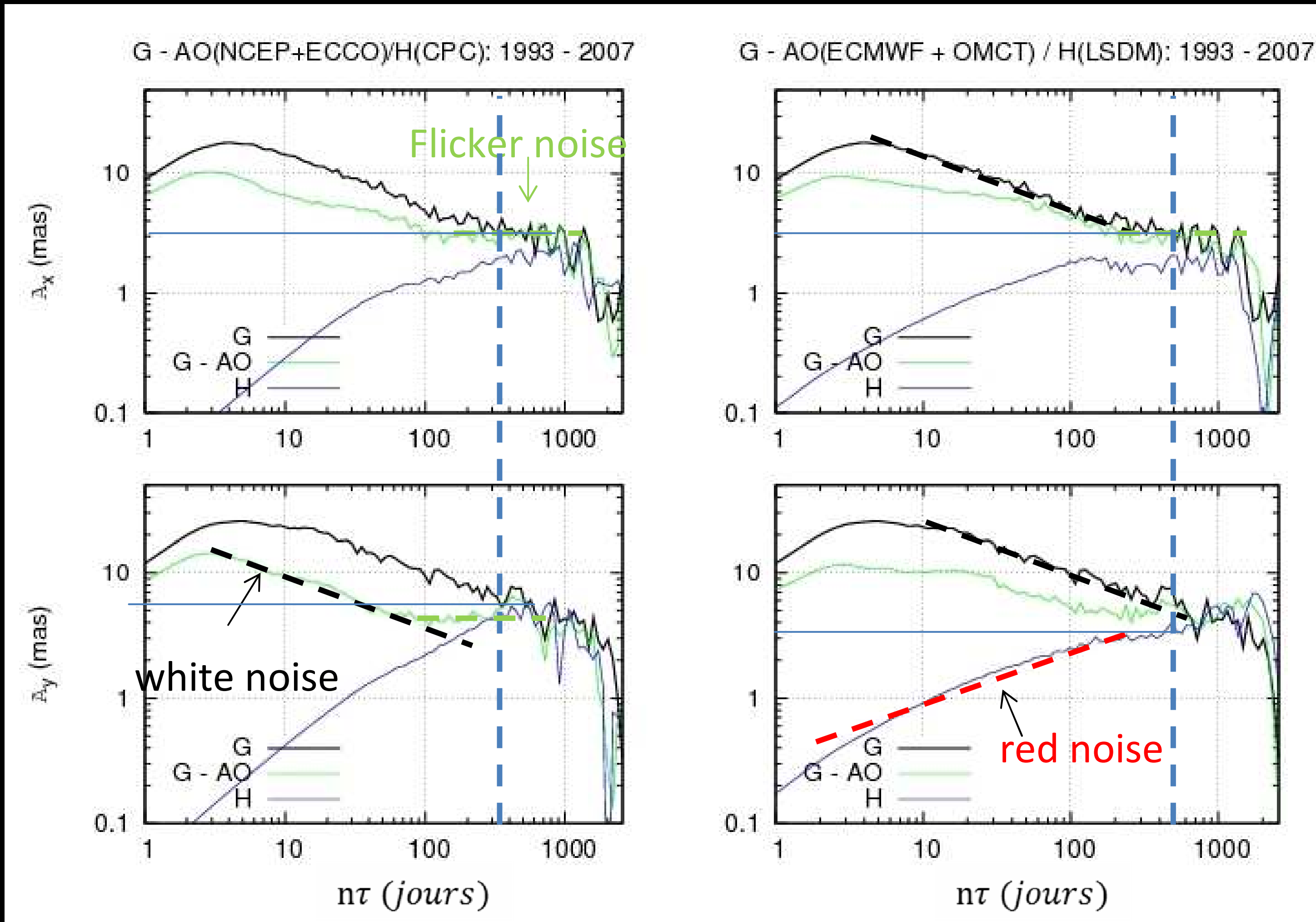
- Allan deviation of Geodetic (G) / Atmospheric (A) + Oceanic (O) + Hydrologic (H) excitations
- Overall good agreement - but A+O too small for sub-seasonal scale.
- Larger instability for y component at seasonal period



Removal of the mean seasonal terms by least square

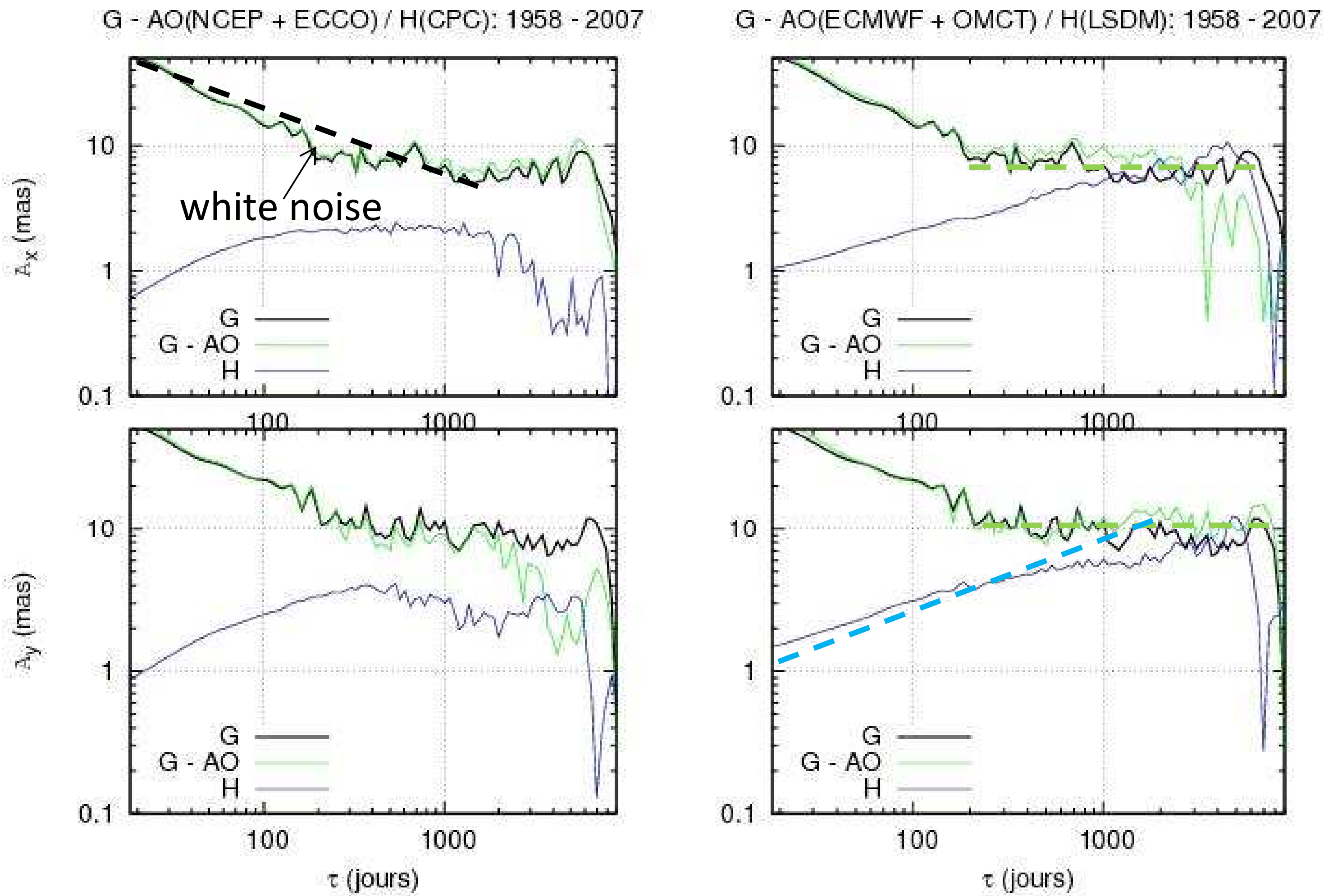
Equatorial component analysis over 1993-2007 & role of the hydrological excitation

- Comparison Geodetic (G) – Atmospheric + Ooceanic (AO) / Hydrological (H) excitations
- Subseasonal / rapid G - AO signal is not explained by hydrological model
- Agreement G - AO / H at seasonal time scale and for longer periods



Equatorial component long term analysis 1958-2000

- Hydrological model clearly accounts for Allan deviation of G-AO after 1000 day (3 years). Both signals ~ Flicker noise.
- Best hydrological model seems to be LSDM



Seismic excitation versus G – AO residuals: 1985-2009

Modeled seismic excitation (step wise function) is random walk, cannot account for polar motion excitation below 3000 days (10 years)

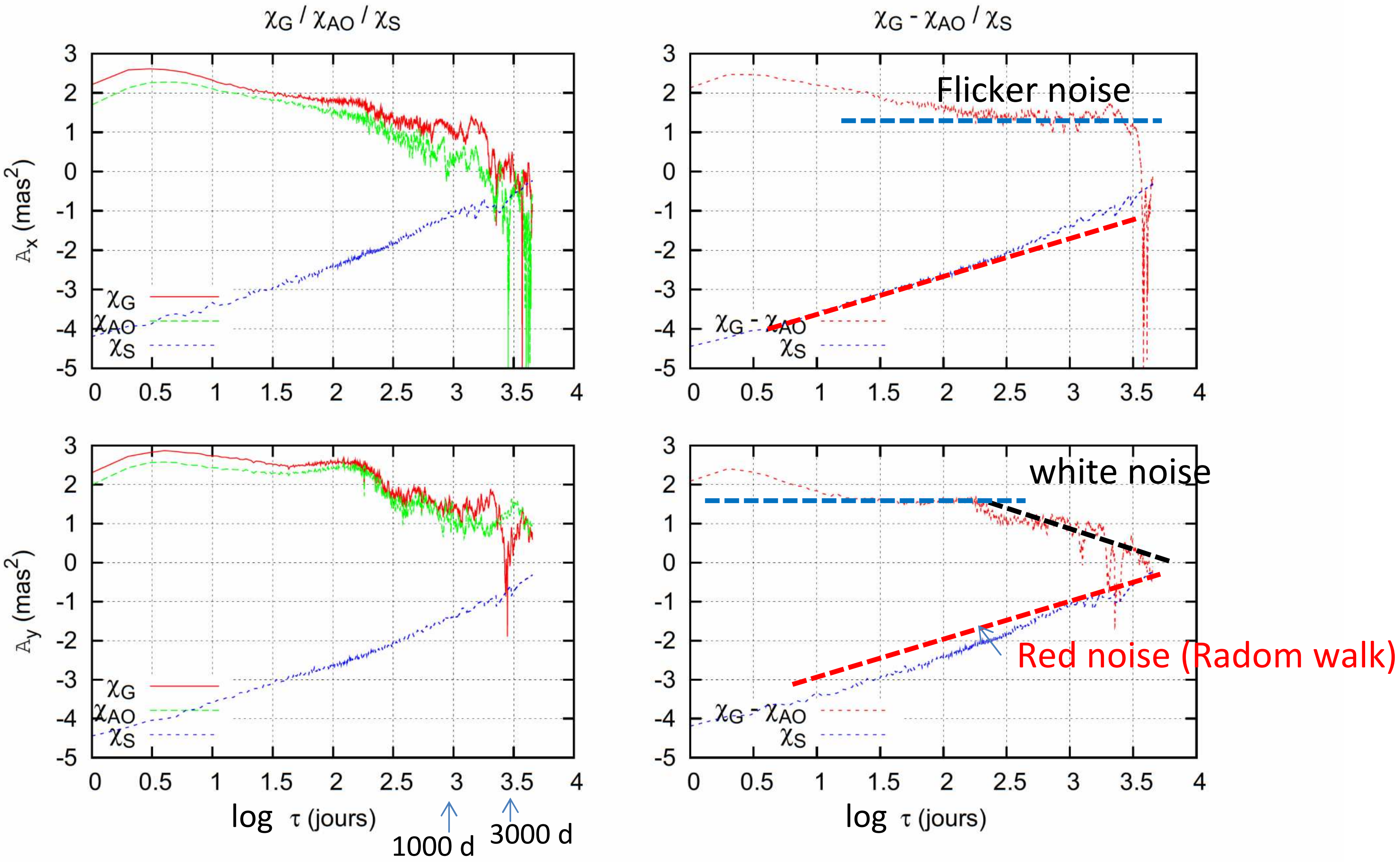
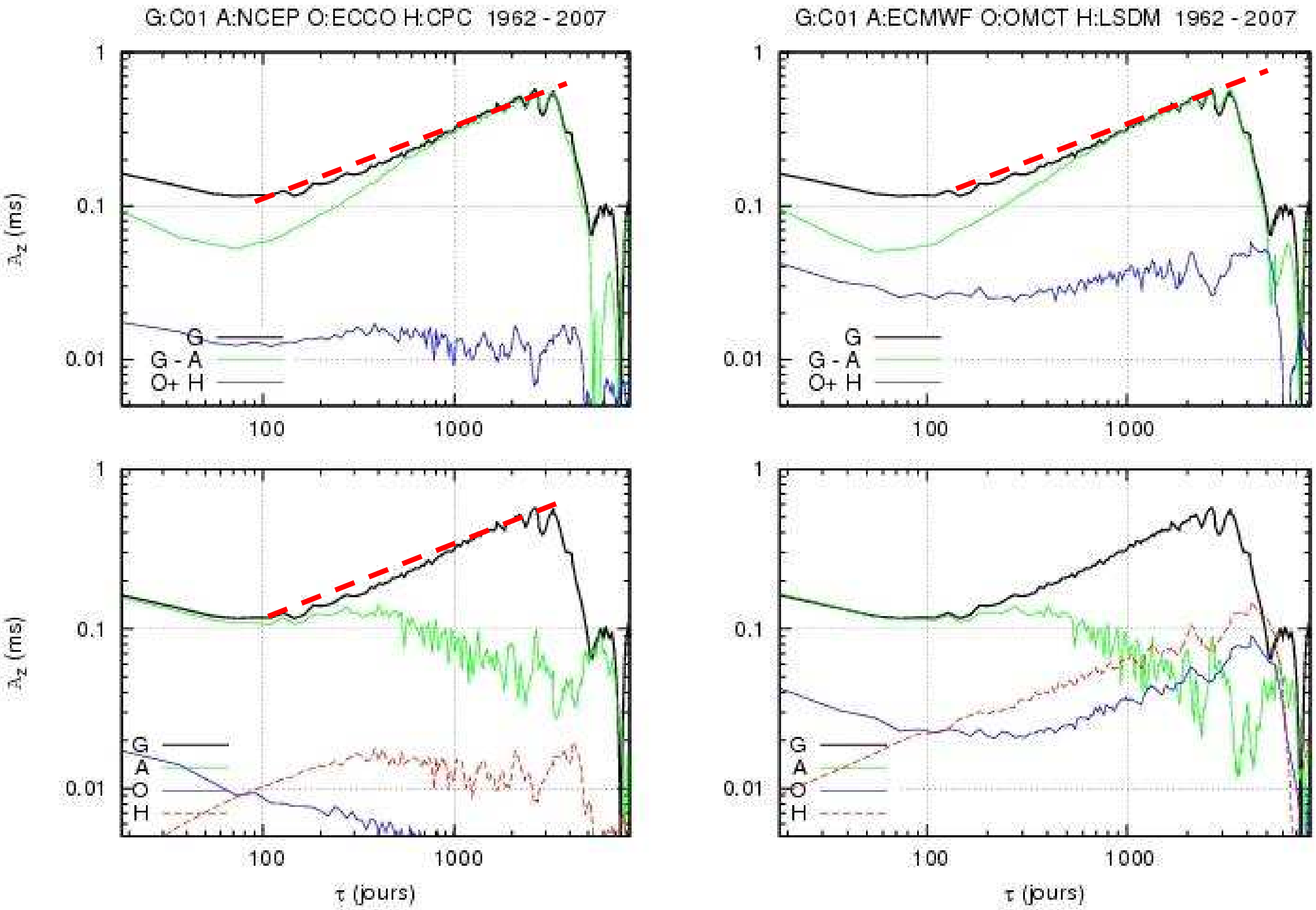


FIG. 12.7: Analyse de variance d'Allan (échelle log-log) : excitations observée χ_G, co-sismique χ_S et atmosphéro-océanique (NCEP+ECCO) χ_{AO} (à gauche) et différences $\chi_G - \chi_{AO}$ (à droite). Composantes x en haut et y en bas

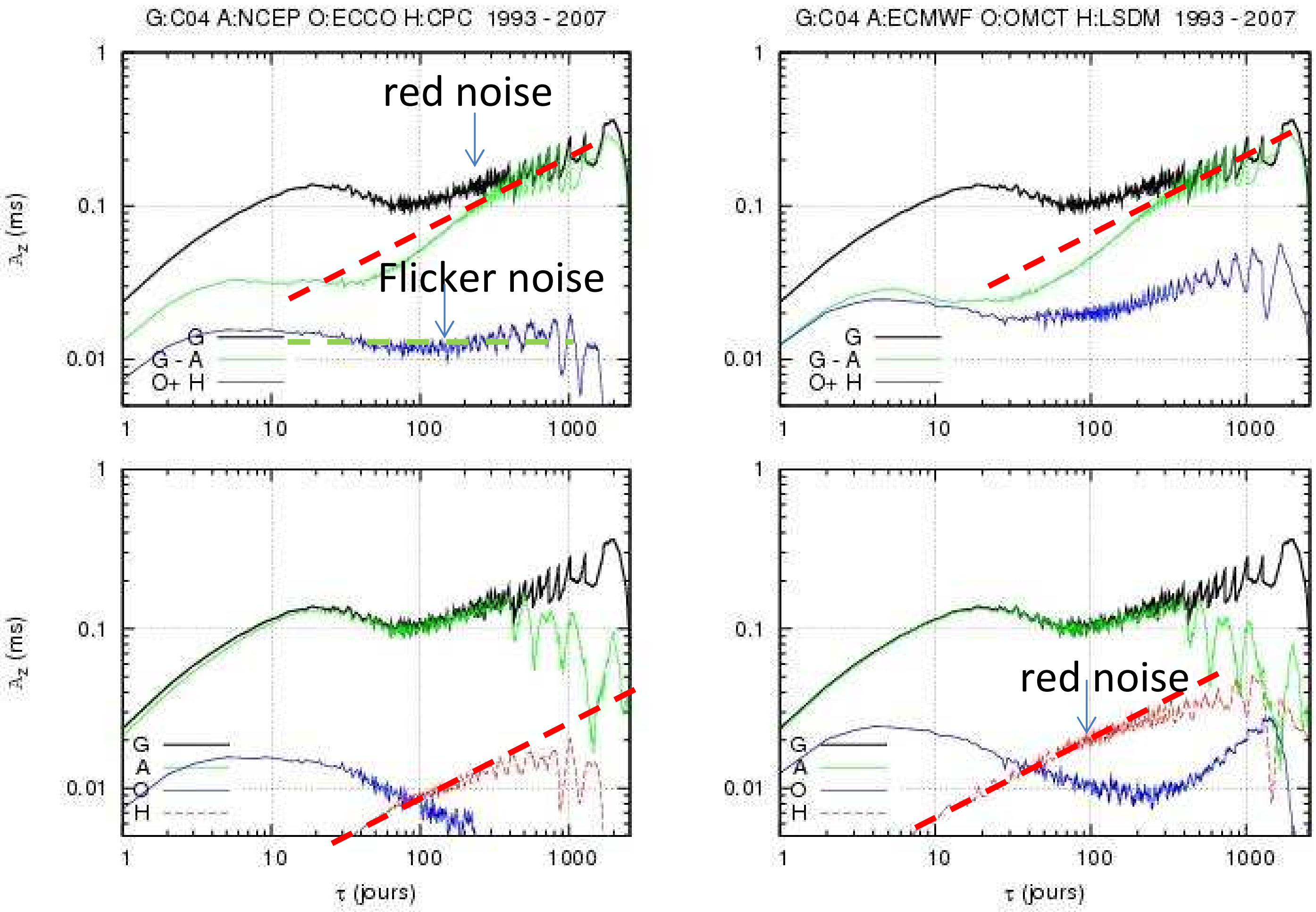
Axial component analysis 1962-2007

- From 1 year the geodetic instability (random walk) increases and is no more related to fluid layers



Axial component analysis 1993-2007

- Good agreement G / A up to 500 days
- Rapid residuals G-A (< 50 days) are explained by oceanic excitation (OMCT)
- But hydrological and oceanic models badly account for G – A residuals above 50 days: defect of these models?



Conclusions

Allan variance analysis is a powerful tool for analysing excitation time series of the Earth rotation, permitting **at a given time scale** :

1. to investigate physical processus at plays

- Equatorial excitation tends to be more stable at long term (~white noise) in contrast to axial excitation (~red noise) → Physical processes are different (surface redistribution versus fluid core motion)
- Over 400 days hydrological processes are fundamental for explaining G – AO residuals – equatorial components.
- Earthquake do not influence PM below 10 years

1. to find the defects in global circulation models :

- Equatorial component : below 100 days hydrological model do not explain the G – AO residuals.
- Axial component: over 50 days G – A residuals do not fit the fluid layer model O+H: defects in both ocean and hydrological models?