# WAVELET BASED COMPARISON OF HIGH FREQUENCY OSCILLATIONS IN THE GEODETIC AND FLUID EXCITATION FUNCTIONS OF POLAR MOTION

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ABSTRACT. It has been already shown that short period oscillations in polar motion, with periods less than 100 days, are very chaotic and are responsible for increase in short-term prediction errors of pole coordinates data. The wavelet technique enables to compare the geodetic and fluid excitation functions in the high frequency band in many different ways, e.g. by looking at the semblance function. The wavelet-based semblance filtering enables determination the common signal in both geodetic and fluid excitation time series. In this paper the considered fluid excitation functions consist of the atmospheric, oceanic and land hydrology excitation functions from ECMWF atmospheric data produced by IERS Associated Product Centre Deutsches GeoForschungsZentrum, Potsdam. The geodetic excitation functions have been computed from the combined IERS pole coordinates data.

## 1. INTRODUCTION

Chaotic oscillations in Earth Orientation Parameters (EOP), driven by exchange of angular momentum between the solid Earth and geophysical fluids, are main causes of increase in their short-term prediction errors (Kosek et al. 2009; Kosek 2010). The EOP predictions are necessary for real time transformation between the terrestrial and celestial reference frames, however the prediction estimates of $x$, $y$ pole coordinates and UT1-UTC data are still not satisfactory. To increase the EOP prediction accuracy the international cooperation have led to the EOPPCC (Earth Orientation Parameters Prediction Comparison Campaign) in 2005-2008 (Kalarus et al. 2010), IERS Working Group on Prediction in 2006-2009 and recently Earth Orientation Parameters Combination Prediction Pilot Project which started in October 2010. The wavelet coefficients related to translation and scale parameters, computed for any time series, describe its non-stationary character. As a result of using them, it is possible to compare two time series in time-frequency domain (Kosek 2010) and it is also feasible to filter common signals in the two time series using wavelet semblance filtering (Cooper and Cowan 2008; Cooper 2009). In this paper, the semblance function based on the wavelet transform coefficients is applied to show (1) time-frequency similarities in pole coordinates data and pole coordinates model data as well as (2) common oscillations in geodetic and fluid excitation functions which consists of the atmospheric, ocean and land hydrology excitation functions.

## 2. DATA

The following data sets were used in the analysis: (1) equatorial components of the effective angular momentum functions of the atmosphere (AAM), ocean (OAM) and land hydrology (HAM) from ECMWF atmospheric data produced by IERS Associated Product Centre Deutsches GeoForschungsZentrum (GFZ) Potsdam, from 1 January 1989 to 31 December 2009 (Dill 2008; Dobslaw and Thomas 2007); (2) IERS pole coordinates data $x$, $y$ EOPC04_IAU2000.62-now, between 1962 and 2010 (IERS 2010).

## 3. COMPARISON OF POLE COORDINATES DATA AND THEIR FLUID EXCITATION FUNCTIONS

In order to show how fluid excitation functions excite polar motion, the geodetic excitation functions can be computed from pole coordinates data or the model pole coordinates data can be calculated from fluid excitation functions. The geodetic excitation functions were computed from the IERSC04 pole coordinates data using the time domain, Wilson and Haubrich (1976), deconvolution formula, in which the Chandler period was equal to 435 days and the quality factor $Q = 100$. The pole coordinates model data were computed from the equatorial components of fluid excitation functions using the trapezoidal rule of numerical integration (Kosek et al. 2009). The fluid excitation functions consist of the AAM or the sums of AAM+OAM and AAM+OAM+HAM excitations.

A comparison of the pole coordinates data and the pole coordinates model data computed from fluid excitation functions was done using the semblance functions. The semblance function of the order $r$ between two complex-valued time series $x(t)$ and $y(t)$ is computed by the following formula:

$$\hat{\vartheta}^r_{xy}(a) = \hat{S}_{xy}(a)cos^r(\Delta\hat{\phi}_{xy}(a))/\sqrt{\hat{S}_{xx}(a)\hat{S}_{yy}(a)} \quad (1)$$

where $\hat{S}_{xy}(a) = \sum_{b=0}^{n-1} \hat{X}(b,a)\overline{\hat{Y}(b,a)}$ is the wavelet cross-spectrum between $x(t)$ and $y(t)$ time series, $\hat{S}_{xx}(a) = \sum_{b=0}^{n-1} |\hat{X}(b,a)|^2$ and $\hat{S}_{yy}(a) = \sum_{b=0}^{n-1} |\hat{Y}(b,a)|^2$ are the wavelet spectra of $x(t)$ and $y(t)$, respectively, and

$$\Delta\hat{\phi}_{xy}(a) = arg\left[\frac{1}{n}\sum_{b=0}^{n-1}[\hat{X}(b,a)\overline{\hat{Y}(b,a)}]/[|\hat{X}(b,a)||\hat{Y}(b,a)|]\right] \quad (2)$$

is the phase synchronization function between $x(t)$ and $y(t)$ time series. The wavelet transform coefficients of $x(t)$ (or $y(t)$) are computed by the following formula:

$$\hat{X}(b,a) = \sqrt{|a|}/n \sum_{\nu=-n/2+1}^{n/2} \hat{x}(\nu)\overline{\tilde{\varphi}}(a2\pi\nu/n)\exp(i2\pi b\nu/n)\ , \quad (3)$$

where $a \neq 0$ and $b = 0, 1, \ldots, n-1$, are dilation and translation parameters, respectively, $\hat{x}(\nu)$ is the Discrete Fourier Transform of $x(t)$, $t = 0, 1, \ldots, n-1$, $\tilde{\varphi}(\omega)$ is the Continuous Fourier Transform of the Morlet wavelet function $\varphi(t) \approx \exp(-t^2/2)\exp(i2\pi t)/\sqrt{2\pi}$ (Goupillaud et al. 1984).

Figure 1 shows the Wavelet Transform semblance functions of the orders 0 and 3 between the considered geodetic and fluid excitation functions. The semblance functions increase when ocean excitation is added to the atmospheric one, and become still greater when the land hydrology excitation is taken into account. Moreover, adding hydrology excitation function to the sum of atmospheric and ocean terms improves the phase agreement between the geodetic and fluid excitation in the annual frequency band. This is because the semblance function of the order 3 attains higher values in the aforementioned frequency band after including land hydrology excitation.

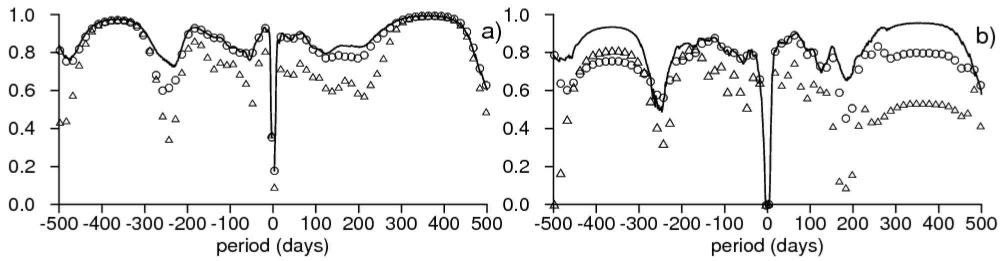


Figure 1: The semblance functions of the orders "0" a) and "3" b) between the geodetic and fluid excitation functions AAM (triangles), AAM+OAM (circles) and AAM+OAM+HAM (solid line).

## 4. WAVELET BASED SEMBLANCE FILTERING

The wavelet semblance function of two discrete complex-valued signals $x(t), y(t),\ \ t = 0, 1, ..., n-1$, where $n = 2^p$, is defined for scale index $j$ and translation index $k$ as

$$\vartheta^{xy}_{j,k} = cos(\theta) = Re\left[S^x_{j,k}S^y_{j,k}/(|S^x_{j,k}||S^y_{j,k}|)\right], \quad (4)$$

where $S^x_{j,k} = \sum_{t=0}^{n-1} x(t)\overline{\varphi}_{j,k}(t)$, $S^y_{j,k} = \sum_{t=0}^{n-1} y(t)\overline{\varphi}_{j,k}(t)$ are the wavelet transform coefficients of $x(t)$, $y(t)$ time series, respectively, $\theta$ is interpreted as the angle between $S^x_{j,k}$ and $S^x_{j,k}$ in a complex plane,

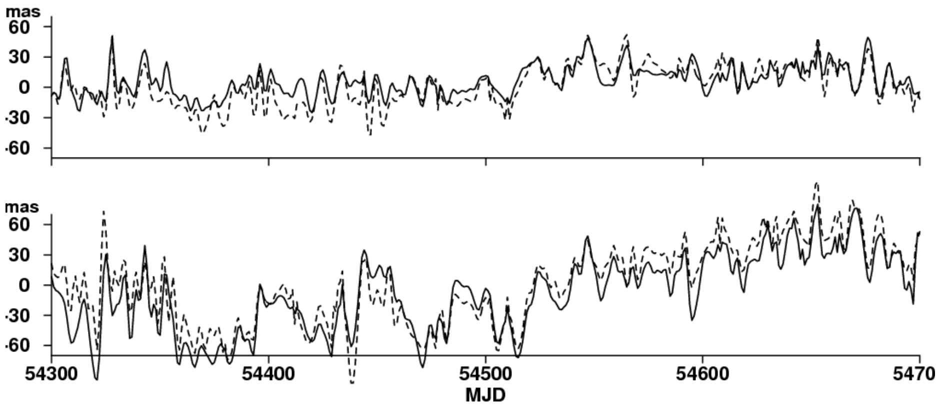


Figure 2: An example of common oscillations in the equatorial components ($\psi_1/\chi_1$ - upper graph, $\psi_2/\chi_2$ - lower graph) of geodetic (solid line) and fluid AAM+OAM+HAM (dashed line) excitation functions, computed using wavelet based semblance filtering with threshold value equal to 0.90.

$\varphi_{j,k}(t)$ is the discrete Shannon wavelet function (Frazier and Torres 1994). For a fixed lowest frequency index $0 \leq j_o \leq p-2$ and time index $k = -2^{j_o}, -2^{j_o}+1, ..., 2^{j_o}-1$, the Shannon wavelet function is given by:

$$\varphi_{j_o}(t) = exp[-i\pi(t-n/2)/n]\frac{sin[2^{j_o+1}\pi(t-n/2)/n]}{n \cdot sin[\pi(t-n/2)/n]}, \qquad \varphi_{j_o}(n/2) = 2^{j_o+1}/n, \tag{5}$$

and for higher frequency index $j = j_o+1, j_o+2, ..., p-1$, and time index $k = -2^{j-1}, -2^{j-1}+1, ..., 2^{j-1}-1$, this function is given by:

$$\varphi_j(t) = exp[-i\pi(t-n/2)/n]\frac{sin[2^j\pi(t-n/2)/n]\{2cos[2^j\pi(t-n/2)/n]-1\}}{n \cdot sin[\pi(t-n/2)/n]}, \qquad \varphi_j(n/2) = 2^j/n. \tag{6}$$

The semblance function varies in the interval $< -1, 1 >$ and detects oscillations in two time series, which are out of phase ($-1$) and in phase ($1$). Semblance filtering is performed by keeping in the reconstruction formulas of both time series $x(t) = \sum_{j=j_o}^{p-1}\sum_{k=-2^{j-1}}^{2^{j-1}-1} S^x_{j,k}\varphi_{j,k}(t)$, $y(t) = \sum_{j=j_o}^{p-1}\sum_{k=-2^{j-1}}^{2^{j-1}-1} S^x_{j,k}\varphi_{j,k}(t)$ only the wavelet transform coefficients $\widetilde{S}^x_{j,k}$, $\widetilde{S}^y_{j,k}$ for which the semblance $\vartheta^{xy}_{j,k}$ exceeds a given threshold (e.g. equal to 0.90 or 0.99). Other wavelet transform coefficients of both time series, for which the semblance is below the adopted threshold, are set to zero. The common signals in $x(t)$ and $y(t)$ are computed using the following reconstruction formulas:

$$\widetilde{x}(t) = \sum_{j=j_o}^{p-1}\sum_{k=-2^{j-1}}^{2^{j-1}-1}\widetilde{S}^x_{j,k}\varphi_{j,k}(t)\ , \qquad \widetilde{y}(t) = \sum_{j=j_o}^{p-1}\sum_{k=-2^{j-1}}^{2^{j-1}-1}\widetilde{S}^y_{j,k}\varphi_{j,k}(t). \tag{7}$$

Using semblance filtering, with the discrete Shannon wavelet functions, it is possible to filter from both time series the variations which are characterized by a good phase agreement. Such an approach enables determination of common oscillations in all possible frequency bands in two complex-valued time series. Figure 2 shows an example of the common oscillations in the geodetic and fluid excitation functions which consist of the sum of atmospheric, ocean and land hydrology excitations. The results of wavelet semblance filtering indicate that the filtered common oscillations in the geodetic and fluid excitation functions in all frequency bands behave very similarly.

Table 1 shows the correlation coefficients in 1989.0-2009.0 between the geodetic and fluid excitation functions, computed using wavelet based semblance filtering for threshold values equal to 0.00, 0.90 and 0.99. When the threshold value is equal to zero, then the excitation functions data are not filtered. It can be noticed that increase in the threshold values increases the correlation coefficients. In addition, correlation coefficients become greater when the fluid excitation function is composed of more components.

| threshold | 0.00 | | 0.90 | | 0.99 | |
|---|---|---|---|---|---|---|
| Fluid excitation | $\psi_1/\chi_1$ | $\psi_2/\chi_2$ | $\psi_1/\chi_1$ | $\psi_2/\chi_2$ | $\psi_1/\chi_1$ | $\psi_2/\chi_2$ |
| AAM | 0.441 | 0.642 | 0.773 | 0.856 | 0.828 | 0.866 |
| AAM+OAM | 0.553 | 0.710 | 0.823 | 0.895 | 0.868 | 0.942 |
| AAM+OAM+HAM | 0.562 | 0.728 | 0.830 | 0.909 | 0.883 | 0.926 |

Table 1: Correlation coefficients in 1989-2009 between the geodetic $\psi_1$, $\psi_2$ and fluid $\chi_1$, $\chi_2$ excitation functions, computed using wavelet based semblance filtering for threshold values equal to 0.0, 0.9 and 0.99.

## 5. CONCLUSIONS

Wavelet based techniques allow one to perform a time-frequency comparison of the geodetic and fluid excitation functions. These functions become the most similar when the fluid excitation function is composed of the atmospheric, ocean and land hydrology excitation functions. Higher order semblance function reveals that addition of hydrology angular momentum to the sum of atmospheric and oceanic ones improves the phase agreement between the geodetic and fluid excitation functions in the annual frequency band. The common oscillations in the geodetic and fluid excitation functions can be detected using wavelet based semblance filtering. Increase in the threshold value in the semblance filtering of the geodetic and fluid excitation functions increases the correlation coefficients values between filtered common oscillations.

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