CHANDLER WOBBLE AND FREE CORE NUTATION OF SINGLE PULSAR

A. GUSEV
Kazan Federal University
18, Kremlevskaya, Kazan, 420008, Russia
e-mail: Alexander.Gusev@ksu.ru

ABSTRACT. PSR B1828-11 has long-term, highly periodic and correlated variations pulse shape and of the rate of slow-down with period variations approximately 1000, 500 and 250 days (Stairs et al., 2000). There are three potential explanations of pulses time-of-arrival from pulsar concerned with the interior of the neutron star, planetary bodies, free precession and nutation. We use the Hamiltonian canonical method of Getino et al., (1999) for the dynamically symmetrical pulsar consisting of the rigid crust, elliptical liquid outer core and solid inner core of PSR B1828-11. Correctly extending theory of differential rotation of a pulsar, we investigated dependence on Chandler wobble period, Inner Chandler Wobble, retrograde Free Core Nutation and prograde Free Inner Core Nutation from ellipticity of inner crystal core, outer liquid core and total pulsar.

1. INTRODUCTION

Observation of pulsars is a powerful source of the information for research of dynamics and internal structure of neutron stars. It's known, that innate feature of pulsar radiation is high stability of time-of-arrival (TOA) of pulses, and therefore the analysis of the TOA fluctuations can be reflection of thin effects of neutron stars dynamics. A small number of neutron stars also exhibit long-term cyclical but not precisely oscillatory variations in their spin (Sedrakian et al., 1999 and references there). For example, the Crab pulsar has very systematically phase residuals with peak-to-peak range of order ± 10 ms and characteristic cycle duration of about 20 months (Gusev,Kitiashvili, 2004). Observation of Vela pulsar's glitch in 1988 showed suppressing of oscillatory phase residuals with a period of order 25 days (Alpar et al., 1990); evidence for oscillations (Tkachenko oscillation) in the frequency derivative of the pulsar both before and after the glitch with a period about 25 days was also reported (Gusev, Kitiashvili, 2005); the evidence of frequency derivative oscillations (Tkachenko oscillation) of the pulsar both before and after the glitch with a period about 25 days was also reported (Gusev,Kitiashvili, 2008). The long-term variations (correlation times 100 days) in the pulse shape of the Vela pulsar has been found in data spanning approximately 4 yr (Sedrakian et al., 1999). The analysis of the pulse shape of PSR 1642-03 has allowed picking out the evidence of cyclical pulse shape variations with a period about 1000 days (Sedrakian et al., 1999). TOA variations of pulsars can be interpreted by three reasons: gravitational perturbation by planetary bodies, peculiarities of a pulsar interior like Tkachenko oscillations and free precession motion, when axis of rotation do not coincide with vectors of the angular moment of solid crust, liquid outer core and crystal core.

Table 1: Some parameters of PSR B1828-11.

Period, ms	405.039883158(8)	
Dispersion measure, cm^{-3}pc	159.7(10)	
Characteristic age, Myr	0.11	
Long-term variation	167	250
in the pulse shape, days	500	1000

The radial velocity of a star is obtained by measuring the magnitude of the Doppler effect in its spectrum. Stars showing a small amplitude variation of the radial velocity can be interpreted as systems having planetary companions. Assuming that the pulsar has a mass of $1.35 M_{\odot}$, the Keplerian orbital radii identified with the three harmonically related sinusoids are 0.9, 1.4 and 2.1 AU and with masses are $3.1M_{\oplus}/sin(i)$, $10.2M_{\oplus}/sin(i)$, $4.6M_{\oplus}/sin(i)$, where i is the orbital inclination.

The second explanation are the periods of Tkachenko oscillations (Tkachenko, 1966) of the neutron superfluid vortex array, which carries much of the angular momentum of the neutron star, depends on the size of the star and the square root of the pulse period. Tkachenko oscillations for the Crab pulsar (PSR 0531+21) are expected 4 months; the period of the Tkachenko oscillations for the PSR B1828-11 are waited 13 months (Ruderman, 1970). Unfortunately, the theory of the Tkachenko oscillations does not easily explain the existence of multiple harmonics in the PSR B1828-11 timing residuals. The Crab pulsar has quasi-periodical variations from 15 to 30 months (Lyne & Graham-Smith, 1998).

The third case, more possible interpretation of TOA variations is a free precession of the pulsar. The time scale for the precession depends on the pulsar deformation degree. The wobble angle θ between the star's symmetry axis and its angular momentum can be estimated by the pulse profile changes. The difference in widths between the "wide" and "narrow" beam profiles is 20 (Link & Epstein, 2001). Interpretation of TOA variations by free precession gives a wobble angle 0.3^0 for the PSR B1828-11 (Stairs et al., 2000) and $0.3^0 - 0.8^0$ for the PSR B1642-03 (Shabanova, 2000; Shabanova et al., 2001). Using observed variations some authors calculated estimates of oblateness for different periods of PSR B1828-11 by: 1000 days $\sim 5 \cdot 10^{-9}$ (Stairs et al., 2000); 500 days $\sim 10^{-8}$ (Link & Cutler, 2002); 511 days - $\epsilon = 9 \cdot 10^{-9}$ (Link & Epstein, 2001).

We propose the explanation for all harmonics of TOA pulses variations as precession of a neutron star owing to differential rotation of crust, outer liquid core and inner crystal core of the pulsar.

2. HAMILTON APPROXIMATION

2.1 Two-layer model

At present time there are investigations of precession and nutation for very different celestial bodies: the Earth (Getino, 1995), Moon (Gusev, 2010), planets of Solar system (Gusev, 2010) and pulsars (Sedrakian et al., 1999; Link & Epstein, 2001; Wasserman, 2003; Cutler et al., 2003).

A correct application of the Hamiltonian method to the problem of the non-rigid Earth (Kinoshita, 1977; Getino, 1995) is a natural way for obtaining an analytical theory of the rotation pulsar which is more appropriate to the real neutron star. Rotation of the Earth-like planets of the Solar system, which have rigid mantle and elliptic liquid core, is characterized by free core nutation (Van Hoolst, 2000).

According to Getino approximation (Getino, 1995), core-mantle interaction can be presented as deformation of inertia tensor of the core (inertional deformation) and free motion of rigid crust and liquid core; interaction can be detected for a definite coordinate frame.

2.2 Three-layer model

The next step in the investigation of rotational variation of the pulsar is the research of the three-layer model. Different authors described rotational variations in the frame of the three-layer model for planets of the Solar system (Dehant et al., 2003; Getino et al., 1999) that have a rigid mantle, a liquid outer core and a solid inner core. The three-layer model is more complicated than the previous case therefore classical methods fail. Getino and Ferrandiz (Getino et al., 1999) developed a canonical formulation for an Earth model, which includes three layers: an axis symmetrical rigid mantle, a fluid outer core (FOC) and a solid inner core (SIC). Flattened of the pulsar, core, FOC and SIC are

$$\begin{aligned} e &= \frac{C-A}{A} - Total pulsar, & e_f &= \frac{C_f - A_f}{A_f} - FOC \\ e_c &= \frac{C_c - A_c}{A_c} - Core, & e_s &= \frac{C_s - A_s}{A_s} - SIC \end{aligned} \tag{1}$$

Here $A, C, A_f, C_f, A_s, C_s, A_c, C_c$ are moments of inertia of the pulsar, fluid outer, solid inner and total cores and of the neutron star accordingly.

$$P_{CW} = P_{PSR}\frac{A_{cr}}{C-A} = P_{PSR}\frac{A_{cr}}{A}\left(\frac{C}{A}-1\right)^{-1} P_{FCN} = -P_{PSR}\frac{A_{cr}}{A}\left(\frac{C_c}{A_c}-1\right)^{-1} \tag{2}$$

We are used four models of pulsar (tabl. 4) for modelling obligates of inner and outer cores and total pulsar (tabl. 5) with mass $\sim 1.4M_{\odot}$ and moment inertia $\sim 10^{44}$ g cm^2. Getino's investigation (Getino et al., 1999) showed that interaction between rigid mantle, FOC and SIC can be characterized by

Table 2: Four models of inner structure for PSR B1828-11

	R_{PSR}, km	h_{cr} km	A_{cr}, g cm^2	h_f, km	A_f, g cm^2	h_s, km	A_s, g cm^2
Model 1	10	0,1	$9 \cdot 10^{39}$	8,9	10^{44}	1	$2 \cdot 10^{40}$
Model 2	11	0,5	$7 \cdot 10^{40}$	10,5	$2,6 \cdot 10^{44}$	1	$2 \cdot 10^{40}$
Model 3	11	1	$9 \cdot 10^{39}$	9	10^{44}	1	10^{41}
Model 4	11	2	$2 \cdot 10^{40}$	8	10^{44}	1	$2 \cdot 10^{41}$

Table 3: Estimates of dynamic ellipticities of crust, mantle and core for PSR B1828-11.

	Model 1	Model 2	Model 3	Model 4
$P_{CW} = 167$ days	$\epsilon = 5,8 \cdot 10^{-12}$	$2 \cdot 10^{-11}$	$4,5 \cdot 10^{-11}$	$6 \cdot 10^{-11}$
$P_{RFCN} = 250$ days	$\epsilon_f = 2 \cdot 10^{-8}$	$2 \cdot 10^{-11}$	$2 \cdot 10^{-11}$	$3,75 \cdot 10^{-11}$
$P_{PFCN} = 500$ days	$\epsilon_s = 1,5 \cdot 10^{-8}$	$1,5 \cdot 10^{-8}$	$1,5 \cdot 10^{-8}$	$1,5 \cdot 10^{-8}$
$P_{ICW} = 1000$ days	$\delta = -9,4 \cdot 10^{-9}$	-10^{-8}	-10^{-8}	-10^{-8}

four modes of periodic variations of rotation pole: Chandler wobble (CW), retrograde free core nutation (FCN), prograde free core nutation (FICN) and inner Chandler wobble (ICW). Thus we can explain all four TOA periodic variations of the PSR B1828-11. In case rotation of a three-layer neutron star we have variations of next types:

- The *Chandler Wobble* (CW) is a motion of the pulsar rotation axis around its dynamical figure due to the bulges of the pulsar. It is the only global rotational mode for completely solid pulsar.
- The *Free Core Nutation* (FCN) is a differential rotation of the liquid core relatively the crust rotation. This mode does exist only if the core is liquid.
- The *Free Inner Core Nutation* (FICN) is a mode related to the differential rotation of the inner core with respect to the other layers of the pulsar. The mode exists only if the pulsar has two-layer core contains outer liquid and inner solid components.
- The *Inner Core Wobble* (ICW) is a differential rotation of the figure axis of the pulsar core with respect to the rotation axis of the pulsar and is due to the flattened of the inner core, having an excess of density with respect to the liquid core. This mode does exist only if there is an ellipsoidal solid inner core inside a liquid core in the pulsar.

2.3 Application of three-layer model for PSR B1828-11

In the frame of the three-layer model we investigate the free rotation of dynamically-symmetrical PSR B1828-11 by Hamilton methods proposed Getino (Getino et al., 1999). Thus, according to our model, the neutron star has rigid the crust, the fluid outer core and the solid inner core. The model explains generation of four modes in the rotation of the pulsar: two modes of Chandler wobble (CW, ICW) and two modes connecting with free core nutation (FCN, FICN) (Gusev & Kitiashvili, 2008). The periods of the described variations can be described in the next way

$$P_{CW} = P_{PSR}\left(\frac{A}{A_{cr}}e - \frac{A_s}{A_{cr}}e_s\right)^{-1}, \qquad P_{RFCN} = P_{PSR}\left(\frac{A}{A_{cr}}e_f\right)^{-1},$$

$$P_{PFCN} = P_{PSR}\left(\delta + \frac{A_s}{A_{cr}}e_f\right)^{-1}, \qquad P_{ICW} = P_{PSR}\left(e_s + \delta\right)^{-1} \tag{3}$$

where P_{PSR} is the period of PSR B1828-11 rotation; $_{,\,cr,\,f,\,s}$ are moments of inertia of the total pulsar, fluid outer and solid inner cores; e_{PSR}, e_f, e_s are dynamical flattening of the pulsar, the fluid outer and the solid inner cores; δ is a small parameter

$$e = \frac{A_{cr}}{A}\left(\frac{P_{PSR}}{P_{CW}} + \frac{A_s}{A_{cr}}e_s\right),\ e_f = \frac{P_{PSR}}{P_{RFCN}}\frac{A_{cr}}{A},\ \ e_s = \frac{P_{PSR}}{P_{ICW}} - \delta,\ \ \delta = -\left(\frac{P_{PSR}}{P_{PFCN}} + \frac{A_s}{A_{cr}}e_f\right) \tag{4}$$

We have got the estimates of dynamical flattening of the crust, the outer liquid and the inner solid cores of the pulsar for known periodic variations of the TOA pulse from PSR B1828-11: 1000, 500, 250, 167 days. Numerical results of the TOA pulse fluctuations are shown in the table 4 for different models of the neutron star interior.

3. CONCLUSION

The observation of PSR B1828-11 has revealed the existence of four periodic variations TOA pulses. In the frame of the three-layer model we proposed the explanation for all pulse fluctuations by differential rotation crust, outer core and inner core of the neutron star and received estimations of dynamical flattening of the pulsar inner and outer cores. We have offered the realistic model of the dynamical pulsar structure and the feature of flattened of the crust, the outer core and the inner core of the pulsar.

4. REFERENCES

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