APPLICATION OF THE SPECTRAL ANALYSIS METHODS FOR THE INVESTIGATION OF THE MOON ROTATION

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ABSTRACT. Dynamics of the rotational motion of the Moon is investigated numerically by using Rodrigues-Hamilton parameters over 418.9 year time interval. The results of the numerical solution of the problem are compared with the composite semi-analytical theory of the Moon rotation (SMR), consisting of Cassini relations and the semi-analytical solutions of the lunar physical libration problem (Eckhardt D.H., 1981), (Moons M., 1982), (Moons M., 1984), (Pešek I., 1982). The initial conditions of the numerical integration are taken from SMR. The investigation of the discrepancies is carried out by the optimal spectral analysis methods for the Newtonian case. All the periodic terms representing the behavior of the residuals are interpreted as corrections to SMR semi-analytical theory. As a result, the Moon Rotation Series (MRS2010) are constructed, which are dynamically adequate to the DE200/LE200 ephemeris over 418.9 year time interval. A numerical solution for the Moon rotation is obtained anew with the new initial conditions calculated by means of MRS2010. The discrepancies between the new numerical solution and MRS2010 do not surpass 20 mas over 418.9 year time interval. The result of the comparison demonstrates that MRS2010 series are more accurately represent the Moon rotation than SMR series.

1. MATHEMATICAL MODEL OF THE PROBLEM

The numerical solution of the problem is obtained by solving the Lagrange differential equations of the second kind for the Moon rotation with respect to the fixed ecliptic and equinox of epoch J2000. The mathematical model of the problem is described in detail in the paper (Eroshkin G.I., Pashkevich V.V., 1997).

The semi-analytical solutions of the lunar physical libration problem include four solutions:
a) MP500 is the 3rd degree solution for the force function (Moons M., 1982);
b) Additional solution for the 4th degree perturbations of the force function (Eckhardt D.H., 1981);
c) Effect of planetary perturbations (Moons M., 1984);
d) Effect of the Earth's flattening (Pešek I., 1982).

As a result of compilation, a semi-analytical theory of the Moon rotation is constructed with respect to the ecliptic and equinox of date. Then the semi-analytical theory is reduced to the fixed ecliptic J2000.0.

The orbital motions of the disturbing bodies are defined by the DE200/LE200 ephemeris. The high-precision numerical integration method (Belikov, 1990) with a number of modifications (Eroshkin et al., 1993) was applied.

2. ALGORITHMS AND RESULTS

The result of the comparison between the numerical solution and semi-analytical solution SMR is studied by means of two variants of the spectral analysis schemes (Figure 1).

Numerical solution of the Moon rotation is implemented with the quadruple precision of the calculations. The initial conditions are computed by the semi-analytical theory of the Moon rotation (SMR), which corresponds to the fixed ecliptic J2000.0. The Moon rotation series SMR consist of 380 periodical terms with the periods from 5.648 days to 84541.30 years. The discrepancies between the numerical

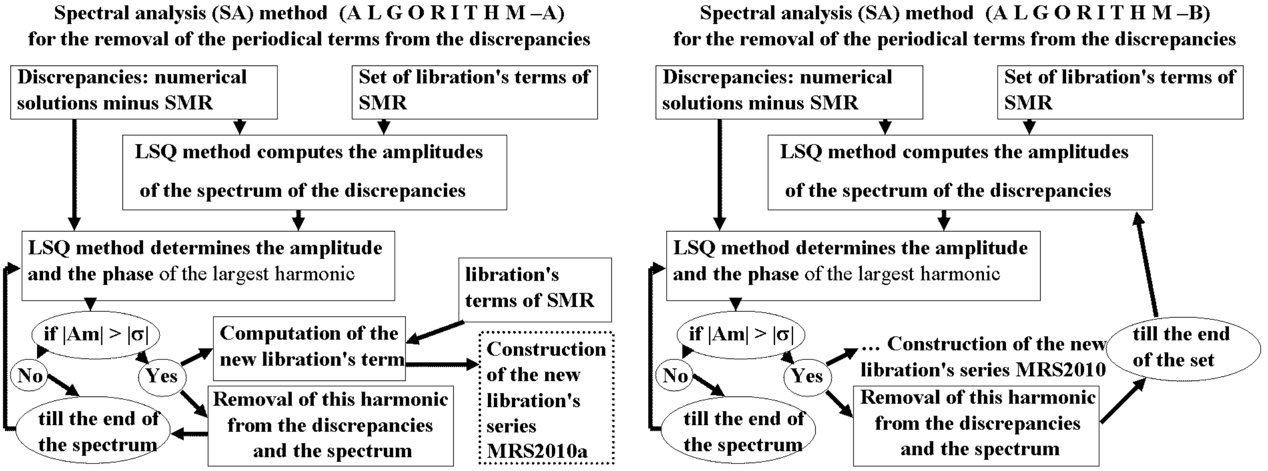

Figure 2: Two types of Algorithms

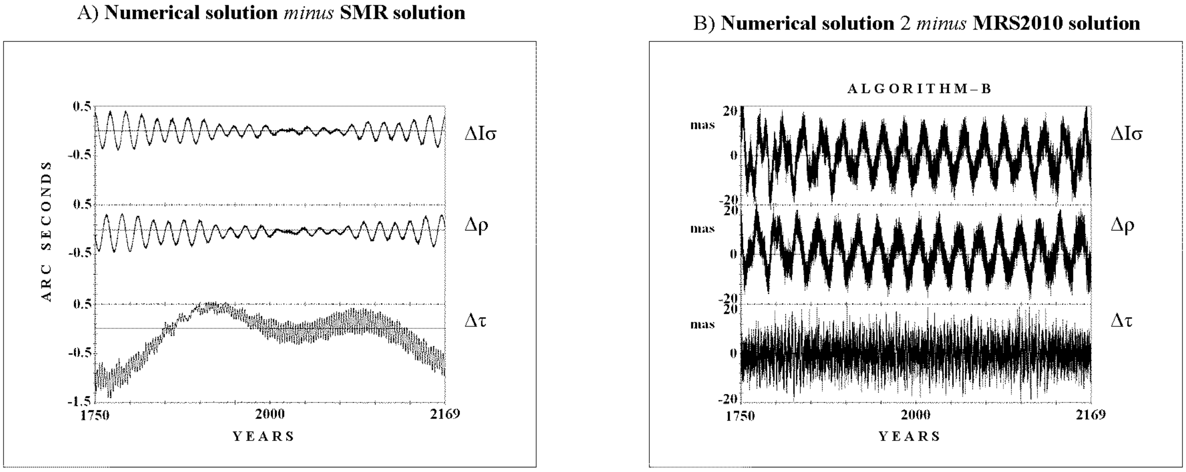

Figure 3: Discrepancies between the numerical and semi-analytical solutions of the Moon rotation

solution and SMR are obtained in Euler angles over 418.9 year time interval with one day spacing (Figure 2A).

The investigation of the discrepancies is carried out by the least squares method (LSQ) and by the spectral analysis method (SA). The set of the frequencies of SMR theory is used without a change. Only the coefficients of the periodic terms are improved.

SA method - A: The spectrum of the discrepancies between the numerical solution and SMR is constructed only once (Figure 3, ALGORITHM - A). Every coefficient of the new periodic term equals the sum of the calculated periodic term coefficient and the coefficient of the corresponding periodic term of SMR. The found new harmonic is removed from the discrepancies and from the spectrum. Starting from the maximum term of the spectrum the procedure is accomplished successively up to its least term. The new periodic and Poisson terms representing the new series MRS2010A are determined.

SA method - B: The spectrum of the discrepancies is constructed anew after the removal of every largest residual harmonic from the discrepancies. Each constructed spectrum is used for the determination of the new coefficient of the periodic term as in SA method - A. This procedure is performed for every harmonic of the set. The new periodic and Poisson terms representing the new series MRS2010 are determined.

The compilation of the power spectrum of the discrepancies between the numerical and semi-analytical SMR solutions for ALGORITHM - B is constructed, beginning from the largest of the residual harmonics, step by step (Figure 3, ALGORITHM - B).

The power spectra represented in Figure 3 are restricted by the terms with the periods from 5.648

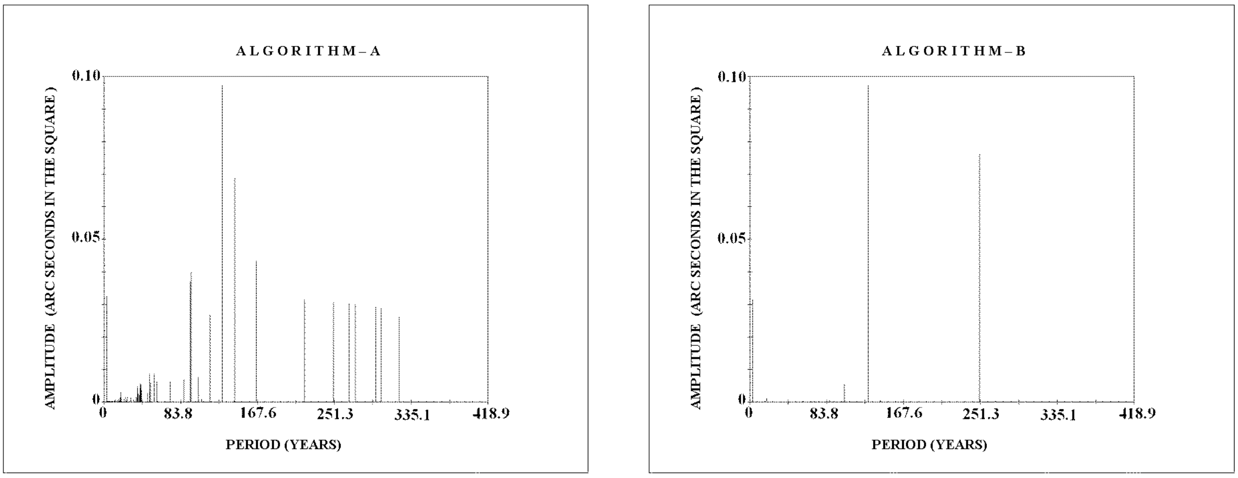

Figure 4: Spectrum of the discrepancies between the numerical and SMR solutions for $\Delta\tau$

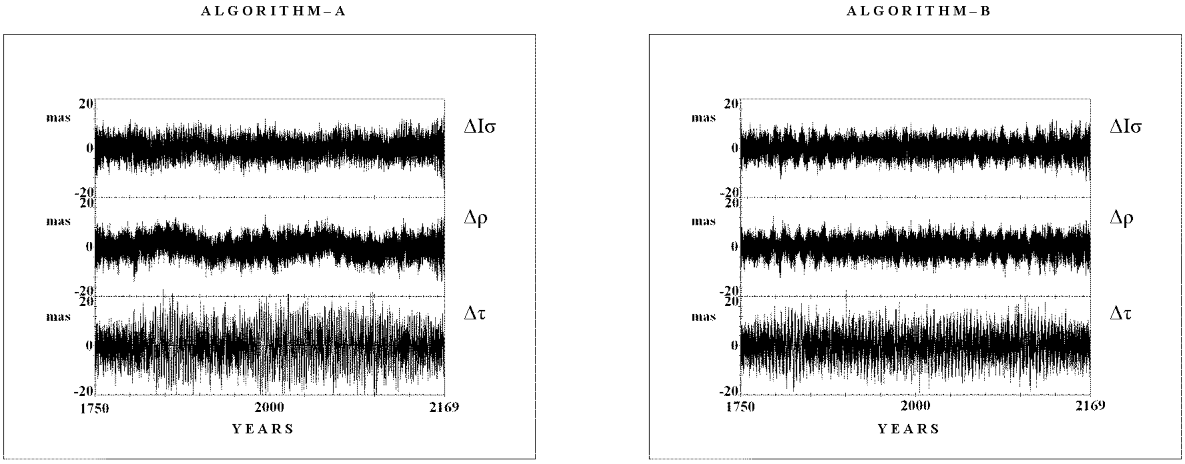

Figure 5: Residuals after the formal removal of 351 periodical terms from the discrepancies between the numerical and semi-analytical (SMR) solutions

days to 418.9 years. Therefore the set of the frequencies of SMR theory, which is used in SA methods, is restricted only by 351 periodical terms.

Figure 3 demonstrates that the composed power spectrum of the discrepancies between the numerical and semi-analytical SMR solutions for ALGORITHM - B is preferential than the power spectrum for ALGORITHM - A, because the close harmonics distinguish better from each other (Figure 3, ALGORITHM - B).

Figure 4 demonstrates that the residuals, after the formal removal of 351 periodical terms from the discrepancies between the numerical and semi-analytical SMR solutions, for ALGORITHM - B are less than the ones for ALGORITHM - A.

This investigation demonstrates that ALGORITHM - B is more accurate than ALGORITHM - A . Thus MRS2010 is better than MRS2010A.

Then the numerical solution for the Moon rotation is constructed anew with the new initial conditions calculated by means of MRS2010, which includes 1433 periodical and Poisson terms. The appearance of Poisson terms in the harmonics is the main difference between MRS2010 series and SMR series.

The results of the comparison of the new numerical integration of the lunar rotation with the new MRS2010 are presented in Figure 2B. The residuals do not reveal secular trends. The behavior of the residuals can be described by the superposition of the harmonics representing both the forced physical libration and the fictive free physical libration. The most essential harmonics of the residuals are the periodic harmonics of the fictive free physical libration with the period of 2.9 years in the longitude ($\Delta\tau$), with the periods of 27.2 days and of 24 years in the inclination ($\Delta\varrho$) and in the node longitude ($\Delta I\sigma$). The appearance of the harmonics of the fictive free physical libration in the numerical solution is explained by the errors of the initial conditions, which are defined by the not enough precise semi-analytical solutions.

Figure 2 demonstrates that the discrepancies of the comparison in the libration angles decrease 25 times for both $\Delta I\sigma$ and $\Delta\varrho$ and 75 times for $\Delta\tau$, after the construction of the new more accurate MRS2010 semi-analytical solution.

The discrepancies of the comparison between the numerical solutions and MRS2010 do not surpass 20 mas over 418.9 year time interval (Figure 2B). It means an essentially better consistency of MRS2010 series with the DE200/LE200 ephemeris.

3. CONCLUSION

It was found that ALGORITHM - B, in which the spectrum is constructed anew after the removal of every largest residual harmonic from the discrepancies, is more accurate than ALGORITHM - A, in which the spectrum is constructed only one time.

ALGORITHM - B is very time-consuming. Hence it is not suitable for the investigation of the long time series, as follows from our previous investigation of the Earth rotation (V.V.Pashkevich and G.I.Eroshkin, 2005). However ALGORITHM - B is very suitable for the investigation of the short time series (the Moon rotation).

The new more accurate series MRS2010, dynamically adequate to the DE200/LE200 ephemeris over 418.9 year time interval, is constructed. MRS2010 includes about 1433 periodical and Poisson terms.

The discrepancies between the numerical solution and MRS2010 do not surpass 20 mas over 418.9 year time interval.

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