On astronomical constants

Michael Soffel & Sergei Klioner

TU Dresden

1. Astronomical constants fall into different categories

2. Constants and underlying framework & model

3. The problem of consistency and accuracy

Astronomical constants fall into different categories:

- natural constants – primary constants

- body constants

- initial values

Natural constants – primary constants

The dynamics of physical systems is decribed by means of a few fundamental interactions

- Gravitation
- Electromagnetism

- Weak force
- Strong force

These interactions are described by means of certain

- fields (G: metric field g; EM: potential A)

- and corresponding Fundamental laws of nature

G : Einstein's field equations (General Relativity)

EM: Maxwell equations & Special Relativity

These laws of nature contain certain physical constants describing the interaction strength, propagation velocities etc.

G: G (Newtonian gravitational constant)
EM: c (vacuum speed of light)

QM: h (Planck‘s constant)

The values of three natural constants can be chosen by law, fixing the basic physical units.

Example: G = c = h = 1

$\rightarrow$ [time] = 5.4 x 10^(-44) s
$\rightarrow$ [length] = 1.6 x 10^(-35) m
$\rightarrow$ [mass] = 2.2 x 10^(-6) kg

Not a convenient system; one tries to ensure historical continuity

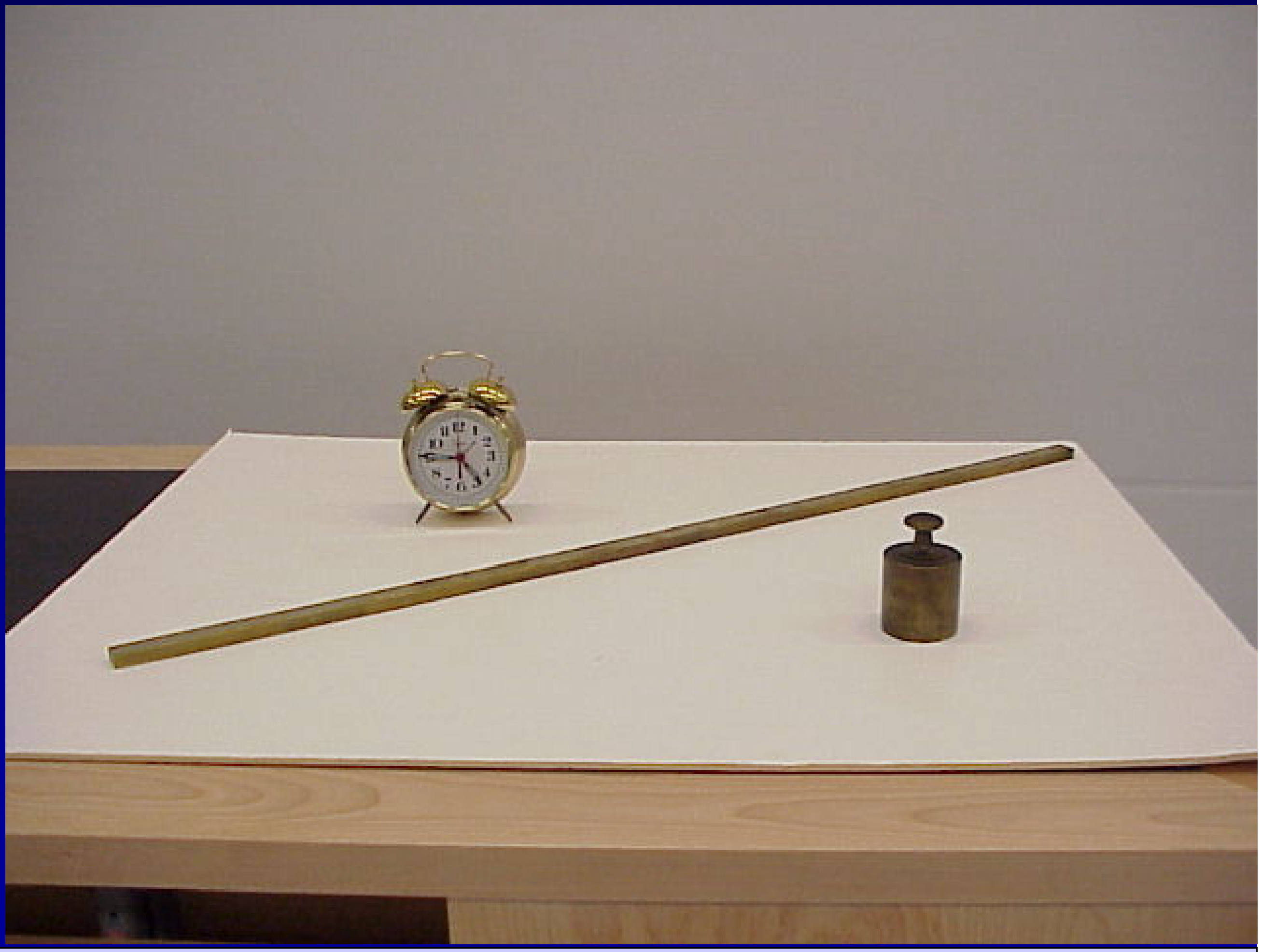
Historically, basic units have been chosen by physical prototypes

or

properties of astronomical bodies

As fundamental units we need

- s (second)
- m (meter)
- kg (kilogram)



Historical definitions on the meter:

1793: length of 10^(-7) of the Earth‘s quadrant passing through Paris

1889: given by the international prototype of platinum-iridium kept at BIPM

Still actual definition of the kilogram:

platinum-iridium prototype

disadvantages of prototypes:

- precise copies are needed
- they made change because of interaction with environment

Copies of the kilo prototype became heavier in course of time; mass differences of 50 μg have been determined.
Likely the prototype lost mass because of cleaning procedures

Historical definitions of the second:

- 1/86 400 of a mean solar day
- 1/31 556 925,9747 of the tropical year 1900 (1956)

For that reason one tries to define the basic units through natural constants

This has been done for s and m;

soon it will be realized for the kg, e.g. by counting the number of atoms of a silicon-sphere (Avogadro method)

The (SI) second:

duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the gound state of the ceasium-133 atom.

The (SI) meter:

the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

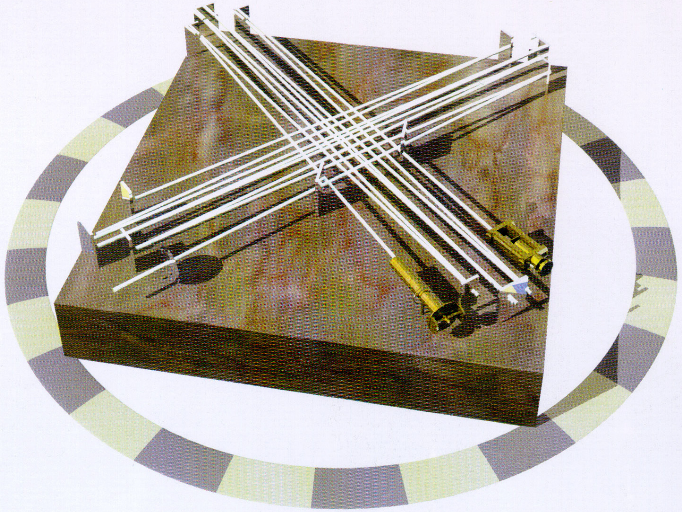
Clearly multiples of s or m or kg can be introduced as primary constants without any problems

PROBLEM:

Such definitions are based upon the validity of the fundamental laws of nature (e.g., Special Relativity)

If some violation will be found, the definitions might become useless

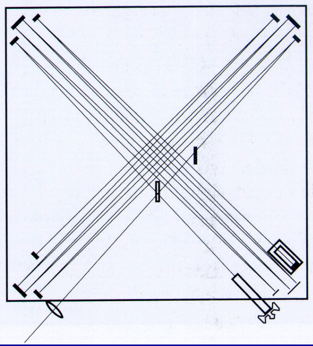
Example: The meter and the isotropy of space



MESSUNG VON UNTERSCHIEDEN IN DER LICHTGESCHWINDIGKEIT

Im Michelson-Morley-Interferometer wird das Licht einer Quelle durch eine halbverspiegelte Glasscheibe in zwei Strahlen aufgeteilt. Das Licht der beiden Strahlen bewegt sich rechtwinklig zueinander und wird am Ende wieder zu einem einzigen Strahl vereinigt, indem es abermals zu der halbverspiegelten Scheibe gelenkt wird. Je nach Strahllänge und nach der Lichtgeschwindigkeit in den beiden Strahlen überlagern sich diese in unterschiedlicher Weise: Trifft Wellenberg auf Wellenberg, verstärken sich die Wellen gegenseitig, trifft Wellenberg auf Wellental, löschen sich die Teilstrahlen aus. Veränderungen, etwa der Übergang von Auslöschung zu Verstärkung, lassen sich beobachten und zeigen an, wenn die relative Lichtgeschwindigkeit in den Teilstrahlen variiert.

Rechts: Diagramm des Experiments nach der Abbildung, die 1887 im Scientific American erschien.



The famous experiment
by Michelson and Morley

1887

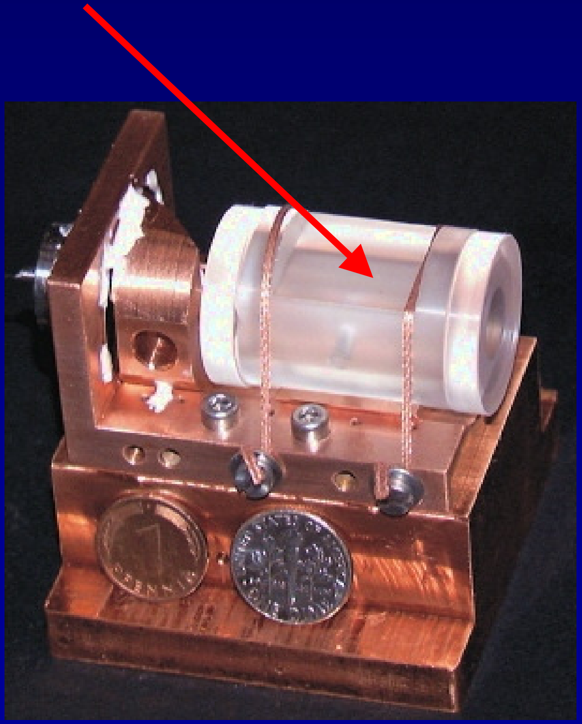
The interference patters
does not depend upon
the orientation of the platform
In space

A modern version of the MM-experiment:

Müller et al., Phys.Rev.Lett. 91, 020401 (2003)

Comparison of the resonance frequency of two orthogonal cryogenic optical resonators (COREs) subject to Earth‘s rotation over one year

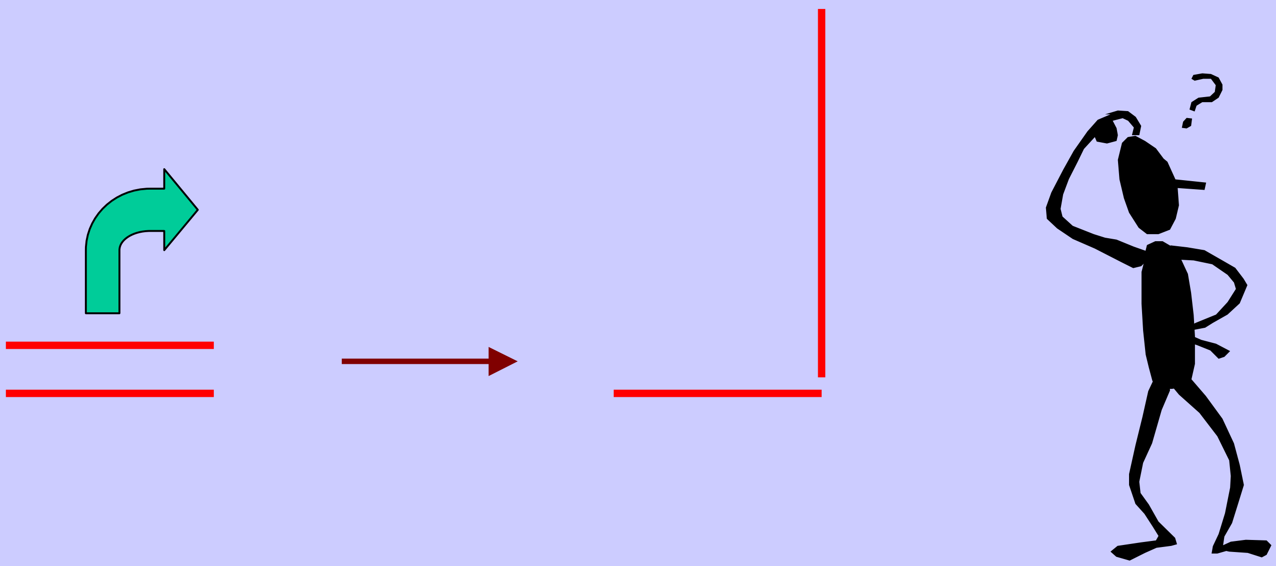
$\Delta c/c$ = (2.6 +/- 1.7) x 10^(-15)



silicon crystal

If principle failed:

the length of a meter stick would depend upon its orientation in space!



- Maxwell‘s theory (geo, optics)
- GRT

fields = A,g; const = c,G

Basic physical laws of nature

GRT is too complex to treat solar-system problems

not even the mass of a body can be defined:

The theory is non-linear and because of E = m c^2
the gravitational fields contribute to the mass
one cannot separate the G-field of body A from that
of body B

One resorts to approximations such as the
first post-Newtonian approximation

- Maxwell‘s theory (geo, optics)
- GRT

fields = A,g; const = c,G

Basic physical laws of nature

- Maxwell's theory (geo, optics)
- GRT

fields = A,g; const = c,G

Basic physical laws of nature

- 1st PN-approximation
- certain class of coordinates (e.g., harmonic)

const = β, γ, $\alpha 1$ ....

Basic (PPN) framework

Body constants

Body constants:

- Mass, potential coefficients
- intrinsic angular momentum (spin)

- principal moments of inertia
- oblateness, dynamical ellipticity

Well defined in Newton‘s theory of gravity

Conceptual problems in GRT:

- E = m c^2
- non-linearity of gravity (G-fields contribute!)

Body constants can be defined in the basic (PPN) framework

Example: mass of a body ($\beta = \gamma = 1$ ..)

Blanchet-Damour mass e.g. of the Earth in the GCRS

$$M_E(T) = \int_E d^3X\Sigma + \frac{1}{6c^2}\frac{d^2}{dT^2}\left(\int_E d^3X\mathbf{X}^2\Sigma\right) - \frac{4}{3c^2}\frac{d}{dT}\left(\int_E d^3X X^a\Sigma^a\right)$$

formally; however, the metric potential in the GCRS is given by

$$W_E = \frac{GM_E}{R} + \dots$$

Time dependence of body constants:

- conceptually: even the masses are time dependent
- they are physically time-dependent for various reasons [energy loss (Sun); dust accretion etc.]

The expected or measured time dependence should be indicated explicitly

For our sun with L = 4 10^(33) erg/s + solar wind

Mdot/M ≈ 10^(-13) per year

Initial values

Bodies with their body constants + initial conditions appear in a dynamical model e.g., for the motion of the gravitational N-body problem (ephemeris equation)

This model will involve additional constants describing certain features if interaction (e.g., lag angle to describe the tidal friction in the E-M system)

Basic DE, INPOP or EPM Equations of motion

- GRT
- Maxwell's theory (geo, optics)

fields = A,g; const = c,G

Basic physical laws of nature

- 1st PN-approximation
- certain class of coordinates (e.g., harmonic)

const = β, γ, $\alpha 1$ ....

Basic (PPN) framework

Bodies with complex interactions

Const = body- + auxiliary const.; initial constants

Basic (dynamical) model

Constants and underlying framework and model

- Some parameters depend upon the underlying framework (PPN- parameters)

- the body constants should have model independent values as functions of time

However: the formal errors depend upon the model;
for an estimation of realistic errors
different models have to be considered

- initial constants are obviously model dependent

Conclusion:

Even for body constants: determined numerical values should always be associated with the formalism and model used for ist definition and determination

For initial constants the underlying model must be stated in greatest detail; for a different model they might loose their meaning

The problem of consistency and accuracy

To ensure consistency realistic errors have to be indicated

(correlations studied; different branches of science consulted etc.)

To get a realistic error might not be easy

Example: Gdot/G

Determination in the frame of the russion ephemeris EPM (E.Pitjeva) 2005 (Astronomy Letters 31, 340-349)

- 10**(-14)/y formal error
- 5 x 10**(-14)/y realistic error (?)

Later, after intensive discussions with Myles Standish and detailed comparisons between DE and EPM it was found:

The determination depended very heavily upon certain sets of observations and adjusted parameters

→

A realistic error for Gdot/G is about 6 10**(-13)/y

A value that agrees excellently with the one from LLR data alone.

Conclusions

- Astronomical constants fall into different categories
- The associated theoretical structure involves:
  - Basic laws of nature
  - Basic framework and
  - Basic model
- For the body constants and for initial constants the associated framework and model has to be indicated clearly
- One has the burden to estimate realistic errors

The END