Latest advances in an astrometric model based on the Time Transfer Functions formalism

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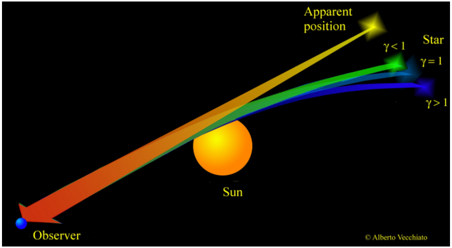
Journées Systèmes de Référence Spatio-Temporels
16 September 2013

Light bending in astrometric observations



body	light bending (μas)	$\psi_{max}(1\ \mu as)$ (deg)
Sun	1.75×10^6	180^o
Mercury	83	$9'$
Venus	493	4.5^o
Earth	574	123^o
Mars	116	$25'$
Jupiter	16270	90^o
Saturn	5780	17^o
Uranus	2080	$71'$
Neptune	2533	$51'$

Classical models for ground based and early space era

$1 - 100\ mas$ accuracy

- **Telescopes**: astrometry, ground based catalogs
- **Hipparcos**: astrometry

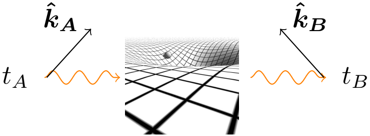
Relativistic models for space era

μas accuracy

- **Gaia**: global map of the Milky way
- **GAME**: tests of General Relativity

Time Transfer Functions (TTF)

- Information about light propagation between 2 points
- Alternative to reconstruction of null-geodesic path



t : coordinate time

$$k_\alpha \equiv g_{\alpha\beta} \frac{dx^\beta}{d\lambda}$$

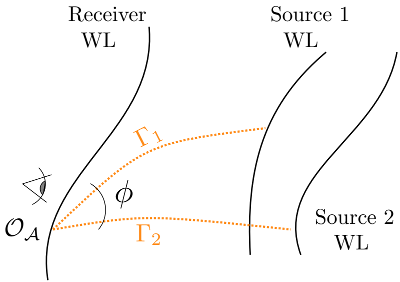
Time of flight $\qquad t_B - t_A = \mathcal{T}_{e/r}(\boldsymbol{x}_A, \boldsymbol{x}_B, t_{A/B})$

Direction triple $\qquad (\hat{k}_i)_{A/B} = \left(\frac{k_i}{k_0}\right)_{A/B} = -c \frac{\partial \mathcal{T}_{e/r}}{\partial x^i_B} \left[1 - \frac{\partial \mathcal{T}_{e/r}}{\partial t_B}\right]^{-1}$

[Le Poncin-Lafitte, Linet and Teyssandier, 2004]

Astrometric observables (1)

Angular distance of two sources



- ϕ: angle between two incoming light ray
- $\left(\widehat{\boldsymbol{k}}\right)_B$: light direction triple at observation coordinates
- $\boldsymbol{\beta}$: observer's velocity in the chosen reference system (RS)
- Pointing astrometry (used by GAME)

Relativistic model for ϕ

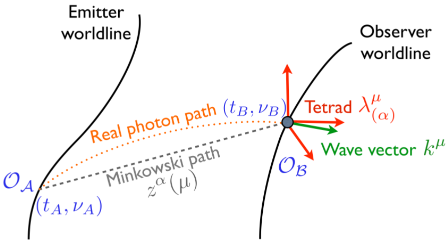
$$\sin^2\frac{\phi}{2} = -\frac{1}{4}\left[\frac{\left(g_{00} + 2g_{0k}\beta^k + g_{kl}\beta^k\beta^l\right) g^{ij}(\hat{k}'_i - \hat{k}_i)(\hat{k}'_j - \hat{k}_j)}{(1+\beta^m\hat{k}_m)(1+\beta^l\hat{k}'_l)}\right]_B$$

[Teyssandier and Le Poncin-Lafitte, 2006]

Astrometric observables (2)

Director cosines of an observation

- $\boldsymbol{n}$: light direction in the observer's reference system (RS)
- $\left(\widehat{\boldsymbol{k}}\right)_B$: light direction triple at observation coordinates
- $\lambda^{\alpha}_{(\mu)}$: transformation matrix to the RS comoving with the observer
- Scanning astrometry (used by Gaia)



$$n^{(i)} = -\frac{\lambda^{0}_{(i)} + \lambda^{j}_{(i)}\hat{k}_j}{\lambda^{0}_{(0)} + \lambda^{j}_{(0)}\hat{k}_j}$$

$$= n^{(i)}(\boldsymbol{x}_B, \mathcal{C}_P; \boldsymbol{x}_A)$$

[Bertone and Le Poncin-Lafitte, 2012]

General closed form 2PM equations

- PM development : $g_{\mu\nu} = \eta_{\mu\nu} + \sum_{i=1}^{\infty} G^i h^{(i)}_{\mu\nu}$, $\mathcal{T}(\boldsymbol{x}_A, \boldsymbol{x}_B, t_{A/B}) = \sum_{i=0}^{\infty} G^i \mathcal{T}^{(i)}$
- Replace in the Hamilton-Jacobi like equation derived from isotropic condition:

$$g^{00}(t_B - \mathcal{T}_r, \boldsymbol{x}_A) + 2cg^{0i}(t_B - \mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x^i_A} + c^2 g^{ij}(t_B - \mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x^i_A}\frac{\partial \mathcal{T}_r}{\partial x^j_A} = 0$$

Minkowskian order + perturbation as integral along straight line z^α (1PM , 2PM)

$$\mathcal{T}_r = \frac{R_{AB}}{c} + \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \mathbf{x}_A, t_B, \mathbf{x}_B\right] d\mu$$

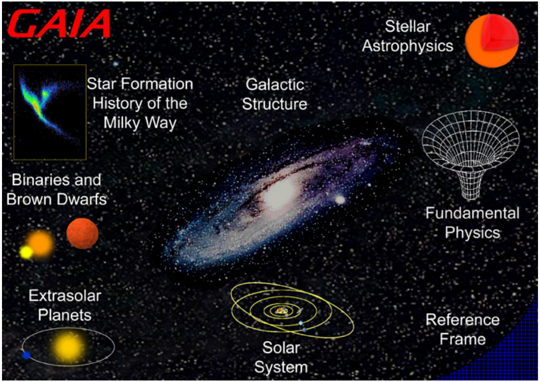
$$+ \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \mathbf{x}_A, t_B, \mathbf{x}_B\right] d\lambda d\mu$$

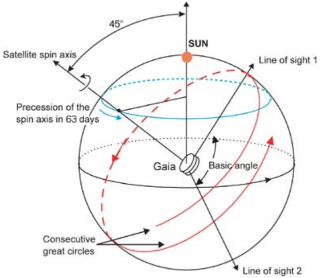
$$\left(\widehat{k}_i\right)_B = -N^i_{AB} + \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \mathbf{x}_A, t_B, \mathbf{x}_B\right] d\mu$$

$$+ \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \mathbf{x}_A, t_B, \mathbf{x}_B\right] d\lambda d\mu$$

[Hees, Bertone and Le Poncin-Lafitte, 2013]

Application to Gaia

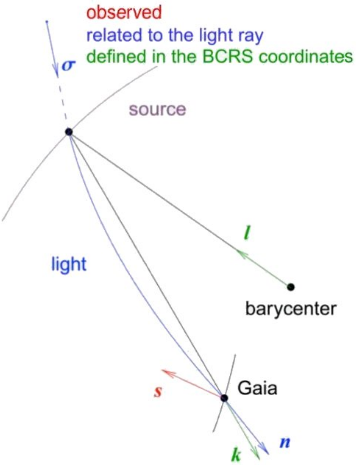




- Scanning the whole sky
- 6h period sky scanning law
- No observations is Sun direction

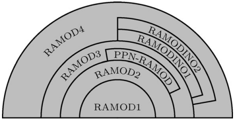
- First fully-relativistic astrometric mission
- No external verification possible at Gaia accuracy
- Data analysis by independent models

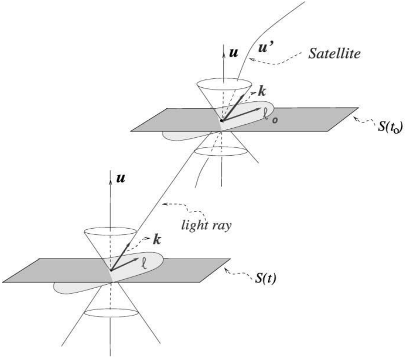
Relativistic astrometric models for Gaia



GREM

[Klioner, 2003]





RAMOD

[de Felice et al., 2004]

Relating the relativistic models

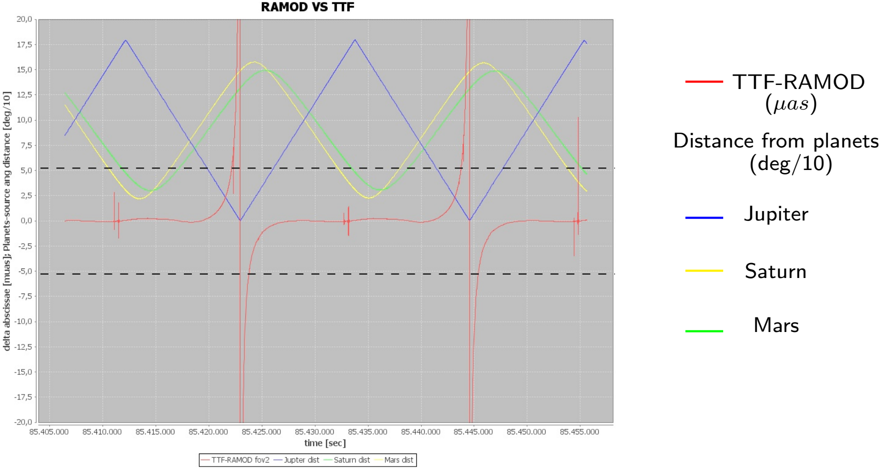
- Aberration *[Crosta and Vecchiato, 2010]*

Additional analytical cross-check using the TTF

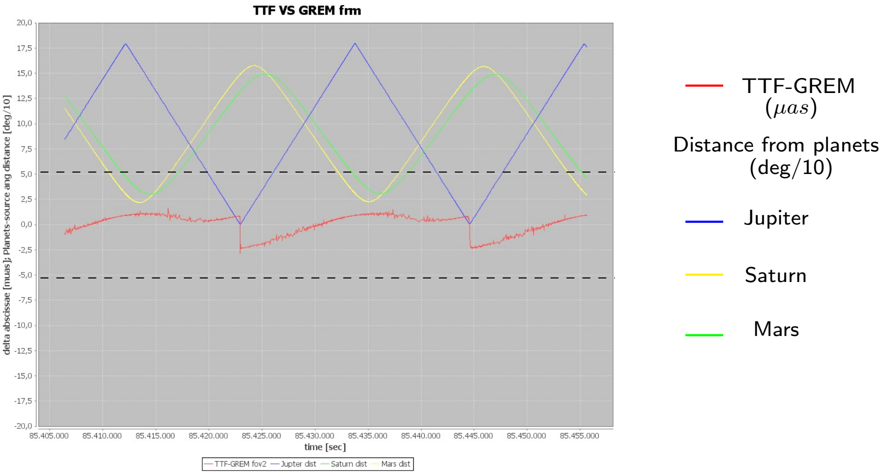
- Coordinate time of flight and light deflection

RAMOD3	$(\hat{k}_i)_B = -\bar{\ell}^i_B \left[1 + \frac{3}{2}(h_{00})_B\right] + \mathcal{O}(c^{-3})$ $\bar{\boldsymbol{\ell}} =$ projection of $\boldsymbol{k}$ in the rest space of the observer
GREM	$(\hat{k}_i)_B = -\sigma^i(1 + 2(h_{00})_B) + \frac{\Delta\dot{x}^i_B}{c} + \mathcal{O}(c^{-3})$ $\boldsymbol{\sigma} =$ direction at past-null infinity $\boldsymbol{\Delta\dot{x}} =$ gravitational deflection

☑ Relations proven for a PPN metric adapted for Gaia



- GSR1+ : PPN-RAMOD + Gaia tetrad + corrections for planets
- Expected deviations (up to $500\ \mu as$) near Jupiter
- Sub-μas difference far from planets
- Future GSR3 : will implement RAMOD3 + Gaia tetrad $\rightarrow$ μas accuracy



- GREM (light deflection + tetrad [Klioner 2003/2004]) accurate at $1\ \mu as$
- Deviation under Gaia accuracy near massive planets ($\leq 1^{\circ}$ from Jupiter)
- Unexpected shape with a periodic trend and sign changes

Conclusions

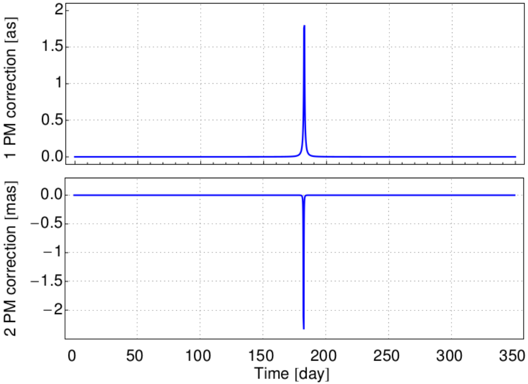
- General covariant description of an astrometric observation
- Sistematic development in the PM approximation of GR
- Application in the Gaia context with analytical comparison of light propagation equations and simulated observation angles
- Differences in the results of TTF and GREM based simulations. This points out the necessity of independent relativistic models

Perspectives

- Further analysis of implemented models
- Sphere reconstruction from TTF model and 5 years of simulated observations

Simulation of a GAME-like observation

- Observation of two sources at 100 AU from Earth
- Earth orbiting around the Sun, one of the sources on the galactic plane
- Maximum bending at conjunction (one source aligned with Sun)



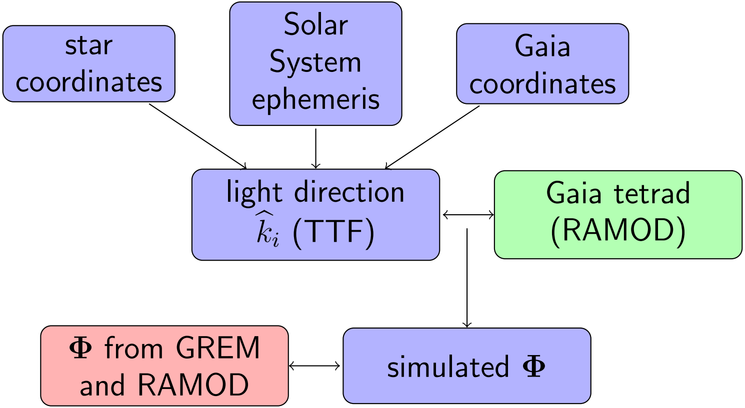
1PM correction
$1.75\ as$

2PM correction
$2.4\ mas$

☑ Comparison to *[Klioner and Zschocke, 2010]*, *[Teyssandier, 2012]*

Implementation of the GSR-TTF software

Simulation of astrometric observations
(at Turin Astrophysical Observatory)



Φ : angle of observation with respect to Gaia axes