AN INERTIAL EFFECT IN SATELLITE MOTION NOT DESCRIBED BY THE CURRENT IERS CONVENTIONS

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ABSTRACT. We recall a known inertial effect in satellite motion caused by the indirect acceleration of the center of integration (the central planet) due to the oblateness of the planet and an attracting third-body. As estimated, the effect leads to perturbations in satellite motion to be well detectable by the modern tools of measuring the satellite orbital parameters. However, the effect is not described by the current IERS Conventions (2003); we suggest to include it to the future IERS Conventions (2010).

1. FORMULATION OF THE EFFECT

Let's consider the motion of an artificial satellite T_1 in the gravitational fields of the oblate central planet T_0 and an attracting oblate third-body T_2. The gravitational potential between T_0 and T_2 is

$$U_{02} = f \iiint_{T_0} dm_0 \iiint_{T_2} \frac{dm_2}{r''_{20}}, \quad (1)$$

where r''_{20} is the distance between an elementary mass dm_2 of the body T_2 and an elementary mass dm_0 of the body T_0, and f is the gravitational constant.

The Eq. (1) can be expanded as follows

$$\begin{aligned} U_{02} &= f\frac{M_0 M_2}{r_{02}}\Bigg\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_0}{r_{02}}\right)^n P_{nm}(\sin\phi_{02})\left[C_{nm}^{(0)}\cos m\lambda_{02} + S_{nm}^{(0)}\sin m\lambda_{02}\right] \\ &+ \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_2}{r_{20}}\right)^n P_{nm}(\sin\phi_{20})\left[C_{nm}^{(2)}\cos m\lambda_{20} + S_{nm}^{(2)}\sin m\lambda_{20}\right] + O\left(C_{20}^{(2)}\frac{R_2^2 R_0}{r_{20}^3}\right)\Bigg\}, \quad (2) \end{aligned}$$

where hereafter r_{ij}, ϕ_{ij}, λ_{ij} are spherical coordinates of the mass center of body T_j in the reference frame of body T_i; $C_{nm}^{(i)}$ and $S_{nm}^{(i)}$ are the coefficients of the expansion of gravitational potential of body T_i in spherical functions; M_i, R_i are the mass and mean equatorial radius of body T_i; P_{nm} are associated Legendre polynomials, and $i, j = 0, 1, 2$.

Then the motion equations of the artificial satellite T_1 in an inertial reference frame are as follows.

$$\ddot{x}_{01} \equiv \ddot{x}_1 - \ddot{x}_0 = -fM_0\frac{x_{01}}{r_{01}^3} + fM_2\left(\frac{x_{02}-x_{01}}{r_{21}^3} - \frac{x_{02}}{r_{20}^3}\right) + \frac{\partial R_{01}}{\partial x_{01}} + \frac{\partial R_{21}}{\partial x_{21}} + \frac{\partial R_{02}}{\partial x_{02}} - \frac{\partial R_{20}}{\partial x_{20}}, \quad (3)$$

where

$$R_{01} \equiv fM_0\frac{1}{r_{01}}\left\{\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_0}{r_{01}}\right)^n P_{nm}(\sin\phi_{01}) \times \left[C_{nm}^{(0)}\cos m\lambda_{01} + S_{nm}^{(0)}\sin m\lambda_{01}\right]\right\}, \quad (4)$$

$$R_{21} \equiv fM_2\frac{1}{r_{21}}\left\{\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_2}{r_{21}}\right)^n P_{nm}(\sin\phi_{21}) \times \left[C_{nm}^{(2)}\cos m\lambda_{21} + S_{nm}^{(2)}\sin m\lambda_{21}\right]\right\}, \quad (5)$$

$$R_{02} \equiv fM_2\frac{1}{r_{02}}\left\{\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R_0}{r_{02}}\right)^n P_{nm}(\sin\phi_{02}) \times \left[C_{nm}^{(0)}\cos m\lambda_{02} + S_{nm}^{(0)}\sin m\lambda_{02}\right]\right\}, \quad (6)$$

$$R_{20} \equiv f M_2 \frac{1}{r_{20}} \left\{ \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left(\frac{R_2}{r_{20}} \right)^n P_{nm} \left(\sin \phi_{20} \right) \times \left[C_{nm}^{(2)} \cos m\lambda_{20} + S_{nm}^{(2)} \sin m\lambda_{20} \right] \right\}, \tag{7}$$

and x_i is Cartesian x-coordinate of body T_i in the inertial reference frame; $x_{ij} \equiv x_j - x_i$.

The last two summands in the right-hand side of Eq. (3) describe the inertial terms in satellite motion caused by the additional accelerations of the central planet due to oblateness of the planet and that of the third-body, respectively. The corresponding expressions for accelerations $\ddot{y}_{01}$ and $\ddot{z}_{01}$ of the other two Cartesian coordinates of the satellite are similar to Eq. (3). Also, from Eq. (6) one can conclude that the inertial term caused by the Moon attraction on the oblate Earth is some 10^3 times more than a similar term caused by the Sun attraction.

2. DISCUSSION OF THE EFFECT

Table 1 presents the effect of the considered inertial terms in Keplerian elements of the LAGEOS and ETALON geodynamical satellites over one year interval. The effect is periodic; variations of maximum amplitude has a period close to that of the lunar orbital motion. Here a, e, ω are the semimajor axis, eccentricity and argument of satellite perigee, respectively; $(e\cos\omega, e\sin\omega)$ is the eccentricity vector.

Satellite	Start of time interval	$\Delta e \times a$ [cm]	$e\Delta\omega \times a$ [cm]	$\frac{\partial}{\partial t}(e\cos\omega)$ [mas/yr]	$\frac{\partial}{\partial t}(e\sin\omega)$ [mas/yr]
LAGEOS-1	1988/01/07	3	6	28	29
LAGEOS-2	1993/01/01	7	5	30	29
ETALON-1	1992/06/01	15	25	32	41
ETALON-2	1992/06/01	27	13	38	44

Table 1: Maximum variations in satellite Keplerian elements due to the inertial effect.

One sees the discussed effect is large enough to be detectable by the current tools of measuring the satellite orbital parameters, e.g. by laser technique. This "indirect oblateness effect" is known in the satellite dynamics in that or another form since the beginning of spaceflights (Sturms 1964, Moyer 1971). However, the current IERS Conventions (McCarthy and Petit 2004) do not describe the corresponding inertial terms. The online documentation of the advanced GEODYN II system for satellite orbit determination (Pavlis et al. 2010) does not mention the discussed terms as well, although the software itself does take them into account (Rowlands 2010, private communication).

The IERS Conventions are very detailed in describing the satellite force model, and for long time are deservedly a standard for developers of satellite dynamics software. The absence of the inertial terms in this standard might mislead its potential users, therefore we suggest to include a description of these terms to the future IERS Conventions (2010).

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