NEW ANALYTICAL PLANETARY THEORIES VSOP2010

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ABSTRACT. The planetary theories VSOP are essentially issued from the research works of P. Bretagnon. After the last version, VSOP2000, he began to make some improvements, but, unfortunately, he did not have time to complete his work. We took up again this work introducing various changes and complements and build two versions : VSOP2010A fitted to DE405 (numerical integration of the JPL) and VSOP2010B, not yet finished, fitted to INPOP08A (numerical integration of the IMCCE at Paris observatory). Over the time interval [1890,2000], the estimated precision is 3 to 10 times better than that of VSOP2000. Over the time interval [-4000,8000], the gain in precision is about 5 times better for the telluric planets and 10 to 50 times better for the outer planets in comparison with VSOP2000.

1. INTRODUCTION

The VSOP theories are analytical solutions for the motion of the planets of the solar system. They are based on the integration of Lagrange differential equations for the elliptic elements : a, semi-major axis, λ mean longitude, k = e cos ϖ, h = e sin ϖ, q = sin $i/2$ cos Ω, p = sin $i/2$ sin Ω where the time is expressed in TDB and e, ϖ, i, and Ω are respectively the eccentricity, the longitude of the perihelion, the inclination and the longitude of the ascending node.

2. HISTORICAL REVIEW

- VSOP82 (Bretagnon, 1982) : Perturbations developed up to the 3rd order of the masses and iterative method up to the power 5 of time in the Poisson series for the 4 outer planets; Perturbations of the Earth-Moon barycenter by the Moon; Relativity introduced by the Schwarzschild problem; Fitted to the numerical integration of the JPL DE200 (Standish, 1982).
- VSOP87 (Bretagnon, 1988) : Extension of VSOP82 in several versions: rectangular or spheric variables, ecliptic reference frame J2000 or of the date, heliocentric or barycentric coordinates.
- VSOP2000 (Moisson and Bretagnon, 2001) : Perturbations developed up to the 3rd order of the masses and iterative method up to the power 7 of time in the Poisson series for the 8 planets; Perturbations by the 5 big asteroids introduced during the iterations; Perturbations by the Moon on the telluric planets; Fitted to the numerical integration DE403 (Standish 1995).

3. MAIN CHARACTERISTICS

3.1. *The last improvements of P. Bretagnon*

- Additional iterations up to the power 12 of time in the Poisson series.
- Numerical precision 10 times better than VSOP82.
- Orbits of the 8 planets and the 5 big asteroids analytically computed during the same process.
- Perturbations of the Moon on the 8 planets introduced during the iterations.
- Relativistic corrections included as for the solution VSOP2000.

3.2. *Our new changes and complements*

- For the theories VSOP2010 we have kept a form similar to the solution VSOP2000, i.e., Poisson series where the arguments are : the mean mean longitudes of the planets $\bar{\lambda}_p = \lambda_p^0 + n_p^0$ t where t is the time (TDB), λ_p^0 is the integration constant of the mean longitude λ and n_p^0 the integration constant of the mean motion, the Delaunay angles of the Moon D, F, l, and the argument μ (defined hereunder).
- VSOP2010A is fitted to the numerical integration DE405 (Standish, 1998) which is the bases of The Astronomical Almanac since 2003.

• VSOP2010B (not yet finished), is fitted to the recent numerical integration INPOP08A (Fienga et al., 2009) which is the new standard reference for the planetary ephemerides published by IMCCE.

• We have computed the perturbations due to the $J2$ of the Sun on the telluric planets.

• We have introduced the perturbations of asteroids (295 in the case of VSOP2010A, 298 in the case of VSOP2010B) under the form of Poisson series of μ (see hereunder).

• We have improved the precision of the theories of Jupiter and Saturn over a large time-span by improving up to the power 20 of time the Poisson developments of some long period arguments starting from the solutions TOP (Simon, 2010). In TOP solutions, the motion of the outer planets and Pluto are computed under the form of Poisson series of only one angular argument μ, linear function of time t given by : $\mu = (n_5 - n_6) \, t \, / \, 880$ where n_5 and n_6 are the mean motions of Jupiter and Saturn respectively.

• We have added the perturbations of Pluto over the outer planets at the first order of the masses under the form of Poisson series of μ.

• We have introduced a complete solution of the motion of Pluto issued from TOP (not yet finished for VSOP2010B).

4. RESULTS AND PRECISION

In VSOP2010A and VSOP2010B, each element (a, λ, k, h, q, p) of each planet has the form of Poisson series developed up to the power 12 of time (power 20 for a et λ of Jupiter and Saturn) where a is expressed in au, λ is expressed in radian and the time is expressed in thousand of years.

Over the interval [1890;2000], the precision of the solutions VSOP2000, VSOP2010A and VSOP2010B can be estimated by their maximum differences with the numerical integrations DE403, DE405 and INPOP08A respectively on the mean longitude λ (see the table hereunder).

λ : maximum differences analytical theory - numerical integration over [1890;2000]. Unit is mas.

Planet	VSOP2000-DE403	VSOP2010A-DE405	VSOP2010B-INPOP08A
Mercury	0.27	0.07	0.04
Venus	0.29	0.08	0.03
Earth-Moon Barycenter	0.35	0.06	0.04
Mars	2.89	2.01	1.13
Jupiter	0.47	0.20	0.06
Saturn	1.75	0.19	0.13
Uranus	1.49	0.86	0.77
Neptune	1.86	0.27	0.10

We can see that the gain in precision related to VSOP2000 is more important for VSOP2010B than for VSOP2010A. That is essentially due to our computation of the perturbations by the asteroids which is closer between VSOP2010B and INPOP08A than in the case of VSOP2010A and DE405.

Over a greater interval [-4000,+8000] a comparison with an internal numerical allows to say that the solutions VSOP2010 are about 5 times better than VSOP2000 for the telluric planets and 10 to 50 times better for the outer planets.

5. REFERENCES

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