VLBI observations of nutation, its geophysical excitations and determination of some Earth model parameters

Jan Vondrák and Cyril Ron, Astronomical Institute, Prague, Czech Republic

- **Introduction:**
  - **Resonances in Earth orientation;**
- **Determination of RFCN period, quality factor and a common multilier from VLBI observations;**
- **Geophysical contribution to nutation:**
  - **Broad-band Liouville equations, numerical integration;**
- **Conclusions.**

Introduction: Resonances in Earth rotation

- Due to the existence of a flattened fluid and rigid inner core, there are strong resonances in near-diurnal (in terrestrial frame) part of the spectrum, leading to
  - Significant modification of nutation amplitudes;
  - a non-negligible influence of geophysical excitations in nutation.
- The strongest resonance in this region is the Retrograde Free Core Nutation (RFCN).

Resonance (Mathews-Herring-Buffet transfer function):

amplitude ratio of non-rigid/rigid Earth model:

$$T(\sigma) = \frac{e_R - \sigma}{e_R + 1} N_0 \left[1 + (1+\sigma)\left(Q_0 + \sum_{j=1}^{4} \frac{Q_j}{\sigma - s_j} \right) \right]$$

where e_R is dynamical ellipticity of rigid Earth, σ is the frequency of nutation (in ITRF), *N, Q* are constants, and s_j are resonance frequencies:

1. **Chandler Wobble - CW ($P_{ter.}$ = 435 d);**
2. **Retrograde Free Core Nutation - RFCN ($P_{cel.}$ = 430 d);**
3. **Prograde Free Core Nutation - PFCN ($P_{cel.}$ = 1020 d);**
4. **Inner Core Wobble - ICW ($P_{ter.}$ = 2400 d).**

V C D Z

C E D R

Estimation of a common multiplier N_o and complex resonant RFCN frequency s_2 from the observed amplitudes and phases of seven nutation terms (6798, 3399, 365.26, 182.62, 121.75, 27.55, and 13.66 days):

- MHB Sun-synchronous correction in prograde annual nutation is removed from the observed value,
- All constants in MHB transfer function are kept, with the exception of N_o, s_2;
- Three parameters (N_o, $\mathrm{Re}(s_2)$, $\mathrm{Im}(s_2)$) are estimated by weighted least-squares:
- $P=0.99727/[(\mathrm{Re}(s)+1]$, $Q= -\mathrm{Re}(s)/2\mathrm{Im}(s)$

VLBI data used:

- Celestial pole offsets in 1-7 day intervals:
  - AUS solution (1984.0-2007.6);
  - GSFC solution (1979.6-2007.2);
  - OPA solution (1984.0-2007.1);
  - USNO solution (1979.6-2007.2);
  - IVS solution (1979.6-2006.9).

Results (VLBI only, no geophysical excitation):

Solution	P	Q	N_o
AUS	−429.98 ±22	19985 ±837	1.00001100 ±357
GSFC	−430.11 ±18	19544 ±633	1.00001046 ±297
OPA	−430.27 ±16	19839 ±593	1.00000826 ±260
USNO	−430.07 ±18	19348 ±646	1.00000818 ±295
IVS	−430.22 ±16	20741 ±645	1.00000921 ±256
MHB	−430.21	19998	1.00001224

Dyn. ellipticity: core (from P) — whole Earth (from N_o)

Atmospheric and oceanic excitations:

- ♦ AAM, OAM are given in terrestrial frame, they must be transformed into celestial (non-rotating) frame;
- ♦ We are interested only in long-periodic motions:
  - Short-periodic signal (P<10 days) was smoothed out.
- ♦ Geophysical excitations, 6-h intervals:
  - Atmospheric Angular Momentum functions (pressure + wind)
    - NCEP/NCAR re-analysis (1983.0-2007.0);
    - ERA (1979.0-2001.00).
  - Oceanic Angular Momentum functions (matter + motion)
    - ECCO model (1993.0-2006.2);
    - OMCT model (1979.0-2001.0);
    - Rui Ponte model (1993.0-2000.5).

VCDZ
CEDR

♦ Following combinations of geophysical excitations are used:

- NCEP AAM with IB correction;
- NCEP AAM without IB correction;
- NCEP AAM(ib) + ECCO OAM;
- NCEP AAM(ib) + PONTE OAM;
- ERA40 AAM + OMCT OAM.

Numerical integration of broad-band Liouville equations:

(complex form, in celestial system, after Brzezinski)

$$\ddot{P} - \mathrm{i}(\sigma'_C + \sigma'_f)\dot{P} - \sigma'_C\sigma'_f P =$$

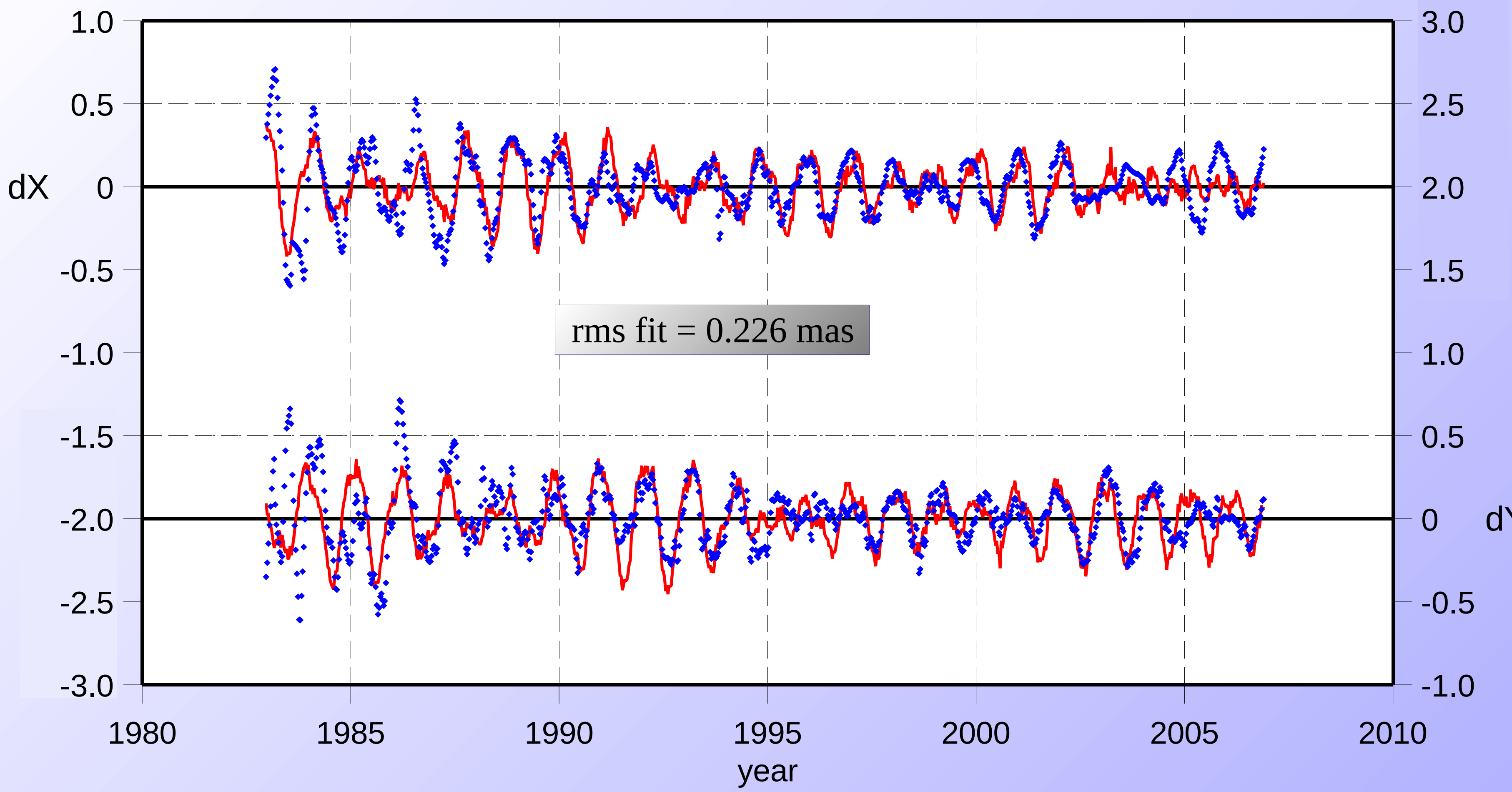
$$= -\sigma_C\left\{\sigma'_f(\chi'_P + \chi'_w) + \sigma'_C(a_p\chi'_P + a_w\chi'_w) + \mathrm{i}\left[(1+a_p)\dot{\chi}'_P + (1+a_w)\dot{\chi}'_w\right]\right\}$$

where

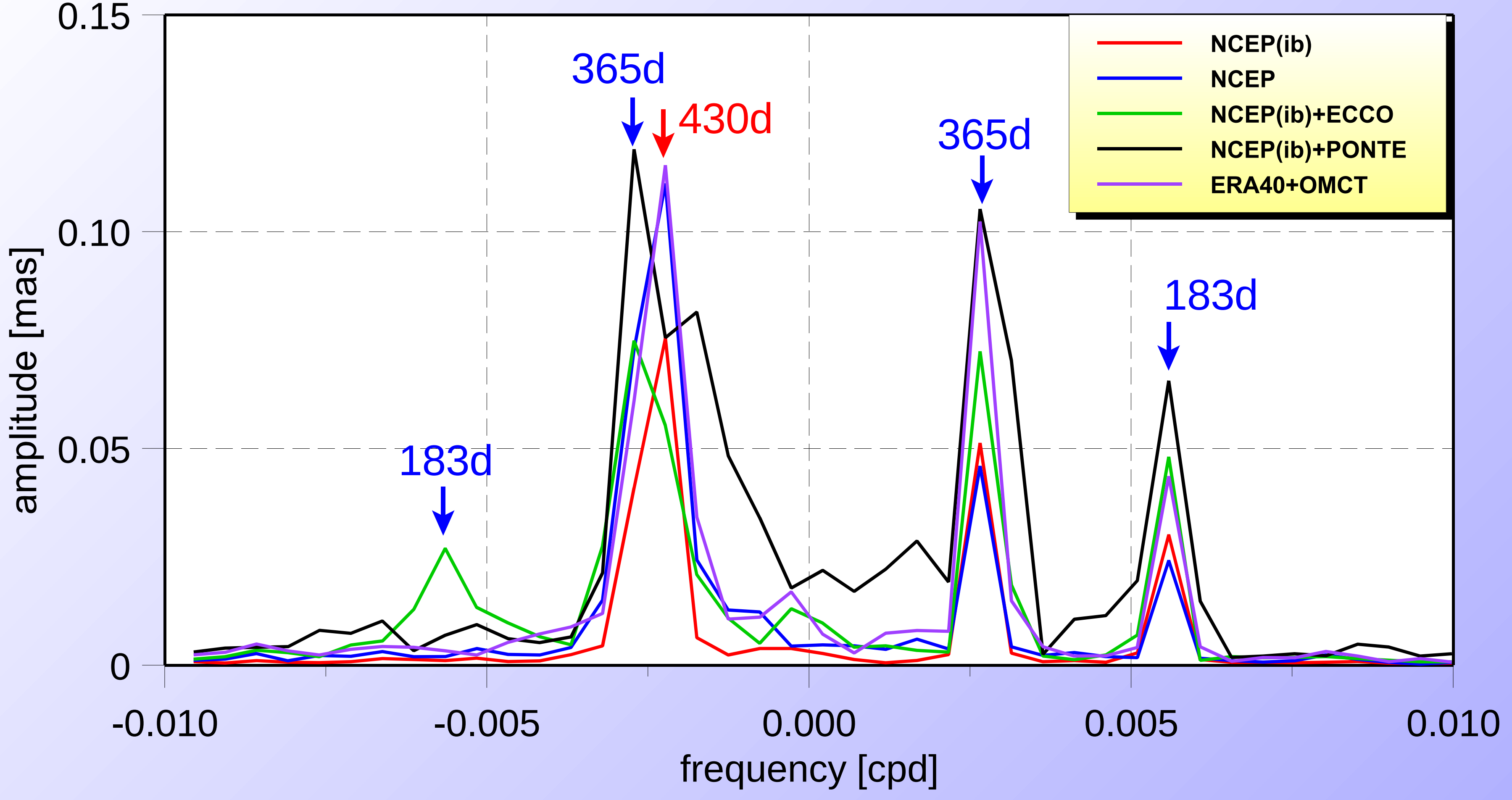
- P is the motion in celestial system;
- σ'_C, σ'_f are Chandler and FCN frequency in celestial frame;
- σ_C is Chandler frequency in terrestrial frame;
- χ'_p, χ'_w are excitations (matter and motion term) in celestial frame;
- $a_p = 9.2\times10^{-2}$, $a_w = 5.5\times10^{-4}$ are numerical constants.

4-order Runge-Kutta method is used

Example: excitation by NCEP AAM (pib+w)



Spectrum of geophysically excited motion of celestial pole



To derive prograde/retrograde complex amplitudes A^+, A^- from least-square fit of dX, dY, we use the following expression:

$$dX + \mathrm{i}dY \approx \Delta\psi \sin\varepsilon_o + \mathrm{i}\Delta\varepsilon = -\mathrm{i}\sum_k (A_k^+ \mathrm{e}^{\mathrm{i}\omega} + A_k^- \mathrm{e}^{-\mathrm{i}\omega})$$

Atmospheric & oceanic contributions to nutation [μas]

Excitation	Annual				Semi-annual				
	prograde		retrograde		prograde		retrograde		
AAM+OAM	Re	Im	Re	Im	Re	Im	Re	Im	σ
NCEP(IB)	−3.4	+109.7	−69.9	−18.7	−44.2	−55.7	−1.8	+0.9	±2.7
NCEP	+13.8	+91.5	−76.8	−52.1	−25.5	−52.5	−6.5	−1.0	±4.9
NCEP+ECCO	−0.6	+110.7	−32.2	−69.1	−46.5	−53.9	−6.2	−21.8	±4.6
NCEP+PONTE	+80.3	+107.4	+86.4	−135.6	−26.3	−62.1	+9.2	−9.6	±8.5
ERA40+OMCT	−64.9	+180.3	−64.3	+5.6	−17.9	−77.3	+0.3	+3.4	±6.8
MHB Sun-syn.	−10.4	+108.2							

- Real geophysical excitations removed from IVS solution, instead of MHB Sun-synchronous term;
- Estimation of Earth parameters N_o, s_2 repeated:

Solution IVS +	P	Q	N_o
MHB Sun-s.	−430.22 ±16	20741 ±645	1.00000921 ±256
NCEP(ib)	−431.00 ±23	20530 ±911	1.00001437 ±368
NCEP	−431.18 ±21	19892 ±781	1.00001506 ±335
NCEP(ib)+ECCO	−430.96 ±19	17584 ±636	1.00001708 ±1700

Theoretical relation between dynamical core flattening e_f and RFCN frequency s_2, following from MHB, is

PREM values

6.160×10^{-4} (→ $\tilde{\beta}$), 0.88621 (→ A_m/A), 0.0064616 (→ A_s/A_f)

$$e_f = \tilde{\beta} - \frac{A_m}{A}[\mathrm{Re}(s_2)+1] - \mathrm{Re}(K^{CMB}) - \frac{A_s}{A_f}\mathrm{Re}(K^{ICB})$$

-0.00231406 ±103 (→ $\mathrm{Re}(s_2)$)

this study (NCEP+ECCO)

2.32×10^{-5} (→ K^{CMB}), 1.11×10^{-3} (→ K^{ICB}), ±10%

MHB values

$$e_f = 0.0026364 \pm 23$$

Dynamical ellipticity of the whole Earth (NCEP):

$$H_d = N_0 H_{dR} = 1.00001506 \times 0.0032737548 =$$
$$= 0.0032738041 \pm 110$$

Conclusions:

- All VLBI solutions yield similar values of estimated parameters N_o, s_2;
- Forced nutations due to excitation by the atmosphere & oceans are significant for annual and semi-annual terms:
  - they are similar for different oceanic models used;
  - prograde annual term is in accordance with the MHB empirical Sun-synchronous correction;
- The application of geophysical excitation yields slightly longer period of RFCN, and larger value of common multiplier than MHB Sun-synchronous correction;
- Determination of the dynamical flattening of the core and of the whole Earth, based on these values, is possible.

V C D Z
C E D R