INFLUENCE OF THE INNER CORE GEOPOTENTIAL VARIATIONS ON THE ROTATION OF THE EARTH

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ABSTRACT. In this investigation we determine a new contribution to the rotation of a three layer Earth model composed by an axial–symmetric mantle, a fluid core, and an axial–symmetric inner core. This contribution emerges as a consequence of the variation of the geopotential induced by the differential rotation of the solid inner core. Within the framework of the Hamiltonian theory of the rotation of the non–rigid Earth, and following the same guidelines as those described in Escapa et al. (2001, 2008), we discuss the influence of this effect on the motion of the Earth figure axis. We also provide numerical estimations for the amplitudes of the nutational motion of this axis.

1. INTRODUCTION

The accurate modeling of the Earth rotational motion requires to consider a three layer structure that reproduces, to some extent, the Earth interior configuration. It is the case, for example, of the last theory of the Earth nutational motion adopted by the International Astronomical Union (IAU), based on the work by Mathews et al. (2002). This three layer structure (Figure 1) is composed of an external solid (mantle) that encloses a fluid (fluid outer core) containing another solid (inner core). In addition, rotational studies also assume that the barycenters of the three constituents are always located in the same point O.

The presence of the inner core introduces three additional degrees of freedom in the description of the system. They are necessary to take into account the possible rotation of the inner core with respect to the mantle. We will refer to this possible rotation as the differential rotation of the inner core. It affects the rotational dynamics of the Earth in different ways. For example, it originates new mechanical interactions of hydrodynamical, gravitational, electromagnetic, or viscous nature. These interactions influence the evolution of the mantle, the fluid, and the inner core itself, and are absent in one or two layer models.

At the same time, the existence of the solid inner core also changes the Earth response to model–independent perturbations like, for example, to the main contribution of the gravitational torque exerted on the Earth by the Moon and the Sun. This perturbation is the same for one, two, or three layer models. However, the response, i.e. the rotational motion, is different according to the particular model under consideration. From the perspective of the dynamical modeling, this change is related with the modification of the nutational normal modes of the Earth (e.g. Moritz & Mueller 1987). The inclusion of the inner core implies the appearance of two new nutational normal modes with respect to the two layer case: the Prograde Free Core Nutation and the Inner Core Wobble (e.g. Escapa et al. 2001).

Among the contributions of the solid inner core to the rotation of the Earth there is one specially peculiar, because it has no equal in one or two layer models. The differential rotation of the inner core modifies the outer gravitational field generated by the Earth. In other words, there exists a variation in the geopotential due to the differential rotation of the inner core. This situation arises even for the models involving rigid layers and an homogeneous fluid. From a mechanical point of view its origin is clear, since a differential rotation of the inner core entails a redistribution of mass inside of Earth. In turn, this redistribution modifies the geopotential.

This effect has been considered previously in some investigations (e.g. Greiner–Mai et al. 2000, 2001, Escapa et al. 2008), but, as far as we know, its influence in the rotational motion of the Earth has not been explicitly evaluated. Due to the minute size of the inner core with respect to the whole Earth and its quasi–sphericity, the numerical values induced by this mechanism should be small. However, some of the current studies of the Earth rotation provide improvements that are at the level of the microarcsecond (Dehant et al. 2008). Hence, in addition to the theoretical interest, it can be also relevant to discuss qualitatively this contribution from the point of view of establishing accurate numerical standards.

2. ANALYTICAL MODELING

We will sketch the main guidelines to evaluate the contribution of the variations of the geopotential to the rotation of the Earth, these variations stemming from the differential rotation of the inner core. For a comprehensive exposition, we refer the reader to Escapa et al. (2011). Since the influence of this effect is expected to be small, it will be enough to consider a simplified three layer Earth model. In particular, we will assume that the Earth is composed of three quasi–spherical, ellipsoidal layers: an axial–symmetrical rigid mantle, a fluid outer core, and an axial–symmetrical rigid inner core.

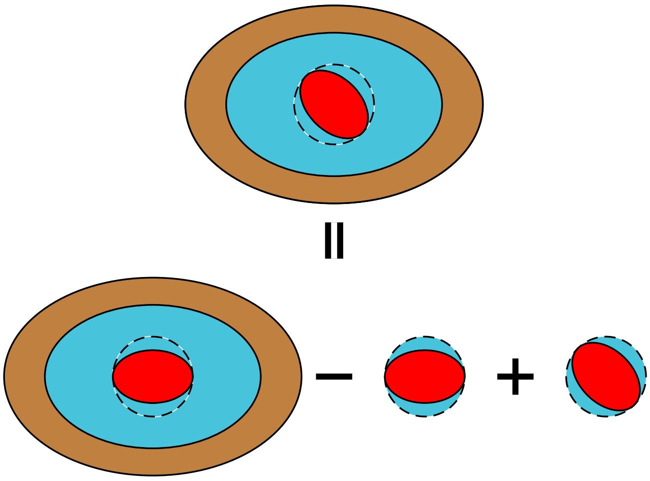

Figure 1: Three layer Earth model.

The rotational dynamics of this system is determined by the hydrodynamical and gravitational interactions. They provide its evolution through the differential equations of motion. We will establish these equations with the help of Analytical Dynamics. Within this framework, it is necessary to construct the rotational kinetic energy and the potential energy of the system. The kinetic energy, $\mathcal{T}$, accounts for the hydrodynamical interaction of the fluid with the solid layers. The potential energy is composed of two terms. One term, $\mathcal{V}_i$, is included to describe the internal gravitational torques among the layers. These torques are consequence of the stratification of the model (e.g. Ramsey 1940). The other one, $\mathcal{V}_e$, is related with the gravitational perturbations caused on the Earth by the external bodies, particulary by the Moon and the Sun.

We will focus our discussion on $\mathcal{V}_e$, since it is the function linked with the geopotential. Usually, it is expanded in a series of spherical harmonics, the main contribution coming from the second degree term. This term can be divided in two parts, accordingly their dependence on the differential rotation of the inner core. Although other approaches are possible (e.g. Escapa et al. 2008), a convenient way to obtain this division is with the aid of MacCullagh's formula

$$\mathcal{V}_e = G\frac{m}{2r^5}\left[3\begin{pmatrix} x \\ y \\ z \end{pmatrix}^t \Pi \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \operatorname{trace}(\Pi)\ r^2\right]. \tag{1}$$

Here, G represents the gravitational constant. The symbol m denotes the mass of the perturber; r its distance to O; and x, y, and z its coordinates relative to the T_m reference system, whose vectorial basis is constituted by the principal axes of the mantle and with origin at O. The matrix Π is the matrix of inertia of the whole Earth.

Therefore, the expression providing the variation of the geopotential can be constructed once we have determined the matrix of inertia of the Earth. The form of this matrix is derived by considering the decomposition given in Figure 1 (see also Escapa et al. 2001). We get

$$\begin{aligned}\Pi &= A\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1+e\end{pmatrix} + A_s\,(e_s-\delta)\begin{pmatrix}k_1^2 & k_1k_2 & k_1k_3\\ k_1k_2 & k_2^2 & k_2k_3\\ k_1k_3 & k_2k_3 & k_3^2-1\end{pmatrix} = \\ &= \Pi_0 + \Delta\Pi.\end{aligned} \tag{2}$$

In this equation A, A_s are the smallest principal moments of inertia of the Earth and the inner core, and e, e_s their ellipticities. The parameter δ is related with the ellipticity of a thin fluid layer surrounding the solid inner core (Escapa et al. 2001). The quantities k_1, k_2, and k_3 are the co–ordinates of the figure axis of the inner core in the mantle reference system T_m. As a consequence of the differential rotation they are functions of time, in contrast to the other parameters appearing in Eq. (2).

In this way, the matrix of inertia depends on the differential rotation through the term $\Delta\Pi$. This dependence is inherited by the geopotential, as it can be shown by substituting Eq. (2) into Eq. (1). Thus, the variation of the geopotential induced by the differential rotation can be derived from

$$\begin{aligned}\mathcal{V}_e &= G\frac{1}{2}\frac{m}{r^5}\left[3\begin{pmatrix}x\\ y\\ z\end{pmatrix}^t \Pi_0 \begin{pmatrix}x\\ y\\ z\end{pmatrix} - \operatorname{trace}(\Pi_0)\, r^2\right] + \frac{3}{2}G\frac{m}{r^5}\begin{pmatrix}x\\ y\\ z\end{pmatrix}^t \Delta\Pi \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \\ &= \mathcal{V}_2 + \Delta\mathcal{V}_2,\end{aligned} \tag{3}$$

since trace$(\Delta\Pi) = 0$. This formula shows that the order of magnitude of the new contribution $\Delta\mathcal{V}_2$ relative to the main part of second degree term $\mathcal{V}_2$ is $A_s(e_s-\delta)/(Ae)$, whose value is about 10^{-5}. Indeed this quantity is small, but it might be not enough to neglect its influence at the microarcsecond level.

3. DISCUSSION

To analyze the influence of $\Delta\mathcal{V}_2$ on the rotational motion of the Earth, we will employ the Hamiltonian formalism (e.g. Getino & Ferrándiz 2001). In our situation the Hamiltonian of the system, given by the sum of the kinetic and potential energies, turns out to be

$$\mathcal{H} = \mathcal{T} + \mathcal{V}_i + \mathcal{V}_2 + \Delta\mathcal{V}_2. \tag{4}$$

The equations of motion are obtained by formulating this Hamiltonian in terms of an Andoyer–like set of canonical variables (Escapa et al. 2001, 2008). Then, it is possible to obtain an analytical solution of these equations by employing a perturbation algorithm developed by Hori (1966). At the first order, this method allows us to separate the contributions due to $\Delta\mathcal{V}_2$ from that originated by $\mathcal{V}_2$.

Following this approach, we have obtained analytical expressions for the the rotational motion of the figure axis. This motion is usually decomposed in two parts (e.g. Kinoshita 1977). The first part is given by the motion of the angular momentum axis. The nutation in longitude and obliquity of the plane perpendicular to this axis is named Poisson terms. The second one represents the motion of the figure axis with respect to the angular momentum axis, in this case the associated nutation is referred as Oppolzer terms.

Specifically, within our order of approximation we have obtained (Escapa et al. 2011) that the motion of the angular momentum axis is not affected by the perturbation $\Delta\mathcal{V}_2$. It means that the Poisson terms are zero. In addition, this fact also implies that the precession of the Earth is not influenced by this perturbation. In contrast, the motion of the figure axis relative to the angular momentum axis is affected, i.e. the Oppolzer terms are different from zero. In this way, the perturbation $\Delta\mathcal{V}_2$ induces a periodic motion in the figure axis. The frequencies of this motion are the same as those derive when considering the $\mathcal{V}_2$ perturbation, that is to say, the induced motion contributes to the long–period nutations.

The numerical representation of this motion is given by providing the amplitudes of the nutation in longitude, $\Delta\psi$, and in obliquity, $\Delta\epsilon$, of the plane perpendicular to the figure axis. They are displayed in Table 1, and have been computed from the numerical values given in Kinoshita (1977), and those reported in Mathews et al. (1991) for the Preliminary Reference Earth Model (PREM). Since at this stage our purpose is to obtain an estimation of the contribution, we have only evaluated the amplitudes of the ten main terms in the trigonometric expansion of the orbital motion of the Moon and the Sun. In this regard, let us point out that the values of the amplitudes can be altered by the particular choice of some parameters of the rheological model, although the order of magnitude is kept.

As it can be seen in Table 1, the amplitudes of the new contributions are above the order of the microarcsecond for many nutational arguments. In particular, the values in the annual and semi–annual band are specially significant. It shows the need of incorporating the influence of the variation of the geopotential induced by the differential rotation of the inner core in the actual standards and models of the Earth rotation.

Argument					Period	Figure axis (μas)	
l_M	l_S	F	D	$\overline{\Omega}$	Days	$\Delta\psi$	$\Delta\epsilon$
+0	+0	+0	+0	+1	-6793.48	2.79	-0.31
+0	+0	+0	+0	+2	-3396.74	0.00	-0.01
+0	+1	+0	+0	+0	365.26	14.95	9.29
+0	-1	+2	-2	+2	365.25	-1.78	0.48
+0	+0	+2	-2	+2	182.63	44.61	-19.92
+0	+1	+2	-2	+2	121.75	1.64	-0.72
+1	+0	+0	+0	+0	27.55	-2.22	0.02
+0	+0	+2	+0	+2	13.66	7.17	-3.08
+0	+0	+2	+0	+1	13.63	1.22	-0.63
+1	+0	+2	+0	+2	9.13	0.96	-0.41

Table 1: Numerical estimation of the nutations of the figure axis due to the influence of the inner core in the geopotential.

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