LIGHT TIME CALCULATIONS FOR DEEP SPACE NAVIGATION

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ABSTRACT. With the recent discovery of few astrometric anomalies derived from the tracking of spacecrafts, such as the Pioneer and Earth flyby anomalies, we propose to reconsider the light time calculation used by Deep Space Navigation. In particular, we show that some traditional approximations can lead to neglect tiny terms that may produce instability in the orbit determination of a probe during Earth flyby.

1. INTRODUCTION

Deep space data processing during the last decade has revealed the presence of some anomalies in the trajectory of probes. Lots of hypothesis have been made trying to solve this puzzle, but they can be summarized in two main approaches : whether these anomalies are the manifestation of some new physics, or something is incorrectly modeled in the data processing. We investigated Moyer's book, which describes the relativistic framework used by space agencies for data processing, and came to the conclusion that the modeling is correct up to the first post-Newtonian approximation of General Relativity. However, some terms, a priori of very small amplitude, have been neglected in the light time calculation. In section 2, we give a brief overview of light time computation as described by the Moyer's book; we show that the transponder's delay, *i.e.* the time delay between the reception and retransmission of the light signal on board the satellite, is not correctly taken into account. In section 3 we present an alternative modeling that corrects this. Finally in section 4 we compare these modelings to highlight the differences and we give some conclusions in section 5. Throughout this work we will suppose that space-time is covered by some global barycentric coordinates system $x^\alpha = (x^0, \boldsymbol{x})$, with $x^0 = ct$, c being the speed of light in vacuum, t a time coordinate and $\boldsymbol{x} = (x^i, i = 1, 2, 3)$. Greek indices run from 0 to 3, and Latin indices from 1 to 3. Here $\boldsymbol{x}_t^b/\boldsymbol{v}_t^b$ represents the position/velocity of b at time t, where b can take the value GS (ground station) or SC (spacecraft). Primed values are related to the *Moyer's modeling*, while we will use non-primed values for the alternative modeling.

2. MOYER NAVIGATION MODEL

Deep space navigation is based on the exchange of light signals between a probe and at least one observing ground station. The calculation of a light time is quite simple : a clock starts counting as an uplink signal is emitted from ground at $\boldsymbol{x}_{1'}^{GS}$. The signal is received by the probe at $\boldsymbol{x}_{2'}^{SC}$ and then reemitted immediately towards the Earth where it is received by a ground station at $\boldsymbol{x}_{3'}^{GS}$. The clock stops counting and gives the round-trip light time ρ', referred to as "two-ways light time" or simply "light time". ρ' can be computed as follow:

$$\rho' = \frac{1}{c}|\boldsymbol{x}_{3'}^{GS} - \boldsymbol{x}_{2'}^{SC}| + \frac{1}{c}|\boldsymbol{x}_{2'}^{SC} - \boldsymbol{x}_{1'}^{GS}| + Shapiro + \delta TS, \tag{1}$$

where *Shapiro* and δTS are the Shapiro delay and other corrections described by Moyer, respectively. Then the light time is used to compute two physical quantities: the *Ranging*, describing the distance between the probe and the ground station, and the *Doppler* related to the radial velocity of the probe with respect to the Earth. *Ranging* signal almost coincides with ρ' when one simply adds a calibrated transponder's delay δt. *Doppler* signal, F, is obtained by differentiating two successive light-time measurements, $\rho'_s = t_{3s} - t_{1s}$ and $\rho'_e = t_{3e} - t_{1e}$, during a given count interval $T_c = t_{3e} - t_{3s}$. Moyer showed that $F = M_2 f_T(t_1)\left(\rho'_e - \rho'_s\right)/T_c$ where M_2 is a transponder's ratio applied to the downlink signal when it is reemitted towards the Earth.

3. ALTERNATIVE NAVIGATION MODEL

In the Moyer's model, the uplink signal is received at $\boldsymbol{x}_{2'}^{SC}$ and immediately transponded towards the Earth : no transponder delay is taken into account. Nevertheless, an electronic delay of some microseconds due to on board processing of the incoming signal is present and we have studied its influence on light time modeling for *Ranging* and *Doppler* calculations. To do that we introduce an alternative model taking into account four events: the emission from the ground station at $\boldsymbol{x}_1^{GS}$, the reception by the probe at $\boldsymbol{x}_2^{SC}$, the reemission at $\boldsymbol{x}_3^{SC}$ and the reception at ground at $\boldsymbol{x}_4^{GS}$. The additional event $\boldsymbol{x}_3^{SC} = \boldsymbol{x}_{2+\delta t_{23}}^{SC}$ accounts for this small delay of δt_{23}=2.5 μs (for modern spacecrafts) :

$$\rho = \frac{1}{c}|\boldsymbol{x}_4^{GS} - \boldsymbol{x}_3^{SC}| + \delta t_{23} + \frac{1}{c}|\boldsymbol{x}_2^{SC} - \boldsymbol{x}_1^{GS}| + Shapiro + \delta TS. \tag{2}$$

4. COMPARISON OF THE TWO MODELINGS

One main difference between the two modelings consists in considering different numbers of epochs. Indeed we introduced the event $\boldsymbol{x}_3^{SC} = \boldsymbol{x}_{2+\delta t_{23}}^{SC}$; this term is implicitly related to δt_{23} by the approximation $\boldsymbol{x}_3^{SC} = \boldsymbol{x}_2^{SC} + \delta t_{23}\boldsymbol{v}_2^{SC} + O(\delta_{23}^2)$. In practice, this delay is calibrated by space agencies; it is added to ρ when computing *Ranging* and has no consequence on the differential *Doppler* F since δ_{23} is a calibrated constant quantity. However, as δ_{23} appears in the expression of $\boldsymbol{x}_3^{SC}$, we can compute the difference between ρ and ρ' by developing to the first order in δt_{23}. We found that this difference is given by:

$$\Delta\rho = \rho - \rho' = \delta t_{23}\frac{1 - \frac{\boldsymbol{v}_2^{SC}\cdot\boldsymbol{N}_{12}}{c}}{1 - \frac{\boldsymbol{v}_1^{GS}\cdot\boldsymbol{N}_{12}}{c}}, \tag{3}$$

where $\boldsymbol{N}_{12} = \frac{\boldsymbol{x}_2^{SC} - \boldsymbol{x}_1^{GS}}{\|\boldsymbol{x}_2^{SC} - \boldsymbol{x}_1^{GS}\|}$.

The last equation highlights the presence of an extra non-constant term directly linked to the transponder delay. It depends also on positions and velocities of both probe and ground station. Neglecting it would actually lead to a wrong determination of the epoch t_1 and to an error in both *Ranging* and *Doppler*. In fact, extracting the downlink signal, we get $|\boldsymbol{x}_{2'}^{SC} - \boldsymbol{x}_{1'}^{GS}| \neq |\boldsymbol{x}_2^{SC} - \boldsymbol{x}_1^{GS}|$ since $t_2 \neq t'_2$ and $t_1 \neq t'_1$.

5. CONCLUSIONS

It is obvious that the influence of the transponder delay has no reason to be only calibrated. It is responsible of a tiny effect on the computation of light time and has an impact on both *Ranging* and *Doppler* determination. Mainly, to summarize, it leads to consider four epochs instead of three. In order to test the amplitude and variability of this effect on real data, we calculated its influence on some real probe-ground station configurations during recent Earth flybys (NEAR, Rosetta, Cassini and Galileo) and we computed the alternative orbit fitted to *Ranging* and *Doppler* data from our light time model. The results showed an amplitude of some mm (*Ranging*) and mm/s (*Doppler*), but also a variability of the signal for different orbital configurations. A more detailed version of these calculations will be presented in a forthcoming article.

6. REFERENCES

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