VARIATIONS OF THE EARTH PRINCIPAL MOMENTS OF INERTIA DUE TO GLACIAL CYCLES FOR THE LAST 800KA

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The Earth shape, gravity and rotation are highly affected by climatic variations associated with the glacial cycles in the late Pleistocene. The processes of glaciation, followed by ice melting, are connected with significant changes of the mean sea level. These processes redistribute great amount of water masses between oceans and ice sheets, which lead to changes of the main moments of inertia. Let consider a homogeneous sphere with radius $R = 6371km$, density $4.9g/cm^3$ which corresponds to equivalent inertial moment of the real earth (the mean Earth density is $5.519g/cm^3$), inertial moment C, mean angular velocity ω_0 and constant mass M. From the conservation of angular momentum, the small changes of the radius R are connected with a corresponding change of the Length of Day (LOD) by the expressions $\Delta LOD = 2\Delta R\omega_0/(\omega_1 R)$, where $\omega_1 = 0.843994809prad/s$. The transfer from homogeneous elastic sphere to the real Earth need to replace the variations of radius R by equivalent changes of the mean sea level ΔMSL. It is necessary to determine transfer coefficient $k = \Delta MSL/\Delta R$ between the homogeneous elastic sphere and the real Earth. Involving coefficient q of ice sheets influence on the moment of inertia, and coefficient p of elastic Earth deformation due to ice loading effect we obtain $\Delta LOD = 2\omega_0(1-p)(1-q)\Delta MSL/(k\omega_1 R)$, $\quad \Delta C = 0.8(1-p)(1-q)MR\Delta MSL/k$. By comparing of the observed amplitudes of 11-year oscillations of the MSL and UT1 we determine p= 0.58 and corresponding effectiveness of MSL on C and LOD variations $1-p = 0.42$. The value of the coefficient k depends on the mean density of sea water at the ocean surface, total ocean surface and the moment of inertia of the thin ellipsoidal shell over the ocean with thickness equal to ΔMSL. In the case of small MSL variations (significantly less than 1m), the water lost occur from all Earth surface, more intensive from the free water surfaces and less intensive from the ground. The coefficient k is approximately equal to 5 in this case (Chapanov and Gambis, 2010). The MSL changes are more than 100m during the glaciations, the level of the water lost from the ground is neglectful, so $k = D_E S_E I_{ES}/D_O S_O I_{OS}$, where D_E is the mean Earth density ($4.9g/cm^3$), D_O - the mean sea water density at the ocean surface, S_E - the total Earth surface, S_O - the global ocean surface, I_{ES} - the moment of inertia of the thin ellipsoidal shell over the Earth, I_{OS} - the moment of inertia of the thin ellipsoidal shell over the ocean. The mean sea surface water density is $1.025g/cm^3$, so $D_E/D_O = 4.78$. The total Earth surface is 510 106 km^2, the global ocean surface - 361 106 km^2, and their ratio - 1.414. The continental ice sheets influence with surface S_{IC} on the axial moment of inertia is $q = S_O I_{IC}/S_{IC} I_{OC}$, where I_{IC} is the total moment of inertia of thin ellipsoidal shell over the continental ice sheets. The sea ice does not affect the moment of inertia changes, due to its hydrostatical equilibrium with the ocean water. Let the surface glacial data is represented over a grid with size $\alpha \times \alpha$ degrees and the geocentric coordinates of the center of the n-th grid element are geocentric distance r_n, longitude λ_n and latitude θ_n. If a and b denote Earth equatorial and polar radii ($a = 6478.137km$, $b = 6356.572km$), then $r_n = \sqrt{a^2b^2(a^2sin^2\theta_n + b^2cos^2\theta_n)}$ and the distance from the center of the n-th grid element to the Earth axis of rotation is $h_n = r_n cos\theta_n$. The surface of the n-th grid element is approximately $s_n = 4\pi^2(\alpha/360)^2 r_n h_n$ and the axial moment of inertia of thin ellipsoidal shell with unit density and thickness over a given area is $I = \sum_n s_n h_n^2$. The axial moment of inertia of ellipsoidal shell with unit density and thickness over the Earth is 1.38×10^{14} km^4 and over the ocean - $1.0\times10^{14}km^4$. Their ratio is 1.38, and the transfer coefficient k in the case of great MSL changes is k=9.3. The continental ice sheets during the last glacial maximum reach latitudes between 40° and 50° in the North America and between 50° and 60° in Euro-Asia. Roughly, this area is almost circular with radius 4400km (corresponding to 40° over the meridian). The center of this area is shifted by 10° from the North Pole to the Greenland direction. The South Pole ice sheet cover almost circular area with latitude above 50° (Paul and Schfer-Neth, 2003; Schfer-Neth and Paul, 2003), thus its radius is approximately the same as the North Pole ice sheet. It is possible to determine coefficient q by means of reconstructed data of global sea surface temperature and salinity during the last glacial maximum (available in the World Data Center for Paleoclimatology, Boulder). A linear $S_O/S_{IC} = 0.084\Delta MSL + 12.45$ and a parabolic $I_{IC}/I_{OC} = 0.0019 + 5 \times 10^{-5}\Delta MSL + 5.71 \times 10^{-6}\Delta MSL^2$ dependencies exist (MSL is expressed in meters), so the coefficient q varies non-linearly between 0.022 and 0.2. The relative glacial steric sea level variations are 0.004 only and therefor it is possible to neglect. Finally the dependencies between LOD in ms (Fig.1, a), inertial moment C in $kg\, m^2$ (Fig.1, c) and glacial MSL variations in meters are

$$\Delta LOD = 2.29(1-p)(1-q)\Delta MSL, \qquad \Delta C = 1.025 \times 10^{12}(1-p)(1-q)I_{OS}\Delta MSL. \tag{1}$$

The moments of inertia of thin ellipsoidal shell with unit density and thickness over the ocean area I_A relative to the x axis and I_B relative to the y axis are $I_A = \sum_n s_n r_n^2(sin^2\theta_n + sin^2\lambda_n cos^2\theta_n)$, $I_B = \sum_n s_n r_n^2(sin^2\theta_n + cos^2\lambda_n cos^2\theta_n)$. The influence of the polar ice on the variations of the axial moments of inertia A and B is $I_P = 1.025(1-p)S_O\Delta MSL\, b'^2$, where b' is the polar Earth radius, corrected with the half mean polar ice hight. Let $I_{H(A,B)}$ denotes the corrections for Himalaya ice, then the variations ΔA and ΔB of the axial moments of inertia A and B (Fig.1, b) are

$$\Delta A = 1.025 \times 10^{12}(1-p)I_A\Delta MSL - I_P - I_{HA},\ \Delta B = 1.025 \times 10^{12}(1-p)I_B\Delta MSL - I_P - I_{HB}. \tag{2}$$

The MSL data for the last $800Ka$ are composed by the reconstructed MSL for the last $380Ka$, determined by the sediments from the Red sea (Siddall et al., 2003) and variations for the period $380Ka-800Ka$ before present (BP), based on the temperature changes, determined by deuterium data from Antarctica ice core (Jouzel et al., 2007).

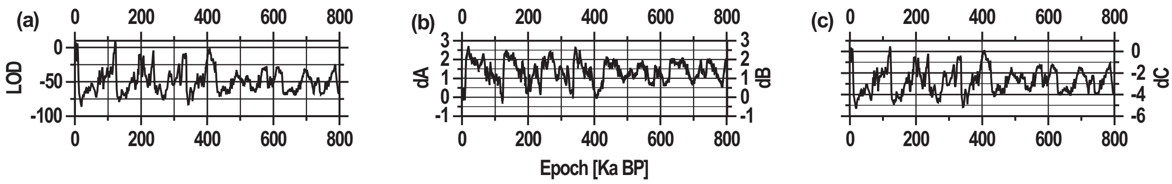

Figure 1: Variations of LOD in ms - (a), inertial moments A (solid line), B (dashed line) in $kg\, m^2 \times 10^{17}$ - (b) and axial moment C in $kg\, m^2 \times 10^{29}$ - (c).

The reconstructed glacial variations of the sea level for the last 800Ka give good opportunity for studying the millennial scale variations of the Earth rotation, shape and axial moments of inertia and to determine long-periodical oscillations of the Length of Day LOD and the Universal Time UT1. The changes of the Earth axial moments of inertia are due to global water redistribution between the ocean and continental ice, which is connected with the sea level variations. The variations of the axial moment of inertia C are synchronized with MSL variations with maximal value $-5.35 \times 10^{29} kg\, m^2$. The variations of the principal moments of inertia A and B are opposite to the MSL variations, due to dominating effect of the polar ice. Their maximal values during the glacial maxima are about $2.54 \times 10^{17} kg\, m^2$. The variations of the principal moment of inertia A are grater by 7% of the variations of the principal moment of inertia B. The periodical part of principal moments of inertia consists of frequencies similar to the frequencies of the Earth orbital variations with periods around $20Ka$, $40Ka$ and $100Ka$.

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