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PROPOSAL FOR THE RE-DEFINITION OF THE ASTRONOMICAL UNIT OF LENGTH THROUGH A FIXED RELATION TO THE SI METRE

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ABSTRACT. As already suggested before the 2009 IAU General assembly (Klioner 2008; Capitaine & Guinot 2009), it is proposed to re-define the ua as a fixed number of SI metres through a defining constant. Such a change of status for the ua would limit its role to that of a unit of length of "convenient" size for some applications. Consequently, $GM_{\rm Sun}$ which would cease to have a "fixed" value in astronomical units and will have to be determined experimentally, which is shown to be desirable for modern dynamics of the solar system. The defining number to be adopted for the conventional definition of the ua should be, for continuity reason, the value for the current best estimate of the ua in m as adopted by IAU 2009 Resolution B2 (i.e. ua = 1.495 978 707 00 x 10^{11} m) (Luzum et al. 2011). Such a change of status of the ua would be a great simplification for the users of the astronomical constants, would let possible variations of the mass of the Sun (and/or G) appear directly, and would avoid an unnecessary deviation from the SI.

1. THE ASTRONOMICAL UNITS

The current System of astronomical constants includes three astronomical units as defined in the IAU 1976 System:

- The astronomical unit of time is a time interval of one day (D) of 86 400 s. It provides a unit of time of "convenient" size for astronomy, which is related to the SI by a defining number.
- The astronomical unit of length (ua) is a specific astronomical unit for expressing distances in the solar system. Its definition is based on the Gaussian gravitational constant k and its value in SI has to be determined experimentally.
- The astronomical unit of mass is the mass of the Sun, $M_{\rm Sun}$. It is a specific astronomical unit for expressing masses; its value in SI has to be derived from the SI values of (1) the heliocentric gravitational constant, $GM_{\rm Sun}$, and (2) the gravitational constant G (which has a current relative uncertainty of 10^{-4}, cf. CODATA 2006). The value of $GM_{\rm Sun}$ in SI is obtained from the SI values for the ua and the day. Therefore, the value in SI of the astronomical unit of mass depends on the ua.

2. DEFINITION AND ROLE OF THE ASTRONOMICAL UNIT OF LENGTH (UA)

In the IAU 1976 System of astronomical constants, the astronomical unit of length has been defined as "that length (A) for which the Gaussian gravitational constant (k) takes the value of 0.017 202 098 95 when the units of measurements are the astronomical units of length, mass and time. The dimensions of k^2 are those of the constant of gravitation (G), i.e., L^3 M^{-1} T^{-2}. The term "unit distance" is also for the length A."

In the SI brochure (cf. Table 7 entitled "Non-SI units whose values in SI units must be obtained experimentally"), which is intended to non-astronomers, the astronomical unit is defined as "approximately

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equal to the mean Earth-Sun distance. It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with a mean motion of 0.017 202 098 95 radians per day (known as the Gaussian constant)."

For any planet, the mean motion n is measured in SI (i.e. rad/s) and can be trivially converted into astronomical units (i.e. rad/D); its semi major axis can be expressed as $a^3 = k^2/n^2$ (if perturbations are neglected). Hence the distances to other planets are in ua and the accuracy of the time measurements is transferred into the relative distances. This definition of the ua was explained by the lack of precise measures of distances in the solar system, while it provides accurate relative distances. It let open the problem of precise scaling of the solar system in the SI.

The status of the ua in the IAU 1976 System of astronomical constants and the IERS Conventions 2003 is such that k is a "defining constant"; the scale distance in the solar system is provided by the value A of the ua in metre as fitted to a planetary ephemeris.

In the IAU 1976 System of astronomical constants, the uncertainty in the estimated ua was of 2 km; The heliocentric gravitational constant was a "derived constant", such that: $GM_{Sun} = k^2A^3/D^2$. In the IERS Conventions 2003, the uncertainty in the ua, as derived from the JPL ephemerides DE403 (Standish 1995) was of 6 m, i.e. more than 2 orders of magnitude better than its IAU 1976 value.

3. RECENT EVOLUTION AND NEW SITUATION

There have been a number of changes and improvements in various contexts since the ua was defined as described in the previous section.

1. The recent IAU Resolutions (1991, 2000, 2006) have specified the definition of the celestial reference systems in the General Relativity framework; in addition, they have recommended improvements in the concepts and models. The definition of the ua should be adapted to comply with this new context.

2. Modern observations in the solar system are based on ranging to planets, spacecraft observations, Very Long Baseline Interferometry (VLBI), etc, which are high accuracy observations. Therefore, ranging observations are so accurate that there is no longer a reason to have a scale-invariant description of the solar system as provided by the current definition of the ua.

3. Recent ephemerides of solar system objects are such that:

- the primary determination of the GM_i of the planets have been obtained directly in $\mathrm{km}^3\mathrm{s}^{-2}$ (TDB-compatible values) and then converted into values in astronomical units in order to comply with the IAU unit distance (Folkner et al. 2008);
- the direct estimation of GM_{Sun} has been tested in the INPOP08 ephemerides (Fienga et al. 2009).

This shows that, in the context of modern ephemerides, the ua appears as an intermediate unit only used for historical purposes.

4. The decrease of the solar mass is expected to be detectable in a near future, i.e. when the accuracy has been improved by a factor 10. With the current definition of the ua, time dependence of M_{Sun} immediately leads to time-dependent astronomical units for length and mass. To investigate the dynamical effects of the time-dependence of M_{Sun} using such time-dependent units is a non-sense.

All these reasons make clear that the status of the ua and GM_{Sun} should be reformed to be more in agreement with the modern context (Klioner 2008, Capitaine & Guinot 2009).

Yet, in the IAU 2009 System of astronomical constants (Luzum et al. 2011) that was recently adopted by the IAU as a replacement of the IAU 1976 System, the ua is still defined from the Gaussian gravitational constant k, which is called an "auxiliary defining constant". However, the numerical values provided for the ua and GM_{Sun} in the table associated with that system have been obtained from the following procedure: (i) the TDB-compatible value of the ua is an average (Pitjeva & Standish 2009) of recent estimates of the ua defined by k, (ii) the TDB-compatible value of GM_{Sun} has been derived from the ua fitted to the DE 421 ephemerides (Folkner et al. 2008). This value of GM_{Sun} is consistent, within uncertainties, with the IAU 2009 value for the ua but has not been directly derived from that value. This shows that the historical definition of the ua is no longer required for modern ephemeris. Note that TDB- and TCB-compatible values are provided for GM_{Sun}.

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4. THE RELATIVISTIC FRAMEWORK

If the historical status of the ua were to be kept, it would be necessary to extend its definition to General relativity. Several options have been considered (Guinot 1995, Capitaine & Guinot 1995, Brumberg & Simon 2004, Standish 2005, Pitjeva 2005, Klioner 2008).

First, it would be necessary to extend the Gaussian gravitational constant to the GR framework; there are several possible options, but the geometrical interpretation through the motion around the Sun would be more delicate. Second, it would be necessary to take into account the use of TCB or TDB, i.e. the scaling factor $F = 1 - L_{\rm B}$ between values of a quantity associated to the use of TCB and TDB; there are several options, such as scaling only the units, scaling only the values, or scaling both, out of which the two most logical possibilities would be:

- option 1: $k^2 = GM_{\rm Sun}$ has the same value with both TCB and TDB, which requires scaling the units of time and length; thus, we would have:
  . for the "unit distance": $\text{ua}_{\rm TDB} = F^{1/3}\ \text{ua}_{\rm TCB}$,
  . for the distance: TDB-compatible value in $\text{ua}_{\rm TDB}$ = $F^{2/3}\times$ TCB-compatible value in $\text{ua}_{\rm TCB}$,
  which would be very confusing because of the unusual scaling;
- option 2: $\text{ua}_{\rm TDB} = \text{ua}_{\rm TCB}$; this requires:
  . for the distance in astronomical units: TDB-compatible value = $F\times$ TCB-compatible value,
  but then $GM_{\rm Sun}$ in astronomical units is no longer k^2 either with TDB or with TCB, which kills the uniqueness of the Gaussian constant k.

This shows that relativity makes the usual definition of the ua even trickier. It would then be better to avoid extending that definition to the GR framework.

5. THE PROPOSAL FOR THE RE-DEFINITION OF THE UA

As already suggested before the 2009 IAU General assembly (Klioner 2008; Capitaine & Guinot 2009), it is proposed to re-define the ua as a fixed number of SI metres through a defining constant.

This would mean:
- dropping the k constant and abandoning the experimental determination of the ua in SI unit,
- determining experimentally $GM_{\rm Sun}$.

Such a change of status for the ua would limit its role to that of a unit of length of "convenient" size for some applications.

Note that defining the ua as a conventional number of metres is in accordance with the adopted way (Klioner et al. 2009) to use the SI units for the relativistic time scales and associated quantities, so that for distances in astronomical units one simply has: TDB-compatible value = $F\times$ TCB-compatible value.

The defining number to be adopted for the conventional definition of the ua should be, for continuity reason, the value for the current best estimate of the ua in m as adopted by IAU 2009 Resolution B2 (i.e. ua = 1.495 978 707 00 x 10^{11} m exactly).

Note also that the CCU declared (CCU 2009) its support to move to a fixed relationship to the SI metre through a defining number determined by continuity.

6. CONCLUSION

A re-definition of the ua is necessary in the modern context in order to make the system of astronomical constants best compliant with modern dynamical astronomy. From the point of view of the principles, the important point is the change of status for the astronomical unit of length (and not the value of its defining number).

Such a change of status of the ua:
- would be a great simplification for the users of the astronomical constants,
- will let possible variations of the mass of the Sun (and/or G) to appear directly (which is the option that has the most physical meaning), and would avoid an unnecessary deviation from the SI.

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This should be largely discussed within the astronomical community in order to be proposed at the next IAU GA (2012).

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