TESTING GRAVITY LAW IN THE SOLAR SYSTEM

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ABSTRACT. The predictions of General relativity (GR) are in good agreement with observations in the solar system. Nevertheless, unexpected anomalies appeared during the last decades, along with the increasing precision of measurements. Those anomalies are present in spacecraft tracking data (Pioneer and flyby anomalies) as well as ephemerides. In addition, the whole theory is challenged at galactic and cosmic scales with the dark matter and dark energy issues. Finally, the unification in the framework of quantum field theories remains an open question, whose solution will certainly lead to modifications of the theory, even at large distances. As long as those "dark sides" of the universe have no universally accepted interpretation nor are they observed through other means than the gravitational anomalies they have been designed to cure, these anomalies may as well be interpreted as deviations from GR. In this context, there is a strong motivation for improved and more systematic tests of GR inside the solar system, with the aim to bridge the gap between gravity experiments in the solar system and observations at much larger scales. We review a family of metric extensions of GR which preserve the equivalence principle but modify the coupling between energy and curvature and provide a phenomenological framework which generalizes the PPN framework and "fifth force" extensions of GR. We briefly discuss some possible observational consequences in relation with highly accurate ephemerides.

1. TESTS OF GENERAL RELATIVITY (GR) IN THE SOLAR SYSTEM

The foundations of GR rely on two main pillars. The first one is the equivalence principle which states the universality of free fall and gives gravitation its geometric nature. This principle is tested in modern experiments at the 10^{-12} level, which makes it one of the best tested properties of nature. The validity of the equivalence principle has also been tested very accurately in the solar system using Lunar Laser Ranging [1] or the Sun-Mars orbit [2]. The resulting bound is too small to allow the comparatively large anomalies observed in the solar system. Therefore, even if a violation of the equivalence principle is nevertheless possible, this strongly indicates that the anomalies, if of gravitational origin, should find an explanation in the framework of metric extensions of GR. We focus on theories which describe gravity by a tensor metric field $g_{\mu\nu}$. Let us mention at this point that the general static and isotropic metric, which can be used as a preliminary description of the solar system, essentially reduces to two functions $g_{00}(r)$ and $g_{rr}(r)$ such that

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = g_{00}(c\mathrm{d}t)^2 + g_{rr}\mathbf{d}\boldsymbol{r}^2 \tag{1}$$

The Einstein curvature tensor $G_{\mu\nu}$, built upon the Ricci curvature, has a null covariant divergence (Bianchi identity)

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \qquad , \qquad D^\mu G_{\mu\nu} = 0 \tag{2}$$

This geometrical identity is often put in correlation with the conservation of the stress tensor, $D^\mu T_{\mu\nu} = 0$. This remark leads to the second pillar of GR which are the equations relating the curvature of spacetime to the energy-momentum content. Those equations allow one to determine the metric tensor from the distribution of energy-momentum in spacetime through the Einstein-Hilbert equation which involves a unique Newton gravitational constant G_N

$$G_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} \tag{3}$$

Note that this form is not imposed by any geometrical argument, so that GR has to be selected out from a large variety of metric theories by comparing the predictions drawn from the Einstein-Hilbert equation to the results of observations or experiments. When performed in the solar system, the tests effectively show a good agreement with the solutions of (3), which means that the metric tensor $g_{\mu\nu}$ has a form close to its GR prediction given, for a static point-like mass M by

$$g_{00} = 1 + 2\phi + 2\phi^2 + \ldots \quad , \quad -g_{rr} = 1 - 2\phi + \ldots \quad \text{with} \quad \phi = -\frac{GM}{rc^2} \tag{4}$$

More precisely, the predictions of the Einstein-Hilbert equation in the solar system are often tested in the PPN framework or a Yukawa fifth force framework. The simplest PPN framework is characterized by two constant parameters β and γ inserted in the previous Taylor expansion (4)

$$g_{00} = 1 + 2\phi + 2\beta\phi^2 + \ldots \quad , \quad -g_{rr} = 1 - 2\gamma\phi + \ldots \tag{5}$$

It turns out that 30 years of tests have put stringent bounds on the parameters β and γ and selected a vicinity of GR when analyzed in the PPN framework, namely $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ and $\beta - 1 = (-2.5 \pm 7.5) \times 10^{-5}$. Concerning γ, the current bound is essentially given by the experiment performed through radar ranging of the Cassini probe during its 2002 solar occultation [3], while the bound on β is obtained via analysis of ephemerides [4].

It is important to note that the previous observations have also been tested in other frameworks. For example the so-called "fifth force" tests which focus on a possible scale-dependent deviation from the gravity force law. Such a deviation, corresponding to an additional massive scalar gravity field, reduces to a modification of the Newtonian potential by an additional Yukawa potential

$$g_{00} = [g_{00}]_{\text{GR}} + 2\phi(r)\alpha \exp\left(-\frac{r}{\lambda}\right) \tag{6}$$

with an amplitude α measured with respect to Newton potential and a range λ related to the mass scale of the hypothetical new particle which would mediate the "fifth force". The presence of such a correction has been looked for on a large range of distances and it turns out that the Yukawa term is excluded with a high accuracy at some ranges, for example $\alpha < 10^{-10}$ at $\lambda \simeq$ Earth-Moon distance and $\alpha < 10^{-9}$ at $\lambda \simeq$ Sun-Mars distance. These bounds, again deduced from Lunar Laser Ranging and tracking of planetary probes, correspond to a remarkable result which approaches the accuracy of equivalence principle tests.

2. NEW FRAMEWORK OF METRIC EXTENSION OF GR

We now present a more general framework, which extends the two previously discussed frameworks. As already emphasized, the form of the coupling between energy-momentum and curvature can still be discussed so that we can generalize Einstein-Hilbert equation (3) such that it takes the form of a non local response relation between Einstein curvature and the energy-momentum tensor (see [5,6] for details)

$$G_{\mu\nu}(x) = \int \mathrm{d}^4x' \chi_{\mu\nu}{}^{\rho\sigma}(x - x') T_{\rho\sigma}(x') \tag{7}$$

As an example, we retrieve GR with the following local expression of the susceptibility $\chi_{\mu\nu}{}^{\rho\sigma}$

$$\left[\chi_{\mu\nu}{}^{\rho\sigma}(x - x')\right]_{\text{GR}} = \frac{4\pi G_N}{c^4} \left(\delta^\rho_\mu \delta^\sigma_\nu + \delta^\rho_\nu \delta^\sigma_\mu\right) \delta^{(4)}(x - x') \tag{8}$$

Let us note at this point that generalized response equations naturally arise from radiative corrections of GR [5,6]. Radiative corrections may induce modifications of GR not only at high energies, but also at large distances [7]. In our simple model of solar system, the stress tensor reads $T_{\rho\sigma}(x) = \delta_{\rho 0}\delta_{\sigma 0} M c^2 \delta^{(3)}(\boldsymbol{x})$ so that the modified Einstein tensor reads

$$\delta G_{\mu\nu}(x) = Mc^2 \int c\mathrm{d}t' \; \delta\chi_{\mu\nu}{}^{00}(x - x') \quad , \quad x' = (ct', \mathbf{0}) \tag{9}$$

An important feature of this framework is that the Einstein tensor $G_{\mu\nu}$, and therefore the Ricci curvature $R_{\mu\nu}$ do no longer vanish outside the sources, as it is the case in GR. Nevertheless, because GR is in good accordance with observations, it is expected that $\delta G_{\mu\nu}$ should stay small. Within a linear approximation, the Bianchi identity can be used to extract the two degrees of freedom $\delta G^{(0)}$ and $\delta G^{(1)}$ which are present :

$$\delta G_{\mu\nu}(r) = \delta G^{(0)}_{\mu\nu}(r) + \delta G^{(1)}_{\mu\nu}(r) = \pi^{(0)}_{\mu\nu 00} \frac{8\pi M \delta G^{(0)}(r)}{c^2} + \pi^{(1)}_{\mu\nu 00} \frac{8\pi M \delta G^{(1)}(r)}{c^2} \tag{10}$$

where $\pi^{(0)}_{\mu\nu 00}$ and $\pi^{(1)}_{\mu\nu 00}$ are operators of projection on the two sectors of different conformal weights, that is to say the traceless and traced part (see [5] for their expression). It has been shown in [6] how these two functions $\delta G^{(0)}$ and $\delta G^{(1)}$ are related to a modification δw of $w \equiv \frac{1}{2} \ln |g_{rr}/g_{00}|$ (which is equivalent to a modification of the Weyl tensor), and a modification δR of the Ricci scalar R

$$\delta G^{(0)}(r) = -\frac{c^2}{8\pi M} \left(\partial_r^2 + \frac{2}{r} \partial_r \right) \delta w(r) \qquad , \qquad \delta G^{(1)}(r) = -\frac{c^2}{8\pi M} \delta R(r) \tag{11}$$

Generally speaking, in the present framework, the gravitational constant G_N is replaced by a non local susceptibility tensor of rank 4 which reduces to only two functions $\delta G^{(0)}$ and $\delta G^{(1)}$. Those two functions correspond to two different sectors for gravitation which, in the simplified situation considered here, can be exactly matched to the two degree of freedom g_{00} and g_{rr} with the help of (11).

$$g_{00} = [g_{00}]_{\mathrm{GR}} + \delta g_{00} \qquad , \qquad g_{rr} = [g_{rr}]_{\mathrm{GR}} + \delta g_{rr} \tag{12}$$

The functions $\delta g_{00}(r)$ and $\delta g_{rr}(r)$ can have a general r-dependence. As a pedagogical example, we will focus in the following on simplified phenomenological models obtained as a Taylor expansion of δg_{00} and δg_{rr} in a vicinity of the Solar system ($\alpha_n r^n \ll 1$ and $\chi_n r^n \ll 1$)

$$\delta g_{00} = 2 \sum_{n>0} \alpha_n r^n \qquad , \qquad \delta g_{rr} = 2 \sum_{n>0} \chi_n r^n \tag{13}$$

More elaborated models can be obtained by adding a cutoff to allow the perturbation to start from a given distance from the sun. The previous models can be viewed equivalently as distance-dependent PPN parameters

$$\beta(r) - 1 = \left(\frac{c^2}{GM} \right)^2 \sum_{n>0} \alpha_n r^{n+2} \qquad , \qquad \gamma(r) - 1 = -\frac{c^2}{GM} \sum_{n>0} \chi_n r^{n+1} \tag{14}$$

3. PHENOMENOLOGICAL CONSEQUENCES ON A SIMPLIFIED MODEL

As emphasized in the previous section, modifications of g_{00} are heavily constrained. Therefore, we will consider in the following a modification of the spatial part g_{rr} alone. Then, even if we can deal with the general case, we further restrict ourselves to the following simple modification

$$\delta g_{rr} = 2\chi_2 r^2 \tag{15}$$

This specific model corresponds to a constant Ricci curvature $R_{rr} = 8\chi_2$, so that χ_2 represents the amount of Ricci curvature outside the source. The geodesic equation is modified, as well as the Shapiro time delay, leading to anomalies in spacecraft tracking and ephemerides. For example, a test particle (spacecraft or planet) is submitted to the following anomalous coordinate acceleration

$$\delta a_r = 2\chi_2 \left(r v_r^2 - r v_\theta^2 - GM \right) \quad , \quad \delta a_\theta = 4\chi_2 \, r v_r v_\theta \tag{16}$$

It is easy to verify that if $\chi_2 > 0$, a purely radial escape trajectory suffers a positive radial acceleration (towards the outside of the solar system) while a circular orbit suffers a negative radial acceleration (towards the sun). Moreover, the orthoradial acceleration δa_θ vanishes for circular orbits and radial escape trajectories. The order of magnitude of the anomalous acceleration is given by

$$\delta a \sim \chi_2 GM \tag{17}$$

This anomalous coordinate acceleration is of course not the observable that is deduced from measurement. Indeed, one has to take into account the effect on light propagation. The modification $\delta\mathcal{T}$ of the one-way light-time $\mathcal{T} = t_2 - t_1$ from position $\boldsymbol{r}_1(t_1)$ to position $\boldsymbol{r}_2(t_2)$ can be computed by integrating $\mathrm{d}s^2 = 0$ along the path, giving

$$c\mathcal{T} = R_{12} + c\delta\mathcal{T}_{\text{Shapiro}} + c\delta\mathcal{T} \quad , \quad c\delta\mathcal{T} = -\frac{1}{3}\chi_2 R_{12}\left(r_1^2 + r_2^2 + \boldsymbol{r}_1 \cdot \boldsymbol{r}_2\right) \tag{18}$$

where $r_i = |\boldsymbol{r}_i| = [r_i]_{\text{GR}} + \delta r_i$ and $R_{12} = |\boldsymbol{r}_2 - \boldsymbol{r}_1| = [R_{12}]_{\text{GR}} + \delta R_{12}$ are modified when compared with their GR values due to the anomalous coordinate acceleration (16). The tracking observables can then be computed from the knowledge of the two-way light-time (a sum of two one-way light-time connected at the level of the spacecraft). For example, up to small relativistic corrections, the Doppler signal is obtained by the time derivative of the light-time while the position in the sky (right ascension and declination) is obtained through spatial derivatives.

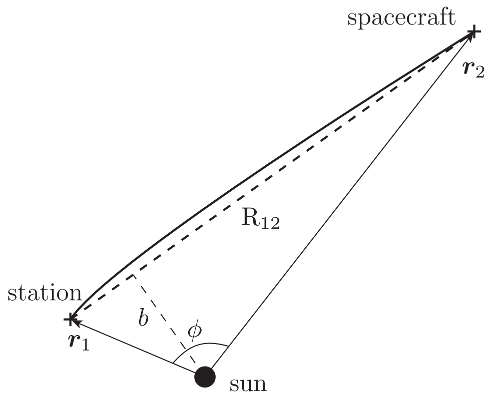

Figure 1: Geometry of spacecraft tracking.

An important feature is that the light-time correction naturally produces daily and semi-annual modulations, due to the geometry of the triangle sun-station-spacecraft. Those modulations will show up in all observables, in particular in ranging. Therefore, correlated anomalies should be observed in ephemerides and analyzed within this framework. As a matter of fact, only a careful comparison of observations in the outer solar system, within the post-Einsteinian phenomenological framework, could allow one to determine whether all gravity tests can be compatible with the anomalies seen in the solar system. This comparison will have to account for the presence of the two sectors as well as for their r-dependences.

4. REFERENCES

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