COMPUTATION OF THE QUANTITIES DESCRIBING LUNAR LIBRATIONS IN THE ASTRONOMICAL ALMANAC

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ABSTRACT. This paper provides a summary of HM Nautical Almanac Office Technical Note 74, on which the poster presented at the Journees 2010 is based. We briefly describe the method used for calculating lunar librations as they are presented in Section D of The Astronomical Almanac. The Euler angles from the JPL DE403/LE403 ephemerides are used to calculate improved lunar librations, beginning with the 2011 edition. Implementation of this ephemeris data supersedes the analytical theory of Eckhardt (1981, 1982) used since 1985.

1. INTRODUCTION

Cassini's laws describe general properties of the Moon's motion: the descending node of the Moon's equator coincides with the ascending node of the Moon's orbit on the ecliptic, the equator maintains a constant inclination to the ecliptic and the rotation rate is such that on average the same side is always facing the Earth. The rotation rate is equal to the rate of motion of the Moon's mean longitude. Dynamical perturbations cause small (of order of a few arcseconds) periodic variations from this mean state, i.e. physical librations. Overlaid are the much larger (of order of several degrees) optical librations which are caused by variations in the rate of the Moon's orbital motion and the inclination of the Moon's equator to its orbital plane. The mean central meridian of the side facing the Earth is specified as the prime meridian to define the rotational state of the Moon. Its direction in space differs by 180° from the mean longitude of the Moon (see Figure 1).

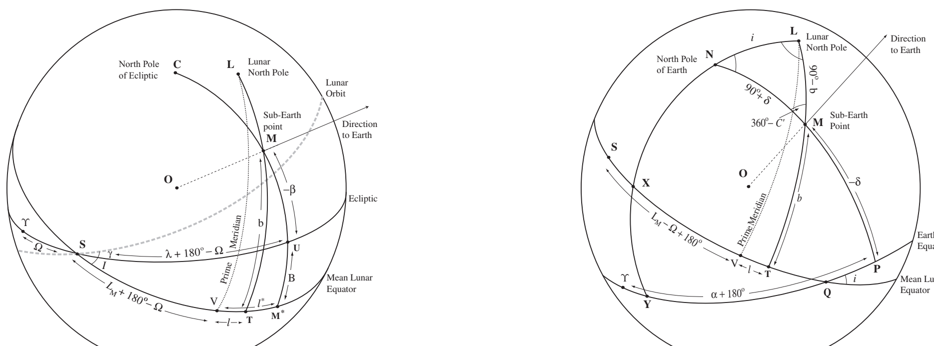

Figure 1: Selenocentric sphere: a) lunar orbit and relationships between sub-Earth point, M, the mean lunar equator and the ecliptic. S is the descending node of the lunar equator on the ecliptic. b) relationships between the sub-Earth point M, the mean lunar equator and the Earth's equator.

2. IMPLEMENTATION OF JPL LUNAR ROTATION ANGLES

The JPL DE403 ephemeris includes an ephemeris for the rotation of the Moon. The Euler angles describing the rotation of the Moon, defined relative to the ICRS equator and equinox, are ϕ, θ and ψ (see Table 1 and Figure 2, also Newhall and Williams, 1997) and describe the orientation of the principal

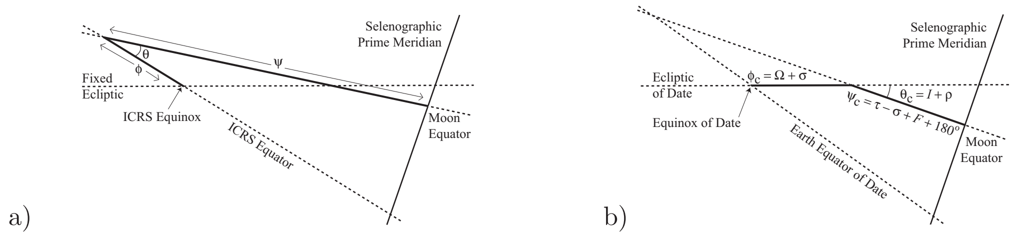

Figure 2: a) the reference frame in the principal moment of inertia (PA) system; b) the reference frame in the mean Earth/rotation axis (ME) system. Description of the angles is given in Table 1.

Table 1: Euler angles as used in Figure 2:

ϕ	Angle along ICRS equator, from ICRS X-axis to ascending node of the lunar equator	ϕ_C	Angle from equinox of date to descending node of lunar equator on ecliptic of date
θ	Inclination between lunar equator and ICRS equator	θ_C	Inclination between lunar equator and ecliptic of date
ψ	Angle along lunar equator from node to lunar prime meridian	ψ_C	Angle along lunar equator from descending node on ecliptic to the lunar prime meridian

axes of inertia of the Moon (PA system, see Figure 2a). Angles given in this system can be transformed to give the Euler angles ϕ_C, θ_C and ψ_C, defined relative to the ecliptic and equinox reference frame of date. They describe the orientation of a slightly different lunar axis system (ME system, see Figure 2b). These quantities are used to compute Ω, I and L_M which are needed to calculate the total librations l_T, b_T and the position angle C_T. For details, see Technical Note 74 and references therein.

3. OVERVIEW OF THE ALGORITHM

First, apparent positions for the Moon (α, δ, d) and the Sun $(\alpha_S, \delta_S, d_S)$ at time t (where α is the right ascension, δ the declination and d the distance) are obtained; as well as the nutation in longitude $(\Delta\psi)$ and obliquity $(\Delta\epsilon)$, also the true obliquity (ϵ). The apparent equatorial coordinates are rotated around the X-axis by the angle ϵ to determine the apparent ecliptic positions of the Sun(λ_S, β_S) and Moon (λ, β). Also $\tau = d/c$, the light time correction for the Moon, is needed (c is the speed of light). The inclination I_m at time $t - \tau$ and the fundamental arguments Ω and L_M (see Simon et al, 1994) are evaluated. From $(\lambda - \Omega - \Delta\psi)$, β, I, and L_M the optical (geometric) librations l_o and b_o are calculated. In order to obtain the position angle of the axis of rotation (C'_o), Ω' and i are determined. Using Euler angles from the ephemeris and applying the transformations needed to go from Figure 2a to Figure 2b, the new Euler angles $(\phi_C, \theta_C, \psi_C)$ are obtained: this transforms from the system of the ephemeris to the true date system by substituting ϕ_C for Ω, θ_C for I, $\psi_C + \phi_C - 180°$ for L_M. This determines the total librations l_T, b_T and C'_T. By subtracting the optical from the total librations, the physical librations δl_p and δb_p are calculated. The quantity $\delta C'_p = C'_o - C'_T$ is obtained as well. These values are tabulated in Section D of The Astronomical Almanac. A detailed step-by-step example of this method can be found in Technical Note 74.

4. REFERENCES

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