PERTURBATION OF A PLANETARY ORBIT BY THE LAMBDA-TERM (DARK ENERGY) IN EINSTEIN EQUATIONS

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1. INTRODUCTION

The problem of cosmological influences at small (e.g. interplanetary) scales is discussed for many decades, starting from the early 1930's, but still remains unsolved definitively by now (Bonnor 2000). It became especially topical in the context of the "dark-energy"-dominated cosmology, because the usual arguments against the local Hubble expansion, such as Einstein–Straus (1945) theorem, are no longer applicable when the most contribution to the energy density of the Universe comes from the perfectly-uniform dark energy (Λ-term). Moreover, there are some empirical evidences in favor of the local cosmological influences. For example, assumption of the local Hubble expansion in the dynamics of the Earth–Moon system is a promising way to resolve a long-standing discrepancy in the rates of secular increase of the lunar semi-major axis measured by the lunar laser ranging, on the one hand, and derived from astrometric observations of the Earth's rotation deceleration, on the other hand (Dumin 2003, 2008, 2009). The aim of the present report is to provide a further support for this idea by a rigorous mathematical treatment of the two-body problem against the cosmological background formed by the Λ-term.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The starting point of our consideration is Kottler metric for a point-like mass embedded in the background formed by the Λ-term, which should be transformed to the Robertson–Walker coordinates to provide the adequate cosmological asymptotics at infinity (Dumin 2007). Keeping only the first-order terms of the Schwarzschild radius $r_g = 2GM/c^2$ and the inverse de Sitter radius $1/r_0 = \sqrt{\Lambda/3}$, we get:

$$\begin{aligned}
&\mathrm{d}s^2 = g_{tt}\, c^2 \mathrm{d}t^2 + 2\, g_{tr}\, c\, \mathrm{d}t\, \mathrm{d}r + g_{rr}\, \mathrm{d}r^2 + g_{\theta\theta}\, \mathrm{d}\theta^2 + g_{\varphi\varphi}\, \mathrm{d}\varphi^2 \,, \\
&g_{tt} \approx -\Big[1 - \frac{2GM}{c^2 r} \Big(1 - \frac{c\sqrt{\Lambda}\, t}{\sqrt{3}} \Big) \Big] \,, \qquad g_{tr} \approx \frac{4\, GM \sqrt{\Lambda}}{\sqrt{3}\, c^2} \,, \\
&g_{rr} \approx \Big[1 + \frac{2GM}{c^2 r} \Big(1 - \frac{c\sqrt{\Lambda}\, t}{\sqrt{3}} \Big) \Big] \Big(1 + \frac{2c\sqrt{\Lambda}\, t}{\sqrt{3}} \Big) \,, \quad g_{\theta\theta} = g_{\varphi\varphi}/\sin^2\theta \approx r^2 \Big(1 + \frac{2c\sqrt{\Lambda}\, t}{\sqrt{3}} \Big) \,.
\end{aligned} \tag{1}$$

The equations of motion of a test particle of infinitely small mass in this metric are (for conciseness, we put $c \equiv 1$, use the quantities r_g and r_0 defined above, and the coordinate system is oriented so that the particle moves in its equatorial plane $\theta = \pi/2 = \mathrm{const}$):

$$\begin{aligned}
&2 \Big[1 - \frac{r_g}{r} \Big(1 - \frac{t}{r_0} \Big) \Big] \ddot{t} - 4 \frac{r_g}{r_0} \ddot{r} + \frac{r_g}{r_0} \frac{1}{r} \dot{t}^{\,2} + 2 \frac{r_g}{r^2} \Big(1 - \frac{t}{r_0} \Big) \dot{t}\, \dot{r} + \frac{1}{r_0} \Big(2 + \frac{r_g}{r} \Big) \dot{r}^{\,2} + 2 \frac{r^2}{r_0} \dot{\varphi}^{\,2} = 0 \,, \\
&4 \frac{r_g}{r_0} \ddot{t} + 2 \Big[1 + 2 \frac{t}{r_0} + \frac{r_g}{r} \Big(1 + \frac{t}{r_0} \Big) \Big] \ddot{r} + \frac{r_g}{r^2} \Big(1 - \frac{t}{r_0} \Big) \dot{t}^{\,2} \\
&\qquad + \frac{2}{r_0} \Big(2 + \frac{r_g}{r} \Big) \dot{t}\, \dot{r} - \frac{r_g}{r^2} \Big(1 + \frac{t}{r_0} \Big) \dot{r}^{\,2} - 2 r \Big(1 + 2 \frac{t}{r_0} \Big) \dot{\varphi}^{\,2} = 0 \,, \\
&r \Big(1 + 2 \frac{t}{r_0} \Big) \ddot{\varphi} + 2 \frac{r}{r_0} \dot{t} \dot{\varphi} + 2 \Big(1 + 2 \frac{t}{r_0} \Big) \dot{r} \dot{\varphi} = 0 \,,
\end{aligned} \tag{2}$$

where dot denotes a derivative with respect to the proper time of the moving particle.

3. RESULTS OF NUMERICAL SOLUTION

Since analytical treatment of the above-written equations is very hard, we shall use here only numerical solutions for the test-particle orbits. Besides, a serious obstacle in the numerical computation is a very

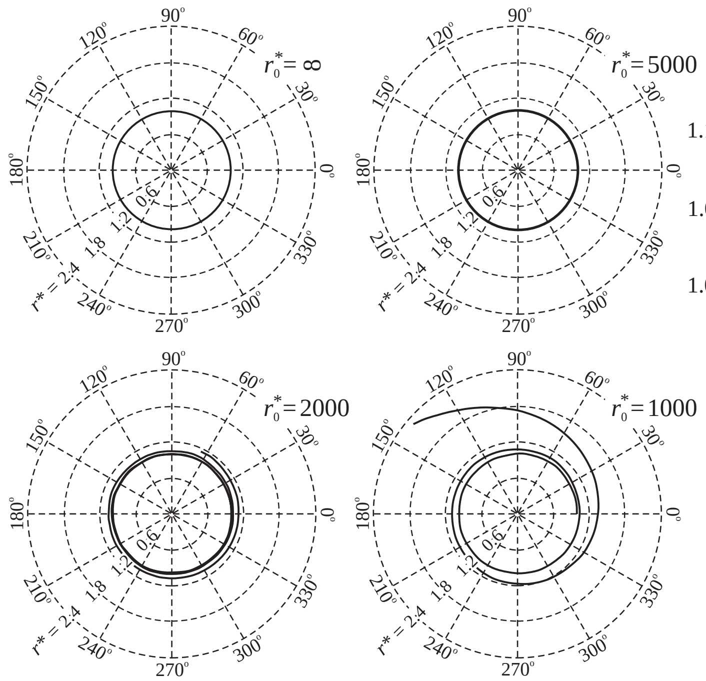

Figure 1: Orbits of the test particles at the specified Schwarzschild radius $r_g^* = 0.01$ and various de Sitter radii r_0^* (i.e., various values of the Λ-term).

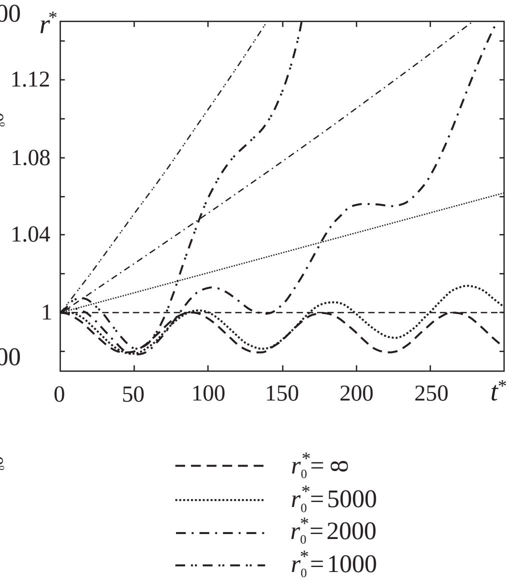

Figure 2: Radii of the orbits as functions of time at the specified Schwarzschild radius of the central body $r_g^* = 0.01$.

much difference in the three characteristic scales of the problem – Schwarzschild radius (e.g., for the Earth as a central body, $\sim 10^{-2}$ m), typical radius of the planetary orbit (e.g., for the Moon, $\sim 10^9$ m), and de Sitter radius ($\sim 10^{27}$ m). So, we present here the results of numerical integration only for a toy model, when difference between the characteristic scales is not so much as in reality. Namely, we take the dimensionless Schwarzschild radius $r_g^* = 0.01$ and de Sitter radii r_0^* about a few thousand. (Here, the quantities with asterisks are normalized to the initial radius of the orbit.)

As is seen in Figure 1, the orbits are almost circular during the first few revolutions if $r_0^* \gtrsim 5000$, but they take a spiral form at $r_0^* \lesssim 2000$ (i.e., when Λ-term is sufficiently large). Figure 2 represents a temporal dependence of radii for the same orbits. (The curves are wavy because the initial unperturbed orbit was chosen to be slightly elliptic.) The almost straight lines in this figure represent the pure Hubble flows (without a massive central body) for the same values of Λ. It is evident that under certain circumstances (depending on the ratio between the above-mentioned characteristic parameters) the orbital radii tend to approach the rates of the Hubble flows. In our opinion, this points to the potential importance of the local Hubble effect for the planetary dynamics, although a more careful analysis (with realistic Earth–Moon parameters and the additional factors affecting the planetary dynamics) is still to be done.

An opposite point of view, that cosmological influences are totally negligible in the Solar system, was put forward by some other authors, e.g., Klioner & Soffel (2005).

4. REFERENCES

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